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Technology Licensing by Advertising Supported Media Platforms: An Application to Internet Search Engines*

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Abstract

We develop a duopoly model with advertising supported platforms and analyze incentives of a superior firm to license its advanced technologies to an inferior rival. We highlight the role of two technologies characteristic for media platforms: The technology to produce content and to place advertisements. Licensing incentives are driven solely by indirect network effects arising from the aversion of users to advertising. We establish a relationship between licensing incentives and the nature of technology, the decision variable on the advertiser side, and the structure of platforms' revenues. Only the technology to place advertisements is licensed. If users are charged for access, licensing incentives vanish. Licensing increases the advertising intensity, benefits advertisers and harms users. Our model provides a rationale for technology-based cooperations between competing platforms, such as the planned *Yahoo-Google* advertising agreement in 2008.

JEL Classification: L13, L24, L86, M37

Keywords: Technology Licensing, Two-Sided Market, Advertising

1 Introduction

Many media firms function as two-sided platforms. They attract audience with content and sell advertising space to businesses. Digital technologies have created several new ways for such platforms both to compete and cooperate. While competition between media platforms is subject to much research, the technological peculiarities of these businesses and technology-based cooperation receive little attention. In this article we highlight two technologies that are of crucial importance for advertising supported platforms: The technology to produce content, and the technology to place advertisements. In addition to investing in the improvement of their own technologies, media firms often engage in cooperation agreements that involve sharing of their know-how with rivals.[1] The competitive effects of such agreements are the focus of this paper.

We consider two media platforms serving advertisers and users, with one possessing superior content producing and advertisement placing technologies and aim to answer two questions. First, what drives a platform endowed with superior technologies to improve its rival by licensing a technology? Second, what welfare effects does such cooperation have? Our results show that a purely advertising financed platform with superior capabilities licenses only its technology to place advertisements, but not the technology to produce content. Licensing incentives are driven by indirect network effects: By improving the competitor on the advertiser side of the market, the superior platform increases its demand on the user side. Our results are robust to whether platforms decide on advertisement quantities or prices. However, if platforms charge users and choose advertisement quantities, then incentives for technology licensing vanish: Licensing of the advertisement placing technology decreases the superior platform's user demand. We consider

[1] This paper is strongly motivated by the cooperation agreements between internet search engine operators. In 2008 search engine operators *Google* and *Yahoo* announced plans for an agreement to cooperate in advertising. The envisaged cooperation would have let *Yahoo* use *Google*'s technology to match advertisements with search keywords in Canada and the U.S. After competition authorities expressed their doubts, the parties officially abandoned the agreement. In 2009 search engine operators *Yahoo and Microsoft* entered an agreement with the latter providing the underlying search technology on some of *Yahoo*'s Web sites. See, for example, BBC News, 13 June, 2008, "Yahoo-Google agree online ad deal," http://news.bbc.co.uk/2/hi/business/7451946.stm, retrieved on 25 June, 2010 and The New York Times, 30 July, 2009, "Yahoo-Microsoft Deal," http://topics.nytimes.com/top/news/business/companies/yahoo_inc/yahoo-microsoft-deal/index.html, retrieved on 9 March, 2011.

welfare implications of technology licensing and find that it is likely to be beneficial for advertisers and detrimental to users. Furthermore, we show that private licensing incentives can be socially suboptimal.

This article makes three main contributions to the research. First, we establish a relationship between the incentives of an advertising supported two-sided platform to license its technologies to a competitor with *i)* the nature of a technology; *ii)* the type of a game played on the advertiser side of the market (quantity or price setting); and *iii)* the structure of platforms' revenues. Second, we provide a rationale for technology licensing that is based purely on indirect network effects. Third, we make predictions about the welfare effects of technology licensing involving advertising supported two-sided platforms, and aim to provide guidance to competition authorities for the evaluation of cases such as the 2008 *Yahoo-Google* and the 2009 *Yahoo-Microsoft* cooperation agreements.

Our article is closely related to the literature on technology licensing in oligopoly. Earlier articles explained a monopolist's incentives to licence a proprietary technology by licensing serving as a commitment device for low future prices (Farrell and Gallini, 1988) or high quality (Shepard, 1987) in a dynamic setting. We add to this strand by explaining how the two-sided nature of the market may drive licensing incentives. As in the models of Farell and Gallini (1988) and Shepard (1987), licensing serves to boost demand for the product of the licensor, which in our case corresponds to advertising space. In our baseline model, by making a more advanced advertisement placing technology available to the inferior rival, the superior platform directly increases the demand for the inferior platform's advertising space. The inferior platform responds by placing more advertisements, which induces advertising-averse users to switch to the superior platform. A larger user base, in turn, increases demand for advertising space on the superior platform as advertisers value a larger audience.

This article also fits into the literature on two-sided markets (Armstrong, 2006; Rochet and Tirole, 2003), particularly the strand focusing on advertising platforms (Anderson and Coate, 2005; Crampes et al., 2009). We contribute to this literature by analyzing asymmetric and vertically differentiated platforms and their incentives to license different technologies. An article particularly close to ours is Crampes et al. (2009). The authors present a model of competition between media platforms financed by both subscriptions and advertising receipts, highlighting the relationship between equilibrium prices, advertising levels and advertising technology. They show that advertising levels may be either too high or too low, depending on the returns to scale in audience size. Our paper also highlights the role of technologies in a context of two-sided platforms, however, our main aim is to explain incentives of media platforms to cooperate in technology licensing and analyze its welfare effects.

The paper proceeds as follows. The next section presents the baseline model and characterizes the equilibrium without technology licensing, assuming quantity setting on the advertiser side and no access fee for users. In Section 3 we apply our framework to the analysis of technology licensing incentives and provide a welfare analysis. In Section 4 we extend our baseline model in two directions in order to derive the results relevant to a broader range of advertising supported media platforms. In particular, we analyze licensing incentives under price setting on the advertiser side and also address the case where users are charged for access. In Section 5 we discuss some of our modelling assumptions. Section 6 concludes.

2 The Baseline Model And Equilibrium Analysis

We analyze a two-sided market in which two horizontally and vertically differentiated platforms $i = \{1, 2\}$ provide content to *users* and sell advertising space to *advertisers*. Our main modelling novelty is that we explicitly distinguish between content producing (CP) and advertisement placing (AP) technologies. The CP technology of a platform is responsible for the intrinsic utility a user draws from consuming content on a platform. The AP technology in turn determines the probability that an advertisement shown on a platform motivates a consumer to buy the advertised product.[2] We assume that both the AP and CP technologies of platform 1 are superior to those of platform 2. In other words, platform 1 can produce both higher quality

[2] For example, the content producing technology corresponds to the quality of television channels' programmes and the relevance of organic search results in the case of internet search engines. The advertisement placing technologies are often proprietary in media markets. For example, U.S. patent No. 7398207 held by a television industry player relates to a technology that adjusts the volume level of an advertisement to that of the program in which the advertisement is embedded. This is to prevent a sudden volume change during advertising breaks. Other technologies held by media firms prevent viewers from disabling advertisements when recording television programmes. Similarly, internet search engines use sophisticated algorithms to match the most relevant advertisements to search keywords thus determining the probability that a click on a sponsored link will result in a successful sale of the advertised good.

content as well as place more relevant advertisements.[3] We will refer to platform 1 as *superior* and to platform 2 as *inferior*.

In our baseline model content consumption at the platforms is free of charge for users, while advertisers pay a price p_i^a for an advertisement slot at platform i.[4] Each platform decides on the number of advertisement slots, a_i, to place, with every advertisement requiring one slot.[5] The platforms provide their services at zero marginal cost and realize profits

$$\pi_i = p_i^a a_i. \tag{1}$$

We assume that users single-home, i.e., every user visits only one platform. Following Peitz and Valletti (2008), we assume that every potential advertiser can place advertisements at just one or both of the platforms, or refrain from advertising. If advertiser k places an advertisement at platform i, its expected profit $E\left(\pi_i^k\right)$ is

$$E\left(\pi_i^k\right) = Pr_i\left\{Sale\right\} n_i p^k - p_i^a - c^k,$$

where n_i, $p^k$and c^k denote the user market share of platform i, the price (net of marginal cost) of advertiser k's product and its costs associated with placing an advertisement, respectively. The price of the advertised product is normalized to unity for all advertisers ($p^k = 1$). The advertising costs c^k capture the advertiser k's fixed costs associated with placing an advertisement other than the price paid for advertising space, such as the costs for designing an advertisement. Advertisers are heterogeneous with respect to costs, which are uniformly distributed on the interval $c^k \in [0, \infty)$.[6] We assume that every user of platform i becomes aware of advertiser k's product after having seen an ad and may buy exactly one unit of the advertised good. $\Pr_i\left\{Sale\right\}$ captures the level of AP technology of platform i: It denotes the probability that a user buys the product after having seen its advertisement on platform i.

[3] The assumption that one platform is superior in both technologies is not crucial for our results. In Section 5 we discuss the case where each platform is superior in one of the technologies.

[4] In Section 4.2 we relax the assumption that platforms do not charge users and analyze platforms' licensing incentives given two sources of revenues: Payments from advertisers and users.

[5] In Section 4.1 we analyze licensing incentives assuming that platforms set slot prices.

[6] The analysis of the case where advertisers sell their products at different prices and have the same advertising costs is available from the authors on request. It is shown that licensing incentives in that case are the same as in the model formulated here.

We assume that $\Pr_i \{Sale\} = 1-\rho_i$, where $\rho_i \in [0,1)$ corresponds to platform i's handicap in ability to place high-quality advertisements (i.e., ones that result in a sure sale of the advertised good). A platform with a lower ρ_i has a better AP technology. For example, $\rho_i = 1/3$ implies that 2/3 of those who have seen an advertisement end up buying the product.

There is a marginal advertiser on platform i with advertising costs $\bar{c}_i$, who is indifferent between placing an ad and not advertising. The expected profit of the marginal advertiser is $(1-\rho_i)n_i - p_i^a - \bar{c}_i = 0$. As every advertiser places one ad, the number of ads on a platform equals the advertising costs of the marginal advertiser, with $a_i = \bar{c}_i$. The inverse demand for advertisement slots at platform i is then given by

$$p_i^a = (1-\rho_i)n_i - a_i. \tag{2}$$

With a superior AP technology, platform 1 can display more relevant advertisements, which increase the probability of a successful sale by advertisers. This translates into a higher willingness to pay for an advertisement slot. We assume for the superior platform that $\rho_1 = 0$, while $\rho_2 \in [0,1)$ reflects the inferior platform's handicap in AP technology.

We now turn to users and the role of CP technology. Users derive a basic utility $u > 0$ and platform-specific utility $\zeta_i q > 0$ from consuming content on platform i, which increases in platform's ability to produce high-quality content. The value $\zeta_i q$ is higher, the better the CP technology of platform i becomes. We assume that content quality is (weakly) higher at platform 1 and $\zeta_1 = 1$ while $\zeta_2 \in (0,1]$. With $\zeta_2 < \zeta_1$ the platforms are vertically differentiated. For notational simplicity, in the following we will often write ζ and ρ instead of ζ_2 and ρ_2, respectively. Let $\Delta \geq 0$ denote the advantage of platform 1 in content quality, with $\Delta := (1-\zeta)q$. If $\Delta = 0$, platforms have CP technologies of same quality, while $\Delta > 0$ means that platform 1 has a superior CP technology.

The platforms are placed on a unit circle and are assumed to be maximally differentiated from each other, such that the address of platform 1 is normalized to $s_1 = 0$ while the address of the other platform is $s_2 = 1/2$. Users are uniformly distributed along the circle with each having an address $t \in [0,1]$ reflecting the preference for the optimal platform. Visiting platform i involves quadratic transportation costs for users, which are positive if the visited platform is not located in the user's ideal position.

We assume that users dislike advertisements. This assumption is often made in the literature on advertising supported two-sided platforms and seems to apply well to most of the markets we have in mind. The user disutility from advertisements depends on the number of ads and is given by a linear function, μa_i, with $\mu > 0$ denoting the strength of disutility per advertisement.[7]

The utility of a user with address t visiting platform i, U_i^t, then takes the form

$$U_i^t = \begin{cases} u + q - [\delta_1(t)]^2 - \mu a_1, & \text{if} \quad i = 1 \\ u + \zeta q - [\delta_2(t)]^2 - \mu a_2, & \text{if} \quad i = 2, \end{cases} \tag{3}$$

$$\text{with } \delta_1(t) = \min\{t, 1-t\} \text{ and } \delta_2(t) = |t - 1/2| \,.$$

The term $\delta_i(t)$ captures the distance between user t and platform i and his transportation costs are $[\delta_i(t)]^2$. We assume that u is high enough, so that in equilibrium every user visits one of the platforms.

The timing of the game is as follows: First, the platforms determine the number of advertisement slots to display. Second, users choose their preferred platform and advertisers buy advertisement slots. We seek for the subgame-perfect Nash equilibrium and solve the game backwards.

Equilibrium Analysis

Every user chooses the platform providing higher utility. We can find two marginal users with addresses t_1 and t_2 which are indifferent between the platforms:

$$\begin{aligned} t_1(a_1, a_2; \zeta, \mu, q) &= \mu(a_2 - a_1) + 1/4 + \Delta, \\ t_2(a_1, a_2; \zeta, \mu, q) &= \mu(a_1 - a_2) + 3/4 - \Delta, \end{aligned} \tag{4}$$

with $t_1 < t_2$. The market shares of the platforms are then $n_1 = 1 - t_2 + t_1$ and $n_2 = t_2 - t_1$. This yields the following user demand at the platforms:

$$\begin{aligned} n_1(a_1, a_2; \zeta, \mu, q) &= 1/2 + 2\left[\Delta - \mu(a_1 - a_2)\right], \\ n_2(a_1, a_2; \zeta, \mu, q) &= 1/2 - 2\left[\Delta - \mu(a_1 - a_2)\right], \end{aligned} \tag{5}$$

with $\partial n_i(a_i, a_j; \cdot)/\partial a_i < 0$ and $\partial n_i(a_i, a_j; \cdot)/\partial a_j > 0$ for $i, j = \{1, 2\}$ and $i \neq j$. By plugging (5) into (2) we get platform i's profit as

$$\pi_i(a_i, a_j; \rho_i, \zeta, \mu, q) = \left[(1 - \rho_i) n_i(a_i, a_j; \cdot) - a_i\right] a_i. \tag{6}$$

[7]The linear specification of disutility from advertising is common in the literature (see Gal-Or and Dukes, 2003; Anderson and Coate, 2005; Peitz and Valletti, 2008).

Platform i maximizes its profit by choosing the number of advertisement slots, a_i. The following lemma states the condition under which both platforms are active on both sides of the market.

Lemma 1. *The necessary and sufficient condition for the platforms to be active on both sides of the market is* $\Delta < \overline{\Delta}$, *with* $\overline{\Delta} := (1+3\mu)/[4(1+\mu)]$. *If this condition holds, the platforms display advertisements, serve some users and realize positive profits.*

Proof. See Appendix.

To guarantee that in equilibrium both platforms are active on both sides of the market, the superior platform's advantage in CP technology should not be too large (i.e., $\Delta < \overline{\Delta}$). The value $\overline{\Delta}$ corresponds to the minimum magnitude of content quality advantage of the superior platform which drives the inferior platform out of the market. If $\Delta = \overline{\Delta}$, then all users choose platform 1 that places advertisements, while the other platform does not advertise. It follows from the platforms' FOCs that a platform placing a positive number of advertisement slots also serves some users:

$$\begin{aligned} n_1^*(\rho,\zeta,\mu,q) &= 2(1+\mu)a_1^*(\rho,\zeta,\mu,q), \\ (1-\rho)n_2^*(\rho,\zeta,\mu,q) &= 2\left[1+\mu(1-\rho)\right]a_2^*(\rho,\zeta,\mu,q). \end{aligned} \tag{7}$$

If a platform places advertisements, it also charges a positive price for them:

$$p_1^{a*}(\rho,\zeta,\mu,q) = (1+2\mu)\,a_1^*(\rho,\zeta,\mu,q), \tag{8}$$

$$p_2^{a*}(\rho,\zeta,\mu,q) = \left[1+2\mu(1-\rho)\right]a_2^*(\rho,\zeta,\mu,q), \tag{9}$$

leading to positive profits. For further analysis in this section we assume $\Delta < \overline{\Delta}$. The following proposition characterizes the equilibrium without technology licensing.

Proposition 1. *The equilibrium without technology licensing has the following properties.*

i) If it has a strict advantage in at least one technology, the superior platform displays more advertisements, charges a higher price for its advertisement slots and realizes larger profits than the inferior platform.

ii) The superior platform has a larger (weakly smaller) market share among users than the inferior platform if $\Delta > \underline{\Delta}$ ($\Delta \leq \underline{\Delta}$), *with* $\underline{\Delta} = \mu\rho/[4(1+\mu)(1+\mu(1-\rho))]$.

Proof. See Appendix.

The superior platform places more advertisements in equilibrium and charges a higher price for them than its competitor if it has a strict advantage in at least one of the technologies. For the intuition behind this result it is helpful to consider the roles of both technologies on advertising decisions. The advantage in CP technology allows the superior platform to place more advertisements because its better content compensates users for the additional nuisance. With a more advanced AP technology, each user is more valuable to advertisers on the superior platform. For the same user market shares advertiser demand is higher at the superior platform, which makes it profitable to place more advertisements. These two insights imply that with a strict advantage in at least one technology, the superior platform displays more advertisements in equilibrium. It follows directly from the equilibrium slot prices in Expressions (8) and (9) that the price of an advertisement slot at the superior platform is higher: $p_1^{a*}(\cdot) > p_2^{a*}(\cdot)$ if $a_1^*(\cdot) > a_2^*(\cdot)$.

The superior platform has a larger market share among users if its advantage in the CP technology is large enough ($\Delta > \underline{\Delta}$). As we showed, the superior platform places more advertisements, it can therefore only have a larger market share among users if it is able to compensate users for the disutility caused by additional advertisements. The only way it can do so is by providing higher quality content. If the content quality advantage is larger than the critical value $\underline{\Delta}$, the superior platform can hold a dominant position among users even though it displays more ads. With a content quality advantage below $\underline{\Delta}$, the superior platform displays more ads than the rival, but it attracts less than half of users, despite having a better CP technology.

We note that the upper bound of the superior platform's quality advantage ($\overline{\Delta}$) depends on the user disutility per advertisement (μ) and does not depend on the inferior platform's handicap in AP technology (ρ). The upper bound is the quality advantage that makes all users prefer the superior platform when it has advertising while the inferior platform does not place any advertisements. However, $\underline{\Delta}$ depends on ρ, and is larger if the inferior platform's ability to place high-quality advertisements is lower. With a lower quality of AP technology, the inferior platform places fewer advertisements in equilibrium. Thus, the superior platform needs a larger advantage in CP technology to attract the majority of users.

Before turning to the analysis of technology licensing incentives, we briefly discuss how our modelling setup applies to the market of internet search engines. Although we omit some unique characteristics of the internet search market, our model takes into account the most important factors determining the choice of search engines by users. According to a survey conducted among internet searchers in 2008, the three most important factors driving user choice are *general search quality*, *home page appeal* and *special features*.[8] These factors are captured in our model by the vertical and horizontal differentiation between the platforms. We do not explicitly model the auction by which search engines allocate advertising space. Instead, we focus on two polar cases: In the baseline model we assume that platforms set advertisement quantities, while in Section 4.1 we investigate the case where platforms decide on the prices of an advertisement slot. We show that licensing incentives are similar in these cases. At this point it is worth noting that our model's prediction on the superior platform converting its technology advantage into higher profits by placing more advertisements is well in line with the observations in the internet search engines market. In the period from December 2008 till March 2010, *Google* placed on average more advertisements per search query than its closets competitor *Yahoo*.[9]

3 Technology Licensing

We are interested in the incentives of a platform holding superior CP and AP technologies to license one or both technologies to the competitor. Technology licensing is a transaction that requires mutual consent of both platforms and we distinguish between the cases when transfers between platforms are allowed and when they are not. If transfers are not allowed, the superior platform licenses its technology (technologies) only if by doing so its individual profit increases. In this case the superior platform chooses the extent to which the competitor can access its proprietary technology by maximizing the superior platform's own profit. The inferior platform accepts any offered technology as it is costless and the improvement leads unambiguously to a higher individual profit.

In case transfers are possible, the superior platform makes a take-it-or-leave-it offer to the competitor involving a payment in exchange for the shared technology. Such an offer allows the licensor to appropriate the entire additional industry profits arising from the improvement of the

[8] See "Search Portals Study," Keynote Systems, 2008, http://www.keynote.com/docs/kcr/KCR_Search_2008.pdf, retrieved on 9 April, 2011.

[9] According to the Search Engine Advertiser Analysis provided by the AdGooroo (available at www.adgooroo.com, retrieved on 9 April, 2011), in the mentioned period *Google* displayed on average 5.24 ads per search keyword in the U.S. and abroad. In the same period *Yahoo* placed on average 4.05 ads per keyword.

inferior platform. It follows that when transfers are allowed, the superior platform determines the optimal level of licensing by maximizing joint profits of both platforms. We now formalize how licensing changes the inferior platform's technologies.

If CP technology is licensed, content quality at the inferior platform increases. We model this by assuming that parameter ζ increases. If the superior platform licenses its AP technology, the inferior platform becomes able to better place advertisements. Formally, ρ decreases and demand for advertisement slots at platform 2 grows. We introduce parameters $\rho_0 \in [0, 1)$ and $\zeta_0 \in (0, 1]$, denoting the initial handicap of the inferior platform in the quality of its AP technology and the initial quality of the inferior platform's CP technology, respectively. We assume that $(1 - \zeta_0)q < \overline{\Delta}$. We start with investigating the effects of AP technology licensing. The following lemma states the effects of a change in ρ on the equilibrium values.

Lemma 2. *As the demand for advertising space at the inferior platform gets larger (i.e., ρ decreases), the following holds:*

i) both platforms provide more advertisement slots,

ii) the superior (inferior) platform gains (loses) market shares among users,

iii) both platforms charge a higher price for advertisement slots,

iv) both platforms make larger profits, therefore, joint profits increase.

Proof. See Appendix.

Although demand for advertising space on a platform is independent of demand on the other platform, both platforms benefit from the increased demand for advertisement slots on the inferior platform due to indirect network effects. The increased advertiser demand at the inferior platform allows it to place more advertisements. In response, some users switch to the superior platform. The increased user demand, in turn, boosts demand for advertising space at the superior platform. Both platforms increase the number of advertisements with the rise in demand for advertisement slots at the inferior platform. The inferior platform displays more advertisements as it is directly affected by the change in advertiser demand. The superior platform is affected indirectly through users being driven away from the competitor due to intensified advertising and increases the number of its advertisement slots too.

We now consider how changes in platforms' advertising levels affect other equilibrium variables. It is instructive to inspect the reaction functions, $a_1(a_2; \zeta, \mu, q)$ and $a_2(a_1; \rho, \zeta, \mu, q)$, which give the optimal number of advertisements placed by each platform in response to the number of advertisements placed by the competitor:

$$a_1(a_2;\cdot) = \frac{1+4(\Delta+\mu a_2)}{4(1+2\mu)},$$
$$a_2(a_1;\cdot) = \max\left\{0, \frac{(1-\rho)\left[1+4(\mu a_1-\Delta)\right]}{4\left[1+2\mu(1-\rho)\right]}\right\}.$$

Note that decisions about the number of advertisements to place are strategic complements as $\partial a_1(a_2;\cdot)/\partial a_2 = \mu/(1+2\mu) > 0$ and $\partial a_2(a_1;\cdot)/\partial a_1 = \mu(1-\rho)/\left[1+2\mu(1-\rho)\right] > 0$ if $a_1 > \Delta/\mu - 1/(4\mu)$.

With a decrease in parameter ρ, the superior platform's reaction function remains unchanged while that of the inferior platform shifts outwards for $a_2 > 0$. Figure 1 illustrates the change in the equilibrium for two situations.[10] In the first situation the reaction function of the inferior platform, $a_2(a_1;\cdot)$, is affected by a decrease in parameter ρ in two ways: Its slope increases and it shifts upwards, resulting in $a_2^{\rho}(a_1;\cdot)$. The equilibrium point moves from F to G. In the second case, the reaction function of the inferior platform, $\widetilde{a}_2(a_1;\cdot)$, rotates around point C, with $\widetilde{a}_2^{\rho}(a_1;\cdot)$ denoting the new function.[11] The equilibrium point shifts from D to E. A decrease in ρ leads to a higher number of advertisements on both platforms.

It is the sum of two effects that determines how the equilibrium number of advertisements changes with AP technology licensing. The *direct effect* corresponds to the change in demand for advertisement slots on the inferior platform and affects only the advertising decision of the latter. The *strategic effect* relates to the fact that decisions on the number of advertisement slots are strategic complements. If one platform displays more advertisements, the other can do so as well. The two effects can be disentangled using the reaction functions:

$$\frac{\partial a_i^*(\cdot)}{\partial \rho} = \underbrace{\left.\frac{\partial a_i(a_j;\cdot)}{\partial \rho}\right|_{a_j^*(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j;\cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \rho}}_{\text{strategic effect}}.$$

[10] The two situations differ in the following way. In the situation where the equilibrium point moves from F to G, the superior platform's advantage in CP technology is not very large: $a_2(0;\cdot) > 0$ implying that $\Delta < 1/4$. In this case the inferior platform advertises even if the superior platform does not. In the case where the equilibrium point moves from D to E, the superior platform's quality advantage is larger: $a_2(0;\cdot) \leq 0$ implying that $\Delta \geq 1/4$. In this case the inferior platform does not advertise if the superior platform places sufficiently few advertisements, namely, if $a_1 \leq (4\Delta-1)/(4\mu)$.

[11] The maximum number of advertisements the superior platform can place to drive the inferior platform out of the user market does not depend on ρ.

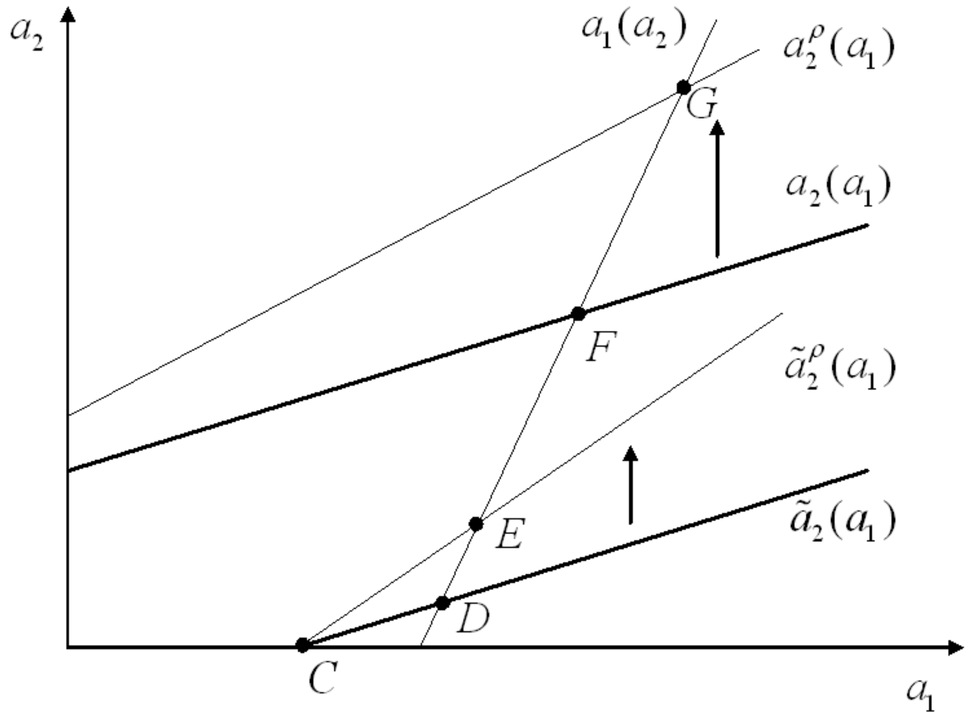

Figure 1: The effect of a decrease in ρ on the reaction functions of the platforms

The inferior platform is affected directly by the AP technology licensing agreement. As its advertising space becomes more valuable, it displays more advertisements. The advertiser demand at the superior platform remains unaffected by this change. The superior platform is affected only indirectly by the change in parameter ρ, through strategic effect. The strategic effect is at work at the inferior platform too and amplifies the positive direct effect. As a result, in the new equilibrium both platforms display more advertisements. Table 1 summarizes these effects.

	Direct effect	Strategic effect	Total effect
$a_1^*(\cdot)$	0	+	+
$a_2^*(\cdot)$	+	+	+

Table 1: The effects of a decrease in ρ on the advertising decisions of the platforms

It follows from Equations (8) and (9) that when platforms place more advertisements as parameter ρ decreases, they also set higher slot prices. Due to indirect network effects both platforms then benefit from the intensified advertiser demand at the inferior platform. It is left to note that the superior platform gains user market shares with licensing. As the inferior platform is affected both directly and through the strategic effect, it increases the number of advertisement slots more than its competitor does, losing thereby some of its users.

We now turn to the effects of licensing the CP technology. The following lemma states our results.

Lemma 3. *As the content quality of the inferior platform improves (i.e., parameter ζ increases):*

i) the superior (inferior) platform displays less (more) advertisements,

ii) the superior (inferior) platform loses (gains) user market shares,

iii) the superior (inferior) platform charges a lower (higher) price for an advertisement slot,

iv) the superior (inferior) platform makes lower (higher) profits,

v) platforms' joint profits decrease.

Proof. See Appendix.

Although the inferior platform makes higher profits with the improved CP technology, the additional profit is not sufficient to compensate the losses suffered by the superior platform. For the intuition behind this result it is useful to inspect how the licensing of CP technology alters the advertising decisions of the platforms. We can again distinguish between the *direct effect* and the *strategic effect* of the change in parameter ζ on the advertising levels:

$$\frac{\partial a_i^*(\cdot)}{\partial \zeta} = \underbrace{\left.\frac{\partial a_i(a_j;\cdot)}{\partial \zeta}\right|_{a_j^*(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j;\cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \zeta}}_{\text{strategic effect}}.$$

For the licensing of CP technology, the direct effect is driven by the change in the content quality advantage of the superior platform. With ζ getting larger, the direct and strategic effects point in opposite directions at both platforms. As the content quality gap between the platforms narrows, the direct effect is positive for the inferior and negative for the superior platform. The content quality at the inferior platform increases, hence, it can place more advertisements in equilibrium without losing users. At the same time, the superior platform's advantage in content quality erodes and it has to reduce the number of advertisements to keep users from switching. In contrast, the strategic effect is negative for the inferior platform. In equilibrium the superior platform decreases its advertising level, and the strategic response of the inferior platform is to show fewer advertisements too. For the superior platform it is the other way around: As the inferior platform shows more advertisements in the new equilibrium, the superior platform displays more advertisements as well. The direct effect is stronger than the strategic effect and the inferior platform increases the number of advertisement slots in the new equilibrium while the superior platform decreases it. Table 2 summarizes these effects.

	Direct effect	Strategic effect	Total effect
$a_1^*(\cdot)$	$-$	$+$	$-$
$a_2^*(\cdot)$	$+$	$-$	$+$

Table 2: The effects of an increase in ζ on the advertising decisions of the platforms

The changes in prices of advertisement slots can again be derived from inspecting Equations (8) and (9). Following a change in parameter ζ, equilibrium prices move in the same direction as advertising levels. Hence, the advertisement slot price rises at the inferior platform while it decreases at the superior platform. The negative effect of an increase in ζ on the superior platform's profit and the positive effect on the inferior platform's profit are then straightforward. Moreover, as the content quality improves at platform 2, it is able to attract users from platform 1 despite increasing the number of advertisements. As a result, the inferior platform increases its user market shares at the expense of the superior platform.

Using Lemmas 2 and 3 we are in position to state platforms' optimal licensing decisions.

Proposition 2. *Regardless whether transfers are allowed or not, the inferior platform gets full access to the superior AP technology, while CP technology is not licensed ($\rho^* = 0$ and $\zeta^* = \zeta_0$).*

Proof. We know from Lemma 2 that both platforms' profits increase when ρ gets smaller, hence, the full licensing of AP technology implying $\rho^* = 0$ is optimal regardless whether transfers are possible or not. We also know from Lemma 3 that both the superior platform's profit as well as joint profits decrease as ζ gets larger. This implies that no transfer exists which compensates the superior platform for its profit loss. Consequently, CP technology is not licensed, regardless whether transfers are allowed or not implying $\zeta^* = \zeta_0$. *Q.E.D.*

Our results show that while AP technology is fully licensed to the competitor, the quality of its CP technology remains unchanged. This result does not depend on the presence of transfers. Technology licensing has an interesting implication for competition policy analysis. The following corollary states our result.

Corollary 1. *Technology licensing agreement intensifies concentration at the user market: The user market shares of the superior platform increase.*

Proof. We know from Proposition 2 that both with and without transfers only the AP technology is licensed, in which case $n_1^*(\cdot)$ increases as stated in Lemma 2. *Q.E.D.*

With the inferior platform improving its AP technology, the market share of the superior platform among users increases, leading to a larger concentration on the user side of the market. Our result provides an additional argument to support concerns of competition authorities about the planned advertising cooperation between *Yahoo* and *Google* in 2008. The DoJ justified its critical approach by claiming that the deal would have virtually eliminated *Yahoo* as a competitor in the advertising market (DoJ, 2008). We argue additionally that although the cooperation was aimed at advertising, it could have strengthened the already dominant position of *Google* among users.

Our results extend earlier insights on licensing incentives (Farrell and Gallini, 1988; Shepard, 1987) to a two-sided market environment. In the mentioned articles the licensor boosts demand for its products directly through technology licensing. In contrast, our model emphasizes the role of indirect network effects. By providing a better advertisement placing technology to the inferior rival, the superior platform does not directly increase demand for its own advertising space. Instead, the superior platform achieves this effect indirectly, by increasing demand for the inferior platform's advertising space. The inferior platform responds by placing more advertisements, which induces advertising-averse users to switch to the rival. The larger user audience at the superior platform, in turn, boosts demand for advertising space as advertisers prefer advertisements placed at a platform with more users.

3.1 Welfare Analysis

In the following we turn to the welfare effects of technology licensing. We analyze the influence of an increase in parameter ζ and a decrease in parameter ρ on advertiser and user surpluses separately and start with the advertiser side. Advertiser surplus (AS) can be derived as:

$$AS(a_1, a_2) = \textstyle\sum_i \left[(1-\rho_i)n_i - p_i^a\right] a_i/2 = \sum_i a_i^2/2. \tag{10}$$

For user surplus, we get from Equation (4) that $t_1(\cdot) = 1 - t_2(\cdot)$, hence, marginal users are located symmetrically on the circle. User surplus (US) follows from Expression (3) as

$$US(a_1, a_2; \zeta, \mu, q, u) = u + 2\textstyle\int_0^{t_1(\cdot)} \left(q - \mu a_1 - [\delta_1(t)]^2\right) dt + 2\int_{t_1(\cdot)}^{1/2} \left(\zeta q - \mu a_2 - [\delta_2(t)]^2\right) dt. \tag{11}$$

The effects of changes in parameters ζ and ρ on advertiser and user surpluses are summarized in the following proposition.

Proposition 3. *Changes in parameters ζ and ρ have contrary effects on user and advertiser surpluses:*

i) with an increase in ζ user (advertiser) surplus increases (decreases),

ii) with a decrease in ρ user (advertiser) surplus decreases (increases).

Proof. See Appendix.

Following an increase in parameter ζ, the inferior platform expands its market shares among users as some users switch from the superior platform. In the resulting equilibrium the inferior (superior) platform advertises more (less). As users switch from the superior platform despite the fact that it places less advertisements, they enjoy a higher surplus. Users who stay with the superior platform win due to a less intense advertising. Users choosing platform 2 are also better off. Although the inferior platform advertisers more, gains from higher content quality outweigh losses from intensified advertising. As a result, users enjoy higher surplus. The effect on the advertiser surplus is straightforward from Expression (10). As the overall advertising intensity $(a_1^*(\cdot) + a_2^*(\cdot))$ decreases and platforms become more symmetric in their advertising levels, advertiser surplus is reduced. In summary, an increase in the content quality of the inferior platform benefits users and negatively effects advertisers.

With a decrease in parameter ρ, the inferior platform faces higher demand for advertisement slots. As a result, both platforms show more advertisements in the new equilibrium, which affects the utility of every user negatively. At the same time, advertiser surplus increases: Advertisers benefit from intensified advertising.[12]

We now turn to the effect of technology licensing on social welfare. Due to high non-linearity of the social welfare function we are not able to derive the socially optimal level of technology licensing. Instead, we focus on the question of whether the privately optimal extent of technology licensing (implying full licensing of AP technology and no licensing of CP technology) can be improved upon from a social welfare perspective. We address this issue by first evaluating the sign of the derivative of the social welfare function ($SW^*(\rho, \zeta, \mu, q, u)$) with respect to parameter ρ at the point $(\rho, \zeta) = (0, \zeta_0)$, which corresponds to the privately optimal extent of licensing. If this derivative is positive, platforms' incentives to license fully AP technology are socially

[12] Our result contrasts with the argument made by competition authorities opposing the *Yahoo-Google* advertising agreement in 2008, which emphasized the negative effect the deal may have had on advertisers.

excessive. However, the non-positive sign of the derivative does not imply that full licensing of AP technology is socially optimal. We then consider the derivative of the social welfare function evaluated at the point $\rho = 0$. Using this derivative we can answer the question whether given full licensing of AP technology social welfare can be improved through additional licensing of CP technology. We introduce $\Delta_0 := (1 - \zeta_0)q$. The following proposition summarizes our results.

Proposition 4. *If $\mu > (1+\sqrt{7})/2$ and $\Delta_0 > \Delta_\rho$, then the privately optimal level of AP technology licensing is socially excessive, with $\Delta_\rho := (3+4\mu)(1+3\mu)^2 \,/\, \left[4(1+\mu)^2(3+4\mu(1+\mu))\right]$. If AP technology is licensed to the privately optimal extent, incentives to additionally license CP technology are insufficient compared to the socially optimal (implying $\zeta^* = 1$) if and only if*

i) $\mu \geq (1+\sqrt{7})/2$ or

ii) $\mu < (1+\sqrt{7})/2$ and $\Delta_0 < \Delta_\zeta$, with $\Delta_\zeta := (1+3\mu)^2 \,/\, [4(4+\mu(6+\mu))]$ or

iii) $\mu < (1+\sqrt{7})/2$, $\Delta_0 \geq \Delta_\zeta$ and $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$

and they are socially optimal otherwise.

Proof. See Appendix.

If user disutility per advertisement (μ) and the asymmetry in the quality of platforms' CP technologies (Δ_0) are large enough, private incentives to license AP technology to the full extent are socially suboptimal. In this case social welfare is higher if platform 2 holds AP technology of a worse quality than the superior platform. The intuition is the following. We showed in Lemma 2 and Proposition 3 that advertisers and platforms win with ρ getting smaller. The only actors in our model who win from an increase in ρ are, therefore, users. If μ is high, users benefit strongly from a relatively low level of advertising due to a handicap of the inferior platform in AP technology. In addition, when Δ_0 is large, platforms differ significantly in their user market shares implying high user transportation costs. A positive ρ due to a less than full licensing of AP technology leads to a more symmetric allocation of users between the platforms, thus lowering transportation costs and increasing user surplus.

If the licensing of AP technology takes place to the full extent, private incentives to license CP technology can be suboptimal. If μ is high enough, the gains of users outweigh the joint losses of advertisers and platforms: Users benefit from a decrease in the overall advertising intensity following the licensing of CP technology. When μ is low, users benefit relatively little from a decrease in advertising intensity. In this case social welfare can only increase if platforms' losses are relatively low compared to user benefits. The superior platform loses less by sharing its CP technology if its initial advantage in the ability to produce content is low. Hence, social welfare increases following the licensing of CP technology if the initial asymmetry in platforms' content qualities is sufficiently small. If, however, platforms differ a lot in the quality of their CP technologies, then social welfare increases only if the condition $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$ holds. Otherwise (if $SW^*(0,1,\cdot) \leq SW^*(0,\zeta_0,\cdot)$), given full licensing of AP technology, social welfare is maximized if CP technology is not shared.

4 Extensions

In this section we extend our analysis in two directions. In the first extension we analyze licensing incentives under the assumption that platforms set advertisement prices instead of choosing advertisement quantities. In the second extension we return to the assumption of a quantity game on the advertiser side, but allow platforms to charge users. The latter applies to a broader range of advertising supported media platforms including TV channels, newspapers, where users are usually charged for excess to a platform's content. To economize on notation, we omit indexing variables with respect to a particular extension.

4.1 Technology Licensing Under Price Setting For Advertisers

In this extension we analyze how platforms' licensing incentives change when platforms play a price game on the advertiser side compared to the quantity game analyzed above. We assume that in the first stage each platform $i = \{1,2\}$ chooses a uniform slot price p_i^a. In the second stage advertisers decide whether to place an advertisement at a particular platform, and users choose which platform to interact with.

By rearranging Expression (2) we obtain the number of advertisements placed at platform i as a function of its audience size, n_i, and slot price, p_i^a:

$$a_i = (1-\rho_i)n_i - p_i^a. \tag{12}$$

By plugging (12) into (5) and solving for n_1 and n_2 we get the user demand at each platform as a function of slot prices:

$$\begin{aligned} n_1(p_1^a, p_2^a; \rho, \zeta, \mu, q) &= \frac{1/2 + 2\Delta + 2\mu(1-\rho) + 2\mu(p_1^a - p_2^a)}{1 + 2\mu(2-\rho)}, \\ n_2(p_1^a, p_2^a; \rho, \zeta, \mu, q) &= \frac{1/2 - 2\Delta + 2\mu - 2\mu(p_1^a - p_2^a)}{1 + 2\mu(2-\rho)}. \end{aligned} \tag{13}$$

Platform i maximizes its profit

$$\pi_i(p_1^a, p_2^a; \rho, \zeta, \mu, q) = [(1-\rho_i)n_i(p_1^a, p_2^a; \cdot) - p_i^a]\, p_i^a \tag{14}$$

with respect to p_i^a. The following proposition characterizes the equilibrium without licensing.

Proposition 5. *If platforms play a price game on the advertiser side, the equilibrium without licensing depends on the superior platform's advantage in CP technology. There exists a threshold* $\overline{\Delta}_{p^a} := \left[1 + \mu(5-2\rho) + 4\mu^2(1-\rho)\right] / [4 + 4\mu(3-2\rho)]$, *such that*

i) if $\Delta \geq \overline{\Delta}_{p^a}$, *only the superior platform is active (on both sides of the market),*

ii) if $\Delta < \overline{\Delta}_{p^a}$, *both platforms are active on both sides of the market. The superior platform charges a higher slot price, displays more advertisements and realizes higher profits than the competitor provided that it has a strict advantage in at least one of the technologies.*

Proof. See Appendix.

The equilibrium without licensing under a price game is qualitatively similar to the results derived under quantity game. The inferior platform is active on both sides of the market only if the quality advantage of the rival in CP technology is not very large ($\Delta < \overline{\Delta}_{p^a}$). The superior platform uses its technology advantage to charge a higher slot price and is able to place more advertisements. We assume that the condition $\Delta < \overline{\Delta}_{p^a}$ is satisfied under $\rho = \rho_0$ and $\Delta = \Delta_0$. In the following lemma we characterize how equilibrium variables change if a technology is licensed.

Lemma 4. *If platforms play a price game on the advertiser side of the market, technology licensing has the following effects on the equilibrium variables.*

i) If AP technology is shared, the inferior platform increases its advertisement price, places more advertisement slots and realizes higher profits. The superior platform places more advertisements, its slot price increases if $\Delta < \Delta_{p^a}$ *and (weakly) decreases otherwise, with* $\Delta_{p^a} := (1+\mu)/\left[4\,(1+3\mu)\right]$. *The superior platform's profit increases (weakly decreases) if* $\Delta < \Delta_\pi(\mu, \rho)$

($\Delta \geq \Delta_\pi(\cdot)$), with

$$\begin{aligned}
\Delta_\pi(\cdot) &:= \chi(\mu,\rho)/\theta(\mu,\rho), \textit{ where}\\
\chi(\mu,\rho) &:= 24\mu^5(1-\rho)^2 + 6\mu^4(1-\rho)(13-7\rho) + 6\mu^3(3-\rho)(5-4\rho)+\\
&\quad + \mu^2(17+\sqrt{105}-4\rho)(17-\sqrt{105}-4\rho)/4 + \mu(11-4\rho)+1,\\
\theta(\mu,\rho) &:= 24\mu^4(1-\rho)(5-3\rho) + \mu^3(29+\sqrt{105}-16\rho)(29-\sqrt{105}-16\rho)/4+\\
&\quad + 8\mu^2(3-2\rho)(5-\rho) + 4\mu(9-4\rho)+4.
\end{aligned}$$

It holds that $\Delta_{p^a} < \Delta_\pi(\cdot)$.

ii) If CP technology is licensed, the inferior (superior) platform raises (reduces) its advertisement price, places more (less) slots and realizes higher (lower) profits.

Proof. See Appendix.

Regardless whether platforms choose advertisement quantities or prices, the equilibrium variables change exactly in the same way when the superior CP technology is licensed. The inferior platform benefits from the resulting competitive scenario, while the superior loses. The former raises its slot price and places more advertisements, and, consequently, its profit increases. We analyze in detail the effect of CP technology licensing on advertisement prices. To do so, consider the reaction functions $p_i^a(p_j^a;\rho,\zeta,\mu,q)$ ($i,j=\{1,2\}$ and $i\neq j$), which state the optimal advertisement price of a platform given the competitor' slot price:

$$\begin{aligned}
p_1^a(p_2^a;\cdot) &= \max\left\{\frac{1/4+\Delta+\mu(1-\rho)-\mu p_2^a}{1+2\mu(1-\rho)},0\right\},\\
p_2^a(p_1^a;\cdot) &= \max\left\{\frac{(1-\rho)\left[1/4-\Delta+\mu(1-p_1^a)\right]}{1+2\mu},0\right\}.
\end{aligned}$$

Advertisement prices are strategic substitutes as $\partial p_i^a(p_j^a;\cdot)/\partial p_j^a < 0$: When a platform raises the price of its advertisement slot, the other platform responds by lowering its own price. A higher advertisement price of a platform implies that it displays fewer advertisements, which in turn allows it to attract some users from the competitor. As the competitor's audience gets smaller, advertisers find that platform less attractive, which puts pressure on it to decrease the slot price. When CP technology is licensed, changes in the equilibrium slot prices are determined by the joint influence of the direct and strategic effects:

$$\frac{\partial p_i^{a*}(\rho,\zeta,\mu,q)}{\partial\zeta} = \underbrace{\left.\frac{\partial p_i^a(p_j^a;\cdot)}{\partial\zeta}\right|_{p_j^{a*}(\cdot)}}_{\text{direct effect}} + \underbrace{\frac{\partial p_i^a(p_j^a;\cdot)}{\partial p_j^a}\frac{\partial p_j^{a*}(\rho,\zeta,\mu,q)}{\partial\zeta}}_{\text{strategic effect}}.$$

Similar to the quantity game, the direct effect is positive for the inferior platform and negative for the superior platform. As the advantage of the superior platform erodes, the inferior platform can attract more users, which in turn strengthens the demand from advertisers and allows it to set a higher advertisement price. Different from the quantity game, the inferior platform benefits not only from the direct effect, but also from the strategic effect. The latter also drives the inferior platform's slot price upwards following a decrease in the competitor's equilibrium price.

We now turn to the intuition behind the result on AP technology licensing. Different from the setup where platforms decide on advertisement quantities, the direct effect is now positive for both platforms. Access to a better AP technology increases the inferior platform's advertiser demand, which is equivalent to more advertisements for any given slot prices. More advertisements at the inferior platform drive users to the competitor, which in turn increases the superior platform's slot price. The direct effect is, therefore, positive for the superior platform. Although user demand at the inferior platform decreases with licensing of AP technology, the term $(1-\rho)\, n_2(p_1^a, p_2^a; \cdot)$ in Expression (14) gets larger for any advertisement prices. Therefore, the direct effect at the inferior platform is also positive.

In contrast, the strategic effect is always negative for the superior platform: The increase of the competitor's equilibrium advertisement price puts a negative pressure on its own slot price. Whether the superior platform's equilibrium slot price gets larger or smaller following a decrease in ρ, depends on the relative magnitudes of the direct and strategic effects. The positive direct effect is stronger when the superior platform's advantage in CP technology is relatively small. Indeed, a decrease in ρ is equivalent to more advertisements placed by the inferior platform, this in turn may only lead to a comparatively large increase in the superior platform's user demand if Δ is small. This is why the superior platform's slot price increases only if $\Delta < \Delta_{p^a}$. The superior platform's profit may, however, increase with licensing of AP technology even if its slot price decreases, which explains the inequality $\Delta_{p^a} < \Delta_{\pi}(\cdot)$ in Lemma 4. This result is different from the one obtained under the quantity game where, following AP technology sharing, platforms' profits move always in the same direction with the number of slots and slot prices. The difference is that under the price game both platforms are influenced directly by AP technology licensing. Being equivalent to an increase of the user demand at the superior platform, the direct effect allows the superior platform's profit to grow even if its slot price decreases as the superior platform always places more advertisements when ρ decreases.

Comparing licensing incentives under the quantity game and the price game we can conclude that the main mechanism which drives the superior platform's incentives to license its AP technology is same in the two setups and works through increasing user demand due to indirect network effects. The difference is, however, that this increase is due to the strategic effect under the quantity game and due to the direct effect under the price game.

We now turn to optimal licensing decisions and focus on the case where transfers are not feasible. We later discuss how the results would change if transfers were possible. It is useful to consider the function $\Delta_\pi(\cdot)$, such that $sign\{\partial\pi_i^*(\cdot)/\partial\rho\} = -sign\{\Delta_\pi(\cdot) - \Delta\}$, as follows from Lemma 4. It holds that $\partial\Delta_\pi(\cdot)/\partial\rho < 0$, therefore, $\Delta_\pi(\cdot)$ increases when ρ gets smaller. If ρ_0 and Δ_0 are initially small, which is the case when platforms hold similar technologies, then the difference $\Delta_\pi(\cdot,\rho_0) - \Delta_0$ is likely to be positive. In the latter case the superior platform shares its AP technology with the rival to the full extent as its profit increases with a decrease in ρ. If, however, the initial asymmetry in platforms' technologies is large enough, then the difference $\Delta_\pi(\cdot,\rho_0) - \Delta_0$ is likely to be negative. In that case the licensing of AP technology may either take place or not as stated in the following proposition.[13]

Proposition 6. *Assume that platforms play a price game on the advertiser side of the market and transfers are not admissible. In this case CP technology is not licensed ($\zeta^* = \zeta_0$). Whether AP technology is licensed depends on the initial asymmetries in technological capabilities of the platforms in the following manner.*

i) If the initial asymmetries in technological capabilities are small (i.e., $\Delta_0 < \Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0)$), AP technology is fully licensed ($\rho^* = 0$).

ii) If the initial asymmetries in technological capabilities are moderate (i.e., $\Delta_\pi(\cdot,\rho_0) < \Delta_0 < \Delta_\pi(\cdot,0)$), AP technology is fully licensed ($\rho^* = 0$) if $\pi_1^*(\rho = 0,\cdot) > \pi_1^*(\rho = \rho_0,\cdot)$, while it is not licensed ($\rho^* = \rho_0$) if $\pi_1^*(\rho = 0,\cdot) \leq \pi_1^*(\rho = \rho_0,\cdot)$.

iii) If the initial asymmetries in technological capabilities are large (i.e., $\Delta_\pi(\cdot,\rho_0) < \Delta_\pi(\cdot,0) < \Delta_0$), AP technology is not licensed ($\rho^* = \rho_0$).

Proof. See Appendix.

Regardless whether platforms decide on advertisement quantities or prices, the superior platform does not share its CP technology with the rival. However, different from the quan-

[13] We omit from the analysis the case where $\Delta_0 = \Delta_\pi(\cdot,\rho_0)$.

tity game, the superior platform only licenses its AP technology if the initial asymmetries in platforms' technologies are not too large. Following the licensing of AP technology, the superior platform's profit is influenced positively by the direct effect and negatively by the strategic effect. Initial asymmetries in platforms' technologies determine the relative strengths of these effects. For instance, the condition $\Delta_\pi(\cdot, \rho_0) < \Delta_\pi(\cdot, 0) < \Delta_0$ is likely to hold if both Δ_0 and ρ_0 are large as $\Delta_\pi(\cdot)$ decreases in ρ, implying a sufficiently large technological advantage of the superior platform. In the latter case the positive direct effect of a decrease in ρ is too small to compensate the negative strategic effect.

It can be shown that CP technology is not licensed in the presence of transfers either: The losses of the superior platform are always larger than the gains of the inferior platform. The presence of transfers, however, is likely to strengthen the incentives for AP technology licensing, such that the latter could be also possible under larger technological asymmetries.

Comparing licensing incentives under quantity and price settings on the advertiser side, we conclude that our main results are valid in both setups: While CP technology is not shared even with transfers, AP technology can be licensed even without transfers. The price game, however, brings an additional dimension into the licensing incentives problem. It points out that besides the difference between CP and AP technologies, the initial asymmetries in technological capabilities of the platforms also matter. If technological capabilities are sufficiently asymmetric, no licensing takes place. Our results imply that in reality we are more likely to observe differences in the quality of advertisement placing technologies of competing platforms in industries, in which these firms set slot prices.

4.2 Technology Licensing With User Prices

In this extension we return to the assumption that platforms decide on advertisement quantities, but allow for the possibility that users are charged for access. We focus on the question of how licensing incentives change when platforms can rely on both groups of customers as sources of revenues. The game proceeds as follows: In the first stage, each platform $i = \{1, 2\}$ decides on the number of advertisements, a_i, and the user access price, p_i^u. In the second stage, advertisers decide whether to place an advertisement at a particular platform, and users choose which platform to interact with. The utility of a user with address t takes the form

$$U_i^t = \begin{cases} u + q - [\delta_1(t)]^2 - \mu a_1 - p_1^u, & \text{if} \quad i = 1 \\ u + \zeta q - [\delta_2(t)]^2 - \mu a_2 - p_2^u, & \text{if} \quad i = 2, \end{cases} \tag{15}$$

with $\delta_1(t) = \min\{t, 1-t\}$ and $\delta_2(t) = |t - 1/2|$.

Using (15) we obtain user demand at each platform:

$$n_1(a_1, p_1^u, a_2, p_2^u; \zeta, \mu, q) = 1/2 + 2\left[\Delta - (p_1^u - p_2^u) - \mu(a_1 - a_2)\right], \tag{16}$$

$$n_2(a_1, p_1^u, a_2, p_2^u; \zeta, \mu, q) = 1/2 - 2\left[\Delta - (p_1^u - p_2^u) - \mu(a_1 - a_2)\right]. \tag{17}$$

Given the demand function $n_i(\cdot)$, platform i maximizes its profit

$$\pi_i(a_1, p_1^u, a_2, p_2^u; \rho_i, \zeta, \mu, q) = \left[(1-\rho_i)n_i(\cdot) - a_i\right] a_i + p_i^u n_i(\cdot)$$

by choosing a_i and p_i^u. In the following proposition we characterize the equilibrium without technology licensing.

Proposition 7. *If platforms can charge users, the equilibrium absent licensing depends on the user disutility per advertisement (μ) and the CP technology advantage of the superior platform (Δ) in the following way.*

i) If $\mu \geq 1$ and $\Delta \geq 3/4$, only the superior platform serves users. It displays no advertisements and relies only on revenues from users.

ii) If $\mu \geq 1$ and $\Delta < 3/4$, both platforms serve users, place no advertisements and rely only on revenues from users.

iii) If $1 - \rho \leq \mu < 1$ and $\Delta \geq \overline{\Delta}_{p^u}$, only the superior platform serves users, with $\overline{\Delta}_{p^u} := \left[1 + 2\mu(2-\mu)\right]/4$. It charges users and places advertisements.

iv) If $1-\rho \leq \mu < 1$ and $\Delta < \overline{\Delta}_{p^u}$, both platforms serve users. The superior platform charges both sides of the market, while the inferior platform relies on revenues from users only.

v) If $\mu < 1 - \rho$ and $\Delta \geq \overline{\Delta}_{p^u}$, only the superior platform serves users. It charges users and places advertisements.

vi) If $\mu < 1 - \rho$ and $\Delta < \overline{\Delta}_{p^u}$, both platforms serve users and charge both sides of the market.

Proof. See Appendix.

If platforms can charge both users and advertisers, depending on the parameters, two different combinations of equilibrium sources of revenues arise. Platforms either rely on revenues from users only or they charge both sides. If users are strongly averse to advertisements (i.e., μ is high), platforms refrain from placing advertisements and rely solely on revenues from users. For each platform there is a threshold for user disutility per advertisement, below which a platform places advertisements. This threshold is larger for the superior platform: It displays ads if $\mu > 1$, while the inferior platform advertisers if $\mu > 1 - \rho$. Due to ist advantage in AP technology, placing advertisements is more profitable for the superior platform and it is ready to sacrifice some users in order to generate revenues from advertisers. For the same reason, the critical value of disutility per advertisement which makes it profitable to place advertisements for the inferior platform $(1 - \rho)$ is lower, the larger its handicap in AP technology becomes. While parameters μ and ρ are decisive for platforms' decisions to place advertisements, parameter Δ determines whether the inferior platform is active at the market. If Δ is sufficiently large, the inferior platform becomes unattractive to users even if it places no advertisements. In the following we restrict attention to the case where the platforms are active on both sides of the market, such that $\mu < 1 - \rho_0$ and $\Delta_0 < \overline{\Delta}_{p^u}$. The following lemma characterizes the effects of technology licensing on equilibrium variables.

Lemma 5. *Assume that platforms charge users and advertisers. Technology licensing has the following effects:*
i) Following the licensing of AP technology the inferior platform reduces (increases) its user (slot) price and displays more advertisements. The superior platform charges a lower price to both users and advertisers, it places less advertisements and its market share among users decreases.
ii) Following the licensing of CP technology the inferior (superior) platform charges both users and advertisers a higher (lower) price, it places more (less) advertisements and its market share among users increases (decreases).
In both cases the inferior (superior) platform's profit increases (decreases) and joint profits decrease.

Proof. See Appendix.

Different from the baseline model where platforms receive revenues from advertisers only, with user prices the incentives to share AP technology vanish regardless whether transfers are allowed or not. To see the intuition behind this result it is again helpful to consider platforms' reaction functions, $a_1(a_2; \rho, \zeta, \mu, q)$ and $a_2(a_1; \rho, \zeta, \mu, q)$:

$$a_1(a_2; \cdot) = \frac{(1-\mu)}{[1+\mu(2-\mu)]} \left[\frac{1}{4} + \Delta + \frac{a_2(\mu+\rho)(2-\rho-\mu)}{1-\rho-\mu} \right],$$

$$a_2(a_1; \cdot) = \max \left\{ \frac{(1-\rho-\mu)}{\left[2-(1-\rho-\mu)^2\right]} \left[\frac{1}{4} - \Delta + \frac{a_1\mu(2-\mu)}{1-\mu} \right], 0 \right\}.$$

As in the baseline model, decisions on the number of advertisement slots are strategic complements: $a_i(a_j; \cdot)$ is an increasing function of a_j (with $i, j = \{1, 2\}$ and $i \neq j$). However, both $a_1(a_2; \cdot)$ and $a_2(a_1; \cdot)$ are now functions of parameter ρ, the inferior platform's disadvantage in AP technology. The decrease in ρ due to the licensing of AP technology affects then both platforms directly. The direct effect is negative for the superior platform and positive for the inferior platform: $\partial \left[(\mu+\rho)(2-\rho-\mu)/\left(1-\rho-\mu\right)\right]/\partial\rho > 0$ implies that as ρ gets smaller for any given number of slots at the competitor, the superior platform places fewer advertisements. Similarly, $\partial \left((1-\rho-\mu)/\left[2-(1-\rho-\mu)^2\right] \right)/\partial\rho < 0$ implies the opposite relation for the inferior platform. The superior platform, however, benefits from a positive strategic effect due to the increased equilibrium number of advertisements at the inferior platform, $a_2^*(\cdot)$. As the direct effect is stronger than the strategic effect, $a_1^*(\cdot)$ decreases.

Why does the direct effect appear for the superior platform following AP technology licensing when platforms charge users? Each platform has now two sources of revenues: users and advertisers. Depending on user disutility per advertisement (μ) and the efficiency of its AP technology ($1-\rho_i$), platform i balances its revenues from the two sides of the market. The superior platform chooses a_1^* and p_1^{u*} such that $p_1^{u*} = a_1^*\mu/(1-\mu)$, while the inferior platform chooses a_2^* and p_2^{u*} such that $p_2^{u*} = a_2^*\left[1-(1-\rho)(1-\rho-\mu)\right]/(1-\rho-\mu)$. As $\partial \left(\left[1-(1-\rho)(1-\rho-\mu)\right]/(1-\rho-\mu)\right)/\partial\rho > 0$, for any given a_2 the inferior platform charges a lower user price when its AP technology improves. With a decrease in ρ, for any given a_2 the inferior platform can charge a higher price for an advertisement slot, while this increase is larger, the more users are served by the platform. This creates incentives for the inferior platform to attract more users by charging a lower user price. A lower user price at the inferior platform in turn reduces user demand at the superior platform giving rise to the negative direct effect.

As all the other equilibrium variables at platform 1 move in the same direction with the number of advertisement slots, the superior platform reduces its prices for both advertisers and users and its market share among users becomes smaller, leading to lower profits. Although the inferior platform reduces the user price, the higher advertisement price, increased number of slots and larger user market shares allow it to realize a higher profit.

The effects of licensing of CP technology on equilibrium variables are the same as in the baseline model, except for an additional effect related to the changes in user prices. While the superior platform has to reduce its user price, the inferior platform charges users a higher price when the quality of its content improves. The following proposition characterizes the licensing incentives of the platforms when users are charged for access.

Proposition 8. *Assume that platforms charge both users and advertisers. Regardless whether transfers are allowed or not, none of the technologies is licensed ($\rho^* = \rho_0$ and $\zeta^* = \zeta_0$).*

Proof. As $\partial\pi_1^*(\cdot)/\partial\rho > 0$ and $\partial\pi_1^*(\cdot)/\partial\zeta < 0$, there is no licensing without transfers. As $\sum_i \partial\pi_i^*(\cdot)/\partial\rho > 0$ and $\sum_i \partial\pi_i^*(\cdot)/\partial\zeta < 0$, there is no licensing in the presence of transfers either. *Q.E.D.*

Our analysis shows that the incentives of a superior media platform to license its AP technology to the inferior rival depend strongly on the sources of revenues available to the platforms. If platforms cannot charge users, the superior platform licenses its AP technology to the full extent. However, the opportunity of user access charges makes licensing incentives vanish. Our results imply that in media industries where advertising supported platforms rely on both sources of revenues (user payments and advertising receipts) the asymmetries in platforms' technological capabilities will persist more than in those industries where platforms charge only advertisers.

5 Discussion

In this section we discuss the implications of some of our modelling assumptions.

Inelastic user demand: We assume throughout the article that user demand is inelastic. In this case licensing has only a business-stealing effect, any potential market expansion (or in case of increased advertising levels, market contraction) effects are ruled out. Allowing user demand to be elastic would most likely weaken the incentives to license AP technology. The fact that some users may refrain from visiting any of the platforms in response to intensified advertising would weaken incentives to advertise more in the new equilibrium. This, in turn, would make licensing of both AP and CP technologies less attractive. As licensing of CP technology does not take place under the inelastic demand, the results regarding CP technology would not change. Our assumption of inelastic user demand applies well to media markets in which changes in advertising levels are likely to have only a moderate influence on the size of the user market. This is the case when users derive a relatively high basic utility from consuming platforms' products. In many media markets, such as search engines, radio or television, it seems realistic that only a relatively small fraction of consumers would decide to completely give up using these media due to excessive advertising. Advertising intensity is likely to be mainly decisive for consumers' choices which platform to use. In most media industries the business-stealing effect of platforms' advertising decisions seems to be more important than the market expansion (contraction) effect.

Single-homing: The assumption that platforms compete for users is crucial for our results. The incentives to license a technology in our model are determined by users switching from one platform to the other depending on the platforms' relative attractiveness. Single-homing seems to be a realistic assumption for media markets where consumers pay an access price. There is also evidence that in the case of free platforms such as internet search engines where users can costlessly multi-home, many users tend to interact with only one platform.[14]

One platform is superior in both technologies: We derived our results under the assumption that one of the platforms is superior in both technologies. We could instead assume that one platform has an advantage in AP technology, while the other platform is superior in CP technology.[15] We show that there is one difference in the effects of technology licensing in this scenario compared to our scenario: Joint profits may increase following the licensing of CP technology. This is the case when platforms hold AP technologies of strictly different qualities and the asymmetry in CP technologies is not very large. Redistribution of users between the platforms following CP technology licensing (from a platform with an inferior AP technology to a platform with a superior AP technology) may increase joint profits as for any given user market share the platform with a superior AP technology is able to charge a higher slot price than the competitor. In the presence of transfers it can be in the interest of a platform with an inferior AP technology to let the competitor 'use' the audience in a more productive way through licensing its advanced CP technology, as a platform with a better AP technology can convert each user into higher revenues from advertisers. The licensing of CP technology in this case serves as a device to redistribute users to the rival in exchange for a transfer. Furthermore,

[14] Evans (2008) provides evidence of user single-homing in the internet search market.

[15] A formal analysis of this scenario is available from the authors upon request. We thank an anonymous referee for urging us to think along these lines.

we show that the losses of the licensor of CP technology are proportional to Δ and they are small when platforms' CP technologies are similar. This creates a potential for CP technology licensing in the presence of transfers if platforms do not differ a lot in content quality. CP technology licensing, however, does not take place in equilibrium. Following AP technology licensing (which always takes place), platforms hold AP technologies of the same quality, such that the redistribution of users between the platforms cannot further increase joint profits.

Expectations: We assumed that advertisers are able to correctly predict platforms' user market shares under any announced advertising levels. Users expect the number of slots at each platform to correspond to the announced one (if under this number the slot price is non-negative). We could alternatively assume that before platforms announce advertising levels, advertisers form expectations about their user market shares such that these expectations are not influenced by the announcements. This assumption would imply that platforms have strong reputations for holding particular market shares among users. For instance, in the case of search engines, *Google* is known to be the dominant firm in many markets. Our results on licensing incentives are robust to an alternative formulation of advertisers' expectations: While AP technology is licensed without transfers, CP technology is not licensed even with transfers.[16]

6 Conclusion

We develop a parsimonious model with two horizontally and vertically differentiated advertising supported media platforms which differ in their technologies to produce content (e.g., TV programs or organic search results) and place advertisements. Our main purpose is to derive conditions under which a platform endowed with a superior technology licences its knowledge to an inferior rival. We highlight the role of two technologies on licensing incentives, which are characteristic for most media platforms. First, the content producing technology, responsible for the utility users draw from consuming content on a platform. Second, the advertisement placing technology, determining the ability of a platform to show advertisements resulting in a high probability of users' purchase of the advertised product. We show that the superior platform licenses its technology to place advertisements, but not its technology to create content.

[16] A formal analysis of licensing incentives under the assumption that advertisers form expectations about user market shares before platforms' announcements is available from the authors on request. We thank an anonymous referee for urging us to think along these lines.

Our explanation is based purely on indirect network effects: By improving the competitor on the advertiser side of the market through licensing its AP technology, the superior platform enhances its own user demand. However, when platforms rely on revenues from users in addition to their advertising receipts, licensing incentives vanish. Our model provides a non-cooperative rationale for technology-based agreements between competing platforms, such as the planned and abandoned *Yahoo-Google* advertising deal in 2008.

Empirical verification of the predictions of our model could be a potentially fruitful avenue for further research, and the market for internet search engines is a good candidate for such an analysis. Data on search engines usage as well as advertising levels for a long period of time is publicly available from sources such as *Comscore* or *Hitwise.* Several cooperation agreements between various search engine operators took place in the recent past (or were planed). The empirical analysis of the effects of these agreements is likely to attract interest from economists as well as policy makers.

Our article provides a tool for competition authorities to analyze the competitive effects of technology-based cooperations in media industries and their welfare implications. Another track for further research could be to extend our model to include more than two platforms. With three active platforms, the model would allow to analyze how competition among three asymmetric search engines is affected if two of them - perhaps the smaller ones - enter into a technology-based cooperation. Such cooperation agreement was recently struck between search engine operators *Yahoo* and *Microsoft* and was approved by competition authorities worldwide. The model could be calibrated based on the operators' real user market shares and advertising levels to be then applied to quantify the competitive and welfare effects of an agreement.

7 Appendix

Proof of Lemma 1. We first derive the equilibrium advertisement levels, $a_i^*(\rho, \zeta, \mu, q)$, $i = \{1, 2\}$. Solving the FOCs of the platforms with respect to a_1 and a_2 simultaneously yields $a_1^*(\cdot)$ and $a_2^*(\cdot)$ as

$$\begin{aligned} a_1^*(\cdot) &= \frac{1 + 3\mu(1-\rho) + 4\Delta\left[1 + \mu(1-\rho)\right]}{4[3\mu^2(1-\rho) + 2\mu(2-\rho) + 1]} \\ a_2^*(\cdot) &= \frac{(1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}. \end{aligned} \tag{18}$$

Given the restriction $0 \leq \rho < 1$, it holds that $a_i^*(\cdot) > 0$ if and only if $0 \leq \Delta < \overline{\Delta}$. The SOCs with respect to a_1 and a_2 are fulfilled as

$$\left.\frac{\partial^2 \pi_1(a_1, a_2; \zeta, \mu, q)}{\partial (a_1)^2}\right|_{a_1^*(\cdot), a_2^*(\cdot)} = -2 - 4\mu < 0,$$

$$\left.\frac{\partial^2 \pi_2(a_1, a_2; \rho, \zeta, \mu, q)}{\partial (a_2)^2}\right|_{a_1^*(\cdot), a_2^*(\cdot)} = -2 - 4\mu(1-\rho) < 0.$$

It follows that both platforms place advertisements in equilibrium if $0 \leq \Delta < \overline{\Delta}$. Using $a_1^*(\cdot)$ and $a_2^*(\cdot)$ we obtain the equilibrium advertisement prices, $p_i^{a*}(\rho, \zeta, \mu, q)$. The difference between the prices can be written as

$$p_1^{a*}(\cdot) - p_2^{a*}(\cdot) = \frac{\rho[1 + 2\mu(1 + (3\mu + 1)(1-\rho))] + 4\Delta[2 - \rho + 2\mu(1 + (1+\mu)(1-\rho)(2-\rho))]}{4\left[3\mu^2(1-\rho) + 2\mu(2-\rho) + 1\right]}.$$

It follows that $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, holding with equality if $\Delta = \rho = 0$. The equilibrium price of platform 2 is

$$p_2^{a*}(\cdot) = \frac{\left[1 + 2\mu(1-\rho)\right](1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}.$$

Given $0 \leq \rho < 1$, $p_2^{a*}(\cdot)$ is positive if and only if $0 \leq \Delta < \overline{\Delta}$. As $p_2^{a*}(\cdot) > 0$ and $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, it must hold that $p_1^{a*}(\cdot) > 0$, hence, both platforms set positive prices for advertisements provided that $0 \leq \Delta < \overline{\Delta}$. As $a_i^*(\cdot) > 0$ and $p_i^{a*}(\cdot) > 0$, both platforms realize positive profits. Turning to the market shares of the platforms among users, we can write platform i's FOC as

$$(1-\rho_i) n_i^*(\rho, \zeta, \mu, q) = \left[2 - (1-\rho_i)\frac{\partial n_i(a_1, a_2; \zeta, \mu, q)}{\partial a_i}\right] a_i^*(\cdot) = 2\left[1 + \mu(1-\rho_i)\right] a_i^*(\cdot),$$

which implies that given $0 \leq \rho_i < 1$ every platform serves some users when it places a positive number of advertisements. *Q.E.D.*

Proof of Proposition 1. i) We shoved in the proof of Lemma 1 that $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, holding with equality only if $\Delta = \rho = 0$. Using equations in (18) we can write the difference between $a_1^*(\cdot)$ and $a_2^*(\cdot)$ as

$$a_1^*(\cdot) - a_2^*(\cdot) = \frac{\rho + 4\Delta\left[2\mu(1-\rho) + 2 - \rho\right]}{4\left[3\mu^2(1-\rho) + 2\mu(2-\rho) + 1\right]},$$

which is non-negative, and equals zero if $\Delta = \rho = 0$. The superior platform having a strict advantage in at least one technology places more advertisements, charges a higher price for them and realizes higher profits.

ii) We turn to the equilibrium user market shares. From Expression (2) we get $n_1^*(\cdot) = p_1^{a*}(\cdot) + a_1^*(\cdot)$ and $n_2^*(\cdot) = \left[p_2^{a*}(\cdot) + a_2^*(\cdot)\right]/(1-\rho)$. Both platforms serve users as $p_i^{a*}(\cdot), a_i^*(\cdot) > 0$,

$i = \{1, 2\}$. Comparing equilibrium user market shares we get

$$n_1^*(\cdot) - n_2^*(\cdot) = \frac{4(1+\mu)\left[1+\mu(1-\rho)\right](\Delta - \underline{\Delta})}{3\mu^2(1-\rho)+2\mu(2-\rho)+1},$$

which is positive if $\Delta > \underline{\Delta}$, negative if $0 \le \Delta < \underline{\Delta}$ and zero when $\Delta = \underline{\Delta}$. The superior platform has a larger market share among users only if its advantage in content quality is large enough. *Q.E.D.*

Proof of Lemma 2. *i)* We start with the effect of a change in ρ on the number of advertisements displayed in equilibrium, $a_i^*(\cdot)$, $i = \{1, 2\}$. Taking derivative of the expressions in (18) with respect to ρ we get

$$\frac{\partial a_1^*(\cdot)}{\partial \rho} = -\frac{\mu(1+\mu)(\overline{\Delta} - \Delta)}{\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]^2}, \tag{19a}$$

$$\frac{\partial a_2^*(\cdot)}{\partial \rho} = -\frac{(1+\mu)(1+2\mu)(\overline{\Delta} - \Delta)}{\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]^2}. \tag{19b}$$

Both derivatives are negative for $0 \le \Delta < \overline{\Delta}$. Thus, if ρ decreases, both platforms show more advertisements.

ii) We proceed with the effect of ρ on the equilibrium market shares, $n_i^*(\cdot) = n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)$, by inspecting the derivative $\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)/\partial \rho$, with $i, j \in \{1, 2\}$, $i \neq j$:

$$\frac{\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{\partial \rho} = \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_i}\frac{\partial a_i^*(\cdot)}{\partial \rho} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \rho}. \tag{20}$$

It follows from (5) that $\partial n_i(a_1, a_2; \cdot)/\partial a_j > 0$ and $\partial n_i(a_1, a_2; \cdot)/\partial a_i = -\partial n_i(a_1, a_2; \cdot)/\partial a_j$. By rearranging Expression (20) we get

$$\frac{\partial n_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{\partial \rho} = \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\left(\frac{\partial a_j^*(\cdot)}{\partial \rho} - \frac{\partial a_i^*(\cdot)}{\partial \rho}\right). \tag{21}$$

Note that $\partial n_1^*(\cdot)/\partial \rho = -\, \partial n_2^*(\cdot)/\partial \rho$. We evaluate the sign of the derivative $\partial(a_2^*(\cdot) - a_1^*(\cdot))/\partial \rho$ by subtracting Expression (19a) from Expression (19b) to get

$$\frac{\partial a_2^*(\cdot)}{\partial \rho} - \frac{\partial a_1^*(\cdot)}{\partial \rho} = -\frac{(1+\mu)^2(\overline{\Delta} - \Delta)}{\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]^2}, \tag{22}$$

which is negative for $0 \le \Delta < \overline{\Delta}$. It follows that $\partial n_1^*(\cdot)/\partial \rho < 0$ and $\partial n_2^*(\cdot)/\partial \rho > 0$.

iii) We turn to the effect of a change in ρ on the prices of advertisement slots and inspect $dp_1^a(n_1(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_1^*(\cdot))/d\rho$ and $dp_2^a(n_2(a_1^*(\cdot), a_2^*(\cdot); \cdot), a_2^*(\cdot); \rho)/d\rho$. Using Expression

(21) these derivatives can be rearranged as

$$\frac{dp_1^a(n_1(a_1^*(\cdot),a_2^*(\cdot);\cdot),a_1^*(\cdot))}{d\rho} = \frac{\partial n_1(a_1,a_2;\cdot)}{\partial a_2}\left(\frac{\partial a_2^*(\cdot)}{\partial\rho} - \frac{\partial a_1^*(\cdot)}{\partial\rho}\right) - \frac{\partial a_1^*(\cdot)}{\partial\rho}, \tag{23a}$$

$$\frac{dp_2^a(n_2(a_1^*(\cdot),a_2^*(\cdot);\cdot),a_2^*(\cdot),\rho)}{d\rho} = (1-\rho)\frac{\partial n_2(a_1,a_2;\cdot)}{\partial a_1}\left(\frac{\partial a_1^*(\cdot)}{\partial\rho} - \frac{\partial a_2^*(\cdot)}{\partial\rho}\right) - n_2^*(\cdot) - \frac{\partial a_2^*(\cdot)}{\partial\rho}. \tag{23b}$$

Taking derivatives of the expressions in (5) with respect to a_1 and a_2 yields

$$\frac{\partial n_i(a_1,a_2;\cdot)}{\partial a_j} = 2\mu. \tag{24}$$

We can now plug Expressions (24), (22) and (19a) into Expression (23a) to get

$$\frac{\partial p_1^{a*}(\cdot)}{\partial\rho} = -\frac{\mu(1+2\mu)(1+\mu)(\overline{\Delta}-\Delta)}{[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2},$$

which is negative for $0 \leq \Delta < \overline{\Delta}$. Plugging Expressions (24), (22), (19b) and $n_2^*(\cdot)$ into Expression (23b) yields

$$\frac{\partial p_2^{a*}(\cdot)}{\partial\rho} = -\frac{[1+2\mu\,(1+\mu\,(1-\rho))\,(1+(3\mu+2)\,(1-\rho))]\,(1+\mu)(\overline{\Delta}-\Delta)}{[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2}.$$

This derivative is negative for $0 \leq \Delta < \overline{\Delta}$.

iv) Finally, to analyze the influence of a change in ρ on the platforms' profits, we inspect the derivative $\partial\pi(p_i^{a*}(\cdot),a_i^*(\cdot))/\partial\rho$:

$$\frac{\partial\pi(p_i^{a*}(\cdot),a_i^*(\cdot))}{\partial\rho} = \frac{\partial p_i^{a*}(\cdot)}{\partial\rho}a_i^*(\cdot) + p_i^{a*}(\cdot)\frac{\partial a_i^*(\cdot)}{\partial\rho}.$$

We know from *i)* and *iii)* that the derivatives $\partial p_i^{a*}(\cdot)/\partial\rho$ and $\partial a_i^*(\cdot)/\partial\rho$ are negative and $a_i^*(\cdot), p_i^{a*}(\cdot) > 0$ if $0 \leq \Delta < \overline{\Delta}$, hence, $\partial\pi_i^*(\rho,\zeta,\mu,q)/\partial\rho < 0$ and the profits of both platforms increase with a decrease in parameter ρ. Note that $\partial p_i^{a*}(\cdot)/\partial\rho < 0$, $\partial a_i^*(\cdot)/\partial\rho < 0$ and $a_i^*(\cdot), p_i^{a*}(\cdot) > 0$ hold for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$. *Q.E.D.*

Proof of Lemma 3. Note first that if $(1-\zeta_0)\,q < \overline{\Delta}$, then following an increase in ζ the condition $\Delta < \overline{\Delta}$ is again fulfilled. *i)* We start with the effect of a change in ζ on the number of advertisements displayed in equilibrium by taking derivatives of $a_1^*(\cdot)$ and $a_2^*(\cdot)$ with respect to ζ:

$$\frac{\partial a_1^*(\cdot)}{\partial\zeta} = -\frac{1+\mu(1-\rho)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}q < 0, \tag{25}$$

$$\frac{\partial a_2^*(\cdot)}{\partial\zeta} = \frac{(1-\rho)(1+\mu)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}q > 0.$$

The superior (inferior) platform displays less (more) advertisements in equilibrium with an increase in ζ, which holds for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$.

ii) Turning to the effect of ζ on the equilibrium market shares, $n_i^*(\cdot)$, we inspect the derivative $dn_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$. Using that $\partial n_i(a_1, a_2; \cdot)/\partial a_i = -\partial n_i(a_1, a_2; \cdot)/\partial a_j$ we have

$$\begin{aligned}\frac{dn_i(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{d\zeta} &= \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_i}\frac{\partial a_i^*(\cdot)}{\partial \zeta} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\frac{\partial a_j^*(\cdot)}{\partial \zeta} + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial \zeta} \\ &= \frac{\partial n_i(a_1, a_2; \cdot)}{\partial a_j}\left(\frac{\partial a_j^*(\cdot)}{\partial \zeta} - \frac{\partial a_i^*(\cdot)}{\partial \zeta}\right) + \frac{\partial n_i(a_1, a_2; \cdot)}{\partial \zeta}. \end{aligned} \tag{26}$$

The covered user market assumption implies $dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta = -dn_2(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$. We focus on the sign of $dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)/d\zeta$. From (25) we obtain

$$\frac{\partial a_2^*(\cdot)}{\partial \zeta} - \frac{\partial a_1^*(\cdot)}{\partial \zeta} = \frac{1 + (1-\rho)(1+2\mu)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}q. \tag{27}$$

We now turn to the value of $\partial n_1(a_1, a_2; \cdot)/\partial \zeta$. Taking derivative of the first expression in (5) with respect to ζ yields

$$\frac{\partial n_1(a_1, a_2; \cdot)}{\partial \zeta} = -2q. \tag{28}$$

By plugging Expressions (24), (27) and (28) into Expression (26) we get

$$\frac{dn_1(a_1^*(\cdot), a_2^*(\cdot); \cdot)}{d\zeta} = -\frac{2\,(\mu+1)\,[\mu(1-\rho)+1]}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}q. \tag{29}$$

It follows that $\partial n_1^*(\cdot)/\partial \zeta < 0$ and $\partial n_2^*(\cdot)/\partial \zeta > 0$ for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$.

iii) We now turn to the effect of an increase in ζ on the equilibrium prices:

$$\begin{aligned}\frac{\partial p_1^a(n_1^*(\cdot), a_1^*(\cdot))}{\partial \zeta} &= \frac{\partial n_1^*(\cdot)}{\partial \zeta} - \frac{\partial a_1^*(\cdot)}{\partial \zeta}, \\ \frac{\partial p_2^a(n_2^*(\cdot), a_2^*(\cdot))}{\partial \zeta} &= (1-\rho)\frac{\partial n_2^*(\cdot)}{\partial \zeta} - \frac{\partial a_2^*(\cdot)}{\partial \zeta}.\end{aligned}$$

The covered user market assumption yields $\partial n_1^*(\cdot)/\partial \zeta = -\,\partial n_2^*(\cdot)/\partial \zeta$. Using (25) and (29) we get

$$\begin{aligned}\frac{\partial p_1^{a*}(\cdot)}{\partial \zeta} &= -\frac{(2\mu+1)\,[1+\mu(1-\rho)]}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}q < 0, \\ \frac{\partial p_2^{a*}(\cdot)}{\partial \zeta} &= \frac{(1-\rho)\,(\mu+1)\,[1+2\mu(1-\rho)]}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}q > 0.\end{aligned} \tag{30}$$

With an increase in parameter ζ, the superior (inferior) platform chargers a lower (higher) price for advertisements for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$.

iv) Finally, to analyze the influence of a change in ζ on platforms' profits we inspect the derivative $\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))/\partial\zeta$:

$$\frac{\partial\pi(p_i^{a*}(\cdot), a_i^*(\cdot))}{\partial\zeta} = \frac{\partial p_i^{a*}(\cdot)}{\partial\zeta} a_i^*(\cdot) + p_i^{a*}(\cdot)\frac{\partial a_i^*(\cdot)}{\partial\zeta}.$$

Using the inequalities in (25) and (30), we conclude that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ and $\partial\pi_2^*(\cdot)/\partial\zeta > 0$ for any $0 \leq \rho < 1$ and $0 \leq \Delta < \overline{\Delta}$. With an increase in parameter ζ, the superior (inferior) platform makes lower (higher) profits.

v) The total effect of a change in ζ on platforms' joint profits is non-positive if $|\partial\pi_1^*(\cdot)/\partial\zeta| \geq |\partial\pi_2^*(\cdot)/\partial\zeta|$. This is equivalent to

$$\left|\frac{\partial p_1^{a*}(\cdot)}{\partial\zeta}\right| a_1^*(\cdot) + p_1^{a*}(\cdot)\left|\frac{\partial a_1^*(\cdot)}{\partial\zeta}\right| \geq \left|\frac{\partial p_2^{a*}(\cdot)}{\partial\zeta}\right| a_2^*(\cdot) + p_2^{a*}(\cdot)\left|\frac{\partial a_2^*(\cdot)}{\partial\zeta}\right|. \tag{31}$$

Consider next the differences $|\partial p_1^{a*}(\cdot)/\partial\zeta| - |\partial p_2^{a*}(\cdot)/\partial\zeta|$ and $|\partial a_1^*(\cdot)/\partial\zeta| - |\partial a_2^*(\cdot)/\partial\zeta|$:

$$\begin{aligned} \left|\frac{\partial p_1^{a*}(\cdot)}{\partial\zeta}\right| - \left|\frac{\partial p_2^{a*}(\cdot)}{\partial\zeta}\right| &= \frac{\left[1 + 2\mu(1 + (1+\mu)(1-\rho))\right]\rho}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1} q, \\ \left|\frac{\partial a_1^*(\cdot)}{\partial\zeta}\right| - \left|\frac{\partial a_2^*(\cdot)}{\partial\zeta}\right| &= \frac{\rho}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1} q. \end{aligned} \tag{32}$$

Note that $|\partial p_1^{a*}(\cdot)/\partial\zeta| \geq |\partial p_2^{a*}(\cdot)/\partial\zeta|$ and $|\partial a_1^*(\cdot)/\partial\zeta| \geq |\partial a_2^*(\cdot)/\partial\zeta|$, holding with equality if $\rho = 0$. Furthermore, $a_1^*(\cdot) \geq a_2^*(\cdot)$ and $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$, holding with equality if $\Delta = \rho = 0$. It follows that Inequality (31) is fulfilled for any $\zeta \in (0,1]$ and $\rho \in [0,1)$ and it holds with equality if $\Delta = \rho = 0$. As ζ cannot be further increased if $\zeta = 1$ ($\Delta = 0$), platforms' joint profits decrease with an increase in ζ. *Q.E.D.*

Proof of Proposition 3. *i)* We first turn to the influence of changes in parameters ρ and ζ on the advertiser surplus. It follows from Expression (10) that $AS(a_1^*(\cdot), a_2^*(\cdot))$ increases in $a_1^*(\cdot)$ and $a_2^*(\cdot)$. In Lemma 2 we showed that $a_1^*(\cdot)$ and $a_2^*(\cdot)$ increase with a decrease in ρ. It follows that advertiser surplus gets larger as ρ decreases. To analyze the effect of a change in parameter ζ on the advertiser surplus, we first take derivative of Expression (10) evaluated at equilibrium values with respect to ζ:

$$\frac{\partial AS(a_1^*(\cdot), a_2^*(\cdot))}{\partial\zeta} = a_1^*(\cdot)\frac{\partial a_1^*(\cdot)}{\partial\zeta} + a_2^*(\cdot)\frac{\partial a_2^*(\cdot)}{\partial\zeta}.$$

We showed in Lemma 3 that $\partial a_1^*(\cdot)/\partial\zeta < 0$ and $\partial a_2^*(\cdot)/\partial\zeta > 0$. From Expression (32) we have that $|\partial a_1^*(\cdot)/\partial\zeta| \geq |\partial a_2^*(\cdot)/\partial\zeta|$, which is fulfilled with equality if $\rho = 0$. As stated in Proposition 1, $a_1^*(\cdot) \geq a_2^*(\cdot)$, holding with equality only if $\Delta = \rho = 0$. It follows that $\partial AS(a_1^*(\cdot), a_2^*(\cdot))/\partial\zeta \leq 0$, holding with equality only if $\Delta = \rho = 0$ (in which case ζ cannot be further increased). Hence, advertiser surplus decreases as parameter ζ gets larger.

ii) We now turn to the user surplus. It is useful to distinguish between two groups of users: Those who do not switch from the original platform in response to a change in parameters ρ or ζ and those who do. We will refer to the former group of users as *switchers* and to the latter group as *non-switchers*. We start with the effect of a change in ρ on the utility of switchers. Let t_1^ρ and t_2^ρ denote the locations of the marginal users (i.e., those who are indifferent between the two platforms) and $U_i^{\rho t}$ the utility of a user t choosing platform i after the change in parameter ρ. We showed in Lemma 2 that $n_1^*(\cdot)$ increases in response to a reduction in ρ, hence, $t_1^\rho > t_1$ and $t_2^\rho < t_2$. Due to symmetry, we can restrict attention to switchers with locations $t \in [t_1, t_1^\rho]$. For these users we have $U_1^t < U_2^t$ as before the change in ρ they preferred the superior platform. We also know from Lemma 2 that $a_1^*(\cdot)$ increases with a decrease in ρ. It follows that $U_1^{\rho t} < U_1^t$ for any t. Combining the two inequalities we get $U_1^{\rho t} < U_1^t < U_2^t$ for $t \in [t_1, t_1^\rho]$. The utility of switchers decreases as ρ gets smaller.

We now turn to the effect of a change in ρ on the utility of non-switchers. From Equation (3) we can distinguish three components of the user utility (apart from the basic utility): content quality ($\zeta_i q$), disutility from advertisements (μa_i) and transportation costs (δ_i^2). For non-switchers, only the disutility from advertisements is affected by a change in ρ. We showed in Lemma 2 that both $a_1^*(\cdot)$ and $a_2^*(\cdot)$ increase with a decrease in ρ, which results in a reduction of the utility of non-switchers. It follows that both switchers and non-switchers are worse-off due to a decrease in parameter ρ.

We now consider the effect of an increase in ζ on the utility of switchers. Let t_1^ζ and t_2^ζ denote the locations of the marginal users and $U_i^{\zeta t}$ the utility of a user t choosing platform i after a change in parameter ζ. We showed in Lemma 3 that $n_1^*(\cdot)$ decreases in response to an increase of ζ, hence, $t_1^\zeta < t_1$ and $t_2^\zeta > t_2$. Due to symmetry, we restrict attention to switchers with locations $t \in [t_1^\zeta, t_1]$. A user switches if doing so increases his utility, hence, for $t \in [t_1^\zeta, t_1]$ we have $U_2^{\zeta t} > U_1^{\zeta t}$. We also know from Lemma 3 that $a_1^*(\cdot)$ decreases with an increase in ζ, so that $U_1^{\zeta t} > U_1^t$ holds for any t. Combining the two inequalities we get $U_2^{\zeta t} > U_1^{\zeta t} > U_1^t$ for $t \in [t_1^\zeta, t_1]$. The utility of switchers increases due to an increase of ζ.

We finally turn to the effect of a change in ζ on the utility of non-switchers. Non-switchers on platform 1 benefit from an increase in ζ as $a_1^*(\cdot)$ decreases in the new equilibrium. The utility of non-switchers at platform 2 also increases as

$$\frac{\partial U_2^{t*}}{\partial \zeta} = q - \mu \frac{\partial a_2^*(\cdot)}{\partial \zeta} = \frac{(1+2\mu)\left[1+\mu(1-\rho)\right]}{3\mu^2(1-\rho)+2\mu(2-\rho)+1} q > 0.$$

We get, hence, that non-switchers benefit from an increase in ζ. As switchers also benefit, we conclude that user surplus increases in response to an increase in parameter ζ. *Q.E.D.*

Proof of Proposition 4. Summing up advertiser surplus, user surplus and platforms' profits given in (10), (11) and (1), respectively, evaluated at equilibrium values, yields social welfare in equilibrium ($SW^*(\rho, \zeta, \mu, q, u)$). Taking derivative of $SW^*(\cdot)$ with respect to ρ and evaluating at $(\rho, \zeta) = (0, \zeta_0)$ gives

$$\left.\frac{\partial SW^*(\cdot)}{\partial \rho}\right|_{(\rho,\zeta)=(0,\zeta_0)} = -\frac{\left[4\mu\left(\mu+1\right)+3\right](\Delta_0 - \overline{\Delta})(\Delta_0 - \Delta_\rho)}{(1+3\mu)^3}. \tag{33}$$

The comparison of $\overline{\Delta}$ and Δ_ρ yields

$$\overline{\Delta} - \Delta_\rho = \frac{\mu\left(3\mu+1\right)\left[\mu - \left(1+\sqrt{7}\right)/2\right]\left[\mu - \left(1-\sqrt{7}\right)/2\right]}{\left(\mu+1\right)^2\left[3+4\mu(1+\mu)\right]},$$

such that $\overline{\Delta} > \Delta_\rho$ if $\mu > \left(1+\sqrt{7}\right)/2$ and $\overline{\Delta} \leq \Delta_\rho$ if $\mu \leq \left(1+\sqrt{7}\right)/2$. Consider first $\mu > \left(1+\sqrt{7}\right)/2$. It follows from (33) that $\partial SW^*(\cdot)/\partial\rho|_{(\rho,\zeta)=(0,\zeta_0)} > 0$ if $\Delta_0 > \Delta_\rho$. If $\mu \leq \left(1+\sqrt{7}\right)/2$, then there is no such Δ_0 for which $\partial SW^*(\cdot)/\partial\rho|_{(\rho,\zeta)=(0,\zeta_0)} > 0$ holds. We next take derivative of social welfare with respect to ζ and evaluate it at $\rho = 0$:

$$\left.\frac{\partial SW^*(\cdot)}{\partial \zeta}\right|_{\rho=0} = \frac{1}{2\Delta_\zeta}\left(\Delta_\zeta - \Delta\right) q.$$

Comparing $\overline{\Delta}$ and Δ_ζ we obtain

$$\overline{\Delta} - \Delta_\zeta = -\frac{(1+3\mu)\left[\mu - \left(1+\sqrt{7}\right)/2\right]\left[\mu - \left(1-\sqrt{7}\right)/2\right]}{2\left(\mu^3+7\mu^2+10\mu+4\right)},$$

such that $\overline{\Delta} > \Delta_\zeta$ if $\mu < \left(1+\sqrt{7}\right)/2$ and $\overline{\Delta} \leq \Delta_\zeta$ if $\mu \geq \left(1+\sqrt{7}\right)/2$. Consider first $\mu < \left(1+\sqrt{7}\right)/2$. Then $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} > 0$ if $\Delta < \Delta_\zeta$ and $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} \leq 0$ if $\Delta \geq \Delta_\zeta$. Assume first that $\Delta_0 < \Delta_\zeta$. Then with an increase in ζ (decrease in Δ) social welfare increases and the socially optimal amount of CP technology licensing implies $\zeta^* = 1 > \zeta_0$ such that private incentives are insufficient. Assume next that $\Delta_0 \geq \Delta_\zeta$. Then social welfare decreases with an increase in ζ (decrease in Δ) on the interval $\Delta > \Delta_\zeta$ and increases on the interval $\Delta < \Delta_\zeta$. The socially optimal amount of technology licensing implies $\zeta^* = 1 > \zeta_0$ if $SW^*(0,1,\cdot) > SW^*(0,\zeta_0,\cdot)$ and $\zeta^* = \zeta_0$ if $SW^*(0,1,\cdot) \leq SW^*(0,\zeta_0,\cdot)$. It follows that in the former case private incentives are insufficient, while they are optimal in the latter case. Consider finally $\mu \geq \left(1+\sqrt{7}\right)/2$. Then for any $\Delta \leq \Delta_0$ it holds that $\partial SW^*(\cdot)/\partial\zeta|_{\rho=0} > 0$ and socially optimal amount of technology licensing implies $\zeta^* = 1 > \zeta_0$ such that private incentives are insufficient. *Q.E.D.*

Proof of Proposition 5. *i)* We first derive the equilibrium slot prices. Maximizing profits in (14) with respect to advertisement prices and assuming interior solutions yields

$$\begin{aligned} p_1^{a*}(\rho,\zeta,\mu,q) &= \frac{[1+\mu(3-\rho)]\left[\Delta + \frac{4\mu^2(1-\rho)+\mu(5-3\rho)+1}{4(1+\mu(3-\rho))}\right]}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}, \\ p_2^{a*}(\rho,\zeta,\mu,q) &= \frac{(1-\rho)\left[1+\mu(3-2\rho)\right]\left(\overline{\Delta}_{p^a}-\Delta\right)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}. \end{aligned} \tag{34}$$

$p_1^{a*}(\cdot) > 0$ always holds, while $p_2^{a*}(\cdot) > 0$ if $\Delta < \overline{\Delta}_{p^a}$. The SOCs are fulfilled, with $\partial^2\pi_1(p_1^a, p_2^a; \cdot)/\partial(p_1^a)^2 = -\left[4\mu(1-\rho)+2\right]/\left[2\mu(2-\rho)+1\right] < 0$ and $\partial^2\pi_2(p_1^a, p_2^a; \cdot)/\partial(p_2^a)^2 = -\left[2(2\mu+1)\right]/\left[2\mu(2-\rho)+1\right] < 0$. Plugging $p_i^{a*}(\cdot)$ into (13) gives the equilibrium user market shares

$$\begin{aligned} n_1^*(\rho,\zeta,\mu,q) &= \frac{2\left[1+\mu(3-2\rho)\right]}{1+2\mu(2-\rho)} p_1^{a*}(\cdot), \\ n_2^*(\rho,\zeta,\mu,q) &= \frac{2\left[1+\mu(3-\rho)\right]}{\left[1+2\mu(2-\rho)\right](1-\rho)} p_2^{a*}(\cdot). \end{aligned}$$

By plugging $n_i^*(\cdot)$ and $p_i^{a*}(\cdot)$ into (12) we obtain the equilibrium numbers of advertisement slots

$$\begin{aligned} a_1^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu(1-\rho)}{1+2\mu(2-\rho)} p_1^{a*}(\cdot), \\ a_2^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu}{1+2\mu(2-\rho)} p_2^{a*}(\cdot). \end{aligned} \tag{35}$$

Finally, we plug $n_i^*(\cdot)$ and $p_i^{a*}(\cdot)$ into (14) to get the equilibrium profits

$$\begin{aligned} \pi_1^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu(1-\rho)}{1+2\mu(2-\rho)} \left[p_1^{a*}(\cdot)\right]^2, \\ \pi_2^*(\rho,\zeta,\mu,q) &= \frac{1+2\mu}{1+2\mu(2-\rho)} \left[p_2^{a*}(\cdot)\right]^2. \end{aligned} \tag{36}$$

Provided $p_2^{a*}(\cdot) > 0$, it holds that $n_i^*(\cdot), a_i^*(\cdot), \pi_i^*(\cdot) > 0$. Hence, both platforms are active on both sides of the market if $\Delta < \overline{\Delta}_{p^a}$. If $\Delta \geq \overline{\Delta}_{p^a}$, only the superior platform is active at the market.

ii) The comparison of equilibrium slot prices in (34) yields

$$p_1^{a*}(\cdot) - p_2^{a*}(\cdot) = \frac{4\Delta\left[2\mu\rho^2 + 6\mu(1-\rho) + (2-\rho)\right] + 2\mu\rho\left[2-\rho+2\mu(1-\rho)\right] + \rho}{4\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]} \geq 0,$$

holding with equality if $\rho = \Delta = 0$. By comparing the equilibrium numbers of advertisement slots in (35) we get

$$a_1^*(\cdot) - a_2^*(\cdot) = \frac{\Delta f(\mu,\rho) + g(\mu,\rho)}{[1+2\mu(2-\rho)]\,[3\mu^2(1-\rho)+2\mu(2-\rho)+1]}, \text{ with}$$
$$f(\mu,\rho) := 6\mu^2(1-\rho)(2-\rho) + 10\mu(1-\rho) + (2-\rho) + 2\mu\rho^2 > 0,$$
$$g(\mu,\rho) := \mu^2\rho(1-\rho)/2 + \mu\rho(2-\rho)/2 + \rho/4 > 0.$$

This implies that $a_1^*(\cdot) \geq a_2^*(\cdot)$, holding with equality if $\rho = \Delta = 0$. Inequalities $p_1^{a*}(\cdot) \geq p_2^{a*}(\cdot)$ and $a_1^*(\cdot) \geq a_2^*(\cdot)$ yield $\pi_1^*(\cdot) \geq \pi_2^*(\cdot)$, holding with equality if $\rho = \Delta = 0$. *Q.E.D.*

Proof of Lemma 4. Note that $\overline{\Delta}_{p^a}$ increases when ρ gets smaller. It follows that if ρ_0 and ζ_0 are such that $0 \leq \Delta_0 < \overline{\Delta}_{p^a}\big|_{\rho=\rho_0}$, then for any $\rho \leq \rho_0$ and $\zeta \geq \zeta_0$ the condition $0 \leq \Delta < \overline{\Delta}_{p^a}$ is fulfilled. *i)* We start with the effect of an AP technology licensing on the inferior platform. Taking derivative of $p_2^{a*}(\cdot)$ with respect to ρ yields

$$\frac{\partial p_2^{a*}(\cdot)}{\partial \rho} = -\frac{\eta(\mu,\rho)\,[\phi(\mu,\rho)/\eta(\mu,\rho) - \Delta]}{4\,[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2}, \text{ with}$$
$$\eta(\mu,\rho) := 24\mu^3(1-\rho)^2 + 4\mu^2(4+\sqrt{2}-2\rho)(4-\sqrt{2}-2\rho) + 4\mu(7-4\rho) + 4,$$
$$\phi(\mu,\rho) := 12\mu^4(1-\rho)^2 + 2\mu^3(1-\rho)(15-7\rho) +$$
$$+\,\mu^2(6+\sqrt{10}-2\rho)(6-\sqrt{10}-2\rho) + \mu(9-4\rho) + 1.$$

It holds that $\eta(\cdot), \phi(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$. Comparing $\phi(\cdot)/\eta(\cdot)$ and $\overline{\Delta}_{p^a}$ we get

$$\frac{\phi(\cdot)}{\eta(\cdot)} - \overline{\Delta}_{p^a} = \frac{\mu^3(1-\rho)\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]}{[1+\mu(3-2\rho)]\left[6\mu^3(1-\rho)^2 + \mu^2(4+\sqrt{2}-2\rho)(4-\sqrt{2}-2\rho)+\mu(7-4\rho)+1\right]},$$

such that $\phi(\cdot)/\eta(\cdot) > \overline{\Delta}_{p^a}$ for any $\mu > 0$ and $\rho \in [0,1)$, which implies that $\partial p_2^{a*}(\cdot)/\partial\rho < 0$ for any admissible parameters (μ,ρ,Δ, with $0 \leq \Delta < \overline{\Delta}_{p^a}$ and $\rho \in [0,1)$). Taking derivative of $a_2^*(\cdot)$ with respect to ρ yields

$$\frac{\partial a_2^*(\cdot)}{\partial \rho} = -\frac{(1+2\mu)\,\lambda(\mu,\rho)\,[\xi(\mu,\rho)/\lambda(\mu,\rho) - \Delta]}{4\,[1+2\mu(2-\rho)]^2\,[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2}, \text{ where}$$
$$\xi(\mu,\rho) := 24\mu^5(1-\rho)^2 + 2\mu^4(1-\rho)(35-19\rho) + 4\mu^3(11+\sqrt{21}-5\rho)(11-\sqrt{21}-5\rho)/5$$
$$+\,4\mu^2(4+\sqrt{5}-\rho)(4-\sqrt{5}-\rho) + \mu(11-4\rho) + 1,$$
$$\lambda(\mu,\rho) := 24\mu^4(1-\rho)^2 + 16\mu^3(2-\rho)(4-3\rho) + 16\mu^2(3+\sqrt{2}-\rho)(3-\sqrt{2}-\rho) + 4\mu(9-4\rho) + 4,$$

such that $\xi(\cdot), \lambda(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$. It follows that $\partial a_2^*(\cdot)/\partial\rho < 0$ provided that

$\Delta < \xi(\cdot)/\lambda(\cdot)$. The comparison of $\xi(\cdot)/\lambda(\cdot)$ and $\overline{\Delta}_{p^a}$ yields

$$\frac{\xi(\cdot)}{\lambda(\cdot)} - \overline{\Delta}_{p^a} = \frac{4\mu^3 (1-\rho)\, \sigma(\mu,\rho)}{\lambda(\mu,\rho)\, [1+\mu(3-2\rho)]}, \text{ where}$$

$$\sigma(\mu,\rho) := 6\mu^3(1-\rho)(2-\rho) + \mu^2(19+\sqrt{57}-8\rho)(19-\sqrt{57}-8\rho)/16 + 4\mu(2-\rho) + 1.$$

It holds that $\sigma(\cdot) > 0$ for any $\mu > 0$ and $\rho \in [0,1)$, hence, $\xi(\cdot)/\lambda(\cdot) > \overline{\Delta}_{p^a}$ and for any admissible parameters $\partial a_2^*(\cdot)/\partial\rho < 0$. Inequalities $\partial p_2^{a*}(\cdot)/\partial\rho < 0$ and $\partial a_2^*(\cdot)/\partial\rho < 0$ imply that $\partial\pi_2^*(\cdot)/\partial\rho < 0$.

We next consider the superior platform and start with $a_1^*(\cdot)$:

$$\frac{\partial a_1^*(\cdot)}{\partial\rho} = -\frac{1}{4}\frac{\mu\alpha(\mu,\rho)\,[\beta(\mu,\rho)/\alpha(\mu,\rho) - \Delta]}{[2\mu(2-\rho)+1]^2\,[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2}, \text{ where}$$

$$\alpha(\mu,\rho) := 48\mu^4(1-\rho)^2 + 16\mu^3(2-\rho)(2-3\rho) + 16\mu^2(1-\rho)(4-\rho) + 4\mu(7-4\rho) + 4,$$

$$\begin{aligned}\beta(\mu,\rho) := {} & 48\mu^5(1-\rho)^2 + 4\mu^4(1-\rho)(35-19\rho) + 4\mu^3(20+2\sqrt{19}-9\rho)(20-2\sqrt{19}-9\rho)/9 \\ & + \mu^2(11+\sqrt{57}-2\rho)(11-\sqrt{57}-2\rho) + \mu(13-4\rho) + 1,\end{aligned}$$

provided that $\alpha(\mu,\rho) \neq 0$. If $\alpha(\mu,\rho) = 0$, then

$$\frac{\partial a_1^*(\cdot)}{\partial\rho} = -\frac{1}{4}\frac{\mu\beta(\cdot)}{[2\mu(2-\rho)+1]^2\,[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2}.$$

For any $\mu > 0$ and $\rho \in [0,1)$ it holds that $\beta(\cdot) > 0$, hence, $\partial a_1^*(\cdot)/\partial\rho < 0$ if $\alpha(\mu,\rho) = 0$. Assume now that $\alpha(\mu,\rho) \neq 0$. The sign of $\alpha(\cdot)$ is ambiguous. The comparison of $\beta(\cdot)/\alpha(\cdot)$ and $\overline{\Delta}_{p^a}$ yields

$$\frac{\beta(\cdot)}{\alpha(\cdot)} - \overline{\Delta}_{p^a} = \frac{4\mu\tau(\mu,\rho)}{\alpha(\cdot)\,[1+\mu(3-2\rho)]}, \text{ where}$$

$$\begin{aligned}\tau(\mu,\rho) := {} & 12\mu^5(1-\rho)^2(2-\rho) + \mu^4(1-\rho)(43+\sqrt{129}-20\rho)(43-\sqrt{129}-20\rho)/20 \\ & + \mu^3(2-\rho)(13+\sqrt{65}-4\rho)(13-\sqrt{65}-4\rho)/2+ \\ & + \mu^2(53+\sqrt{265}-24\rho)(53-\sqrt{265}-24\rho)/48 + 6\mu(2-\rho) + 1.\end{aligned}$$

For any $\mu > 0$ and $\rho \in [0,1)$ it holds that $\tau(\cdot) > 0$. Depending on the sign of $\alpha(\cdot)$ two cases are possible. If $\alpha(\cdot) < 0$, then $\beta(\cdot)/\alpha(\cdot) < 0$ and $\alpha(\cdot)(\beta(\cdot)/\alpha(\cdot) - \Delta) > 0$, such that $\partial a_1^*(\cdot)/\partial\rho < 0$. If $\alpha(\cdot) > 0$, then $\beta(\cdot)/\alpha(\cdot) > \overline{\Delta}_{p^a}$ and for any $\Delta < \overline{\Delta}_{p^a}$ it holds $\alpha(\cdot)\,[\beta(\cdot)/\alpha(\cdot) - \Delta] > 0$, such that $\partial a_1^*(\cdot)/\partial\rho < 0$.

We next turn to the superior platform's price. Taking derivative of $p_1^{a*}(\cdot)$ with respect to ρ we get

$$\frac{\partial p_1^{a*}(\cdot)}{\partial\rho} = -\frac{\mu\,(1+2\mu)\,(1+3\mu)\,(\Delta_{p^a} - \Delta)}{[3\mu^2(1-\rho)+2\mu(2-\rho)+1]^2},$$

such that $\partial p_1^{a*}(\cdot)/\partial\rho < 0$ if $0 \leq \Delta < \Delta_{p^a}$. The comparison of Δ_{p^a} and $\overline{\Delta}_{p^a}$ yields

$$\overline{\Delta}_{p^a} - \Delta_{p^a} = \frac{\mu\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]}{(1+3\mu)\left[1+\mu(3-2\rho)\right]} > 0,$$

which holds for any $\mu > 0$ and $\rho \in [0,1)$. We conclude that if $\Delta < \overline{\Delta}_{p^a}$ ($\Delta \geq \overline{\Delta}_{p^a}$), then $\partial p_1^{a*}(\cdot)/\partial\rho < 0$ ($\partial p_1^{a*}(\cdot)/\partial\rho \geq 0$). Using the expression for $\pi_1^*(\cdot)$ in (36) and taking derivative of $\pi_1^*(\cdot)$ with respect to ρ we obtain

$$\frac{\partial\pi_1^*(\cdot)}{\partial\rho} = -\frac{4\mu^2\left[p_1^{a,*}(\cdot)\right]^2}{\left[1+2\mu(2-\rho)\right]^2} + \frac{2\left[1+2\mu(1-\rho)\right]p_1^{a*}(\cdot)}{1+2\mu(2-\rho)}\frac{\partial p_1^{a*}(\cdot)}{\partial\rho}.$$

It holds that $sign\left\{\partial\pi_1^*(\cdot)/\partial\rho\right\} = sign\left\{\left[\partial\pi_1^*(\cdot)/\partial\rho\right]/p_1^{a*}(\cdot)\right\}$. Dividing $\partial\pi_1^*(\cdot)/\partial\rho$ with $p_1^{a*}(\cdot)$ we get

$$\frac{\partial\pi_1^*(\cdot)/\partial\rho}{p_1^{a*}(\cdot)} = -\frac{\mu\theta(\cdot)\left[\Delta_\pi(\cdot)-\Delta\right]}{2\left[1+2\mu(2-\rho)\right]^2\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]^2}.$$

For any $\mu > 0$ and $\rho \in [0,1)$ it holds that $\theta(\cdot), \chi(\cdot) > 0$, implying $\Delta_\pi(\cdot) > 0$. We next compare $\Delta_\pi(\cdot)$ and $\overline{\Delta}_{p^a}$:

$$\Delta_\pi(\cdot) - \overline{\Delta}_{p^a} = -\frac{4\mu^3(1-\rho)\,\omega(\mu,\rho)}{\left[1+\mu(3-2\rho)\right]\theta(\mu,\rho)}, \text{ where}$$

$$\omega(\mu,\rho) := 6\mu^3(2-\rho)(1-\rho) + \mu^2(19+\sqrt{57}-8\rho)(19-\sqrt{57}-8\rho)/16 + 4\mu(2-\rho) + 1.$$

Note, that $\omega(\cdot)$ is positive for any $\mu > 0$ and $\rho \in [0,1)$, hence, $\Delta_\pi(\cdot) < \overline{\Delta}_{p^a}$. It follows that $\partial\pi_1^*(\cdot)/\partial\rho < 0$ ($\partial\pi_1^*(\cdot)/\partial\rho \geq 0$) if $\Delta < \Delta_\pi(\cdot)$ ($\Delta \geq \Delta_\pi(\cdot)$). We finally compare $\Delta_\pi(\cdot)$ and Δ_{p^a}:

$$\Delta_{p^a} - \Delta_\pi(\cdot) = -\frac{4\mu(1+2\mu)\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]^2}{(1+3\mu)\theta(\cdot)} < 0,$$

which holds for any $\mu > 0$ and $\rho \in [0,1)$.

ii) With $\Delta = (1-\zeta)q$, we have $\partial\Delta/\partial\zeta = -q$. It is straightforward that $\partial p_2^{a*}(\cdot)/\partial\zeta, \partial a_2^*(\cdot)/\partial\zeta > 0$, while $\partial p_1^{a*}(\cdot)/\partial\zeta, \partial a_1^*(\cdot)/\partial\zeta < 0$ for any $\rho \in [0,1)$ and $\zeta \in (0,1]$. This in turn implies that $\partial\pi_2^*(\cdot)/\partial\zeta > 0$ and $\partial\pi_1^*(\cdot)/\partial\zeta < 0$. *Q.E.D.*

Proof of Proposition 6. We showed in part *ii)* of the proof of Lemma 4 that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ for any $\rho \in [0,1)$ and $\zeta \in (0,1]$. The superior platform has no incentives to share its CP technology. *i)* We showed in part *i)* of the proof of Lemma 4 that $sign\left\{\partial\pi_1^*(\cdot)/\partial\rho\right\} = sign\left\{\Delta - \Delta_\pi(\cdot)\right\}$. Taking derivative of $\Delta_\pi(\cdot)$ with respect to ρ yields

$$\frac{\partial\Delta_\pi(\cdot)}{\partial\rho} = -\frac{96\mu^5(1-\rho)(2\mu+1)^2\left[3\mu^2(1-\rho)+2\mu(2-\rho)+1\right]}{\left[\theta(\cdot)\right]^2} < 0.$$

If $\Delta_0 < \Delta_\pi(\cdot, \rho_0)$, then due to $\partial\Delta_\pi(\cdot)/\partial\rho < 0$ for any $\rho \in [0, \rho_0]$ it holds that $\Delta_0 < \Delta_\pi(\cdot, \rho_0) \leqslant \Delta_\pi(\cdot)$, which implies that $\Delta_0 - \Delta_\pi(\cdot) < 0$ and $\partial\pi_1^*(\cdot)/\partial\rho < 0$. Hence, on the interval $\rho \in [0, \rho_0]$, $\pi_1^*(\cdot)$ increases with a decreases in ρ and the superior platform licenses fully its AP technology, so that $\rho^* = 0$. Note finally that with $\partial\Delta_\pi(\cdot)/\partial\rho < 0$ it holds that $\Delta_\pi(\cdot, \rho_0) < \Delta_\pi(\cdot, 0)$.

ii) Assume now that $\Delta_\pi(\cdot, \rho_0) < \Delta_0 < \Delta_\pi(\cdot, 0)$ and let $\widehat{\rho}$ be such that $\Delta_\pi(\cdot, \widehat{\rho}) = \Delta_0$. It must be that $0 < \widehat{\rho} < \rho_0$. Then for any $\rho \in [\widehat{\rho}, \rho_0]$ it holds that $\Delta_0 \geq \Delta_\pi(\cdot)$, hence, $\pi_1^*(\cdot)$ (weakly) increases in ρ on the interval $\rho \in [\widehat{\rho}, \rho_0]$. However, for any $\rho \in [0, \widehat{\rho})$ it holds that $\Delta_0 < \Delta_\pi(\cdot)$, hence, $\pi_1^*(\cdot)$ decreases in ρ on the interval $\rho \in [0, \widehat{\rho})$. Then depending on the relation between $\pi_1^*(\rho = 0, \cdot)$ and $\pi_1^*(\rho = \rho_0, \cdot)$ two cases are possible. If $\pi_1^*(\rho = 0, \cdot) > \pi_1^*(\rho = \rho_0, \cdot)$, AP technology is fully licensed, while there is no licensing if $\pi_1^*(\rho = 0, \cdot) \leq \pi_1^*(\rho = \rho_0, \cdot)$.

iii) Assume finally that $\Delta_\pi(\cdot, \rho_0) < \Delta_\pi(\cdot, 0) < \Delta_0$. As $\partial\Delta_\pi(\cdot)/\partial\rho < 0$, for any $\rho \in [0, \rho_0]$ it holds that $\Delta_\pi(\cdot) < \Delta_0$ and $\partial\pi_1^*(\cdot)/\partial\rho < 0$, no licensing then takes place. *Q.E.D.*

Proof of Proposition 7. Maximizing profits with respect to the number of advertisements and user prices yields the following FOCs

$$n_1^* - 2\mu\left(a_1^* + p_1^{u*}\right) - 2a_1^* \leq 0,\ a_1^* \left.\frac{\partial\pi_1(\cdot)}{\partial a_1}\right|_{a_1^*, p_1^{u*}, a_2^*, p_2^{u*}} = 0, \tag{37}$$

$$n_1^* - 2\left(p_1^{u*} + a_1^*\right) \leq 0,\ p_1^{u*} \left.\frac{\partial\pi_1(\cdot)}{\partial p_1^u}\right|_{a_1^*, p_1^{u*}, a_2^*, p_2^{u*}} = 0, \tag{38}$$

$$(1-\rho)\left(n_2^* - 2\mu a_2^*\right) - 2\left(\mu p_2^{u*} + a_2^*\right) \leq 0,\ a_2^* \left.\frac{\partial\pi_2(\cdot)}{\partial a_2}\right|_{a_1^*, p_1^{u*}, a_2^*, p_2^{u*}} = 0, \tag{39}$$

$$n_2^* - 2p_2^{u*} - 2(1-\rho)a_2^* \leq 0,\ p_2^{u*} \left.\frac{\partial\pi_2(\cdot)}{\partial p_2^u}\right|_{a_1^*, p_1^{u*}, a_2^*, p_2^{u*}} = 0. \tag{40}$$

We first show that there is no equilibrium with $p_1^{u*} = 0$. To see this, assume that there is an equilibrium with $p_1^{u*} = 0$. Condition (38) implies that $n_1^* - 2a_1^* \leq 0$. Assume that in this equilibrium $a_1^* > 0$. For any $a_1^* > 0$ it holds that $n_1^* - 2\mu a_1^* - 2a_1^* < n_1^* - 2a_1^*$, yielding $n_1^* - 2\mu a_1^* - 2a_1^* < 0$. In this case Condition (37) requires $a_1^* = 0$, which is a contradiction. Hence, if $p_1^{u*} = 0$, then $a_1^* = 0$, leading to zero profits for the superior platform, which cannot be in equilibrium.

We next show that $\mu \geq 1$ implies $a_1^* = 0$, while $\mu < 1$ implies $a_1^* > 0$. Assume $a_1^* > 0$. Condition (37) requires that $n_1^* - 2\mu\left(a_1^* + p_1^{u*}\right) - 2a_1^* = 0$. As $p_1^{u*} > 0$, Condition (38) implies $n_1^* - 2\left(p_1^{u*} + a_1^*\right) = 0$. From the latter equalities we get $p_1^{u*}(1-\mu) = \mu a_1^*$, such that $a_1^* > 0$ if and only if $\mu < 1$ (as $p_1^{u*} > 0$). Assume next that $a_1^* = 0$. As $p_1^{u*} > 0$, Condition (38) then yields $p_1^{u*} = n_1^*/2$, which we plug into the inequality of Condition (37) to get $(1-\mu)n_1^* \leq 0$. The latter inequality is fulfilled if and only if $\mu \geq 1$ (as $p_1^{u*} > 0$ implies $n_1^* > 0$).

We now turn to the inferior platform and show that if it is active on the user side, then it places advertisements if and only if $\mu < 1-\rho$. Note that if $p_2^{u*} = 0$, then $a_2^* = 0$, and if $p_2^{u*} > 0$, then $a_2^* \geq 0$. Assume an equilibrium with $p_2^{u*} > 0$ and $a_2^* = 0$. From Condition (40) we get $p_2^{u*} = n_2^*/2$, which implies that $n_2^* > 0$. Plugging p_2^{u*} into the inequality of Condition (39) yields $(1-\rho-\mu)n_2^* \leq 0$, which holds if and only if $\mu \geq 1-\rho$ (as $n_2^* > 0$). Assume an equilibrium where $p_2^{u*}, a_2^* > 0$. Conditions (39) and (40) yield $p_2^{u*}(1-\rho-\mu) = a_2^*\left[1-(1-\rho)(1-\rho-\mu)\right]$. $p_2^{u*} > 0$ and $a_2^* > 0$ can hold together only if and only if $\mu < 1-\rho$.

It follows that depending on μ the following six equilibria are possible:

Case 1a) $\mu \geq 1$: $a_1^* = 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 1b) $\mu \geq 1$: $a_1^* = 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} > 0$.

Case 2a) $1-\rho \leq \mu < 1$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 2b) $1-\rho \leq \mu < 1$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} > 0$.

Case 3a) $\mu < 1-\rho$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* = 0$, $p_2^{u*} = 0$.

Case 3b) $\mu < 1-\rho$: $a_1^* > 0$, $p_1^{u*} > 0$, $a_2^* > 0$, $p_2^{u*} > 0$.

We first assume $\mu \geq 1$ and consider *Case 1b*. Plugging $a_1^* = 0$ and $a_2^* = 0$ into Conditions (38) and (40) yields $p_1^{u*} = n_1^*/2$ and $p_2^{u*} = n_2^*/2$, respectively. Plugging p_1^{u*} and p_2^{u*} into Equation (16) and using $n_1^* + n_2^* = 1$ yields $n_1^*(\zeta, q) = 1/2 + 2\Delta/3$. Moreover, $n_2^*(\zeta, q) = 1/2 - 2\Delta/3$, $p_1^{u*}(\zeta, q) = 1/4 + \Delta/3$ and $p_2^{u*}(\zeta, q) = 1/4 - \Delta/3$. It holds that $n_1^*(\cdot) < 1$ and $p_2^{u*}(\cdot) > 0$ if $\Delta < 3/4$. Plugging the equilibrium values into Conditions (37) and (39) yields $(1/2+2\Delta/3)(1-\mu) \leq 0$ and $(1/2-2\Delta/3)(1-\rho-\mu) \leq 0$, which are fulfilled if $\Delta < 3/4$. It is straightforward to show that if $\Delta \geq 3/4$, the equilibrium in *Case 1a* applies, with $p_1^{u*} = \Delta - 1/4$ and $n_1^* = 1$.

We next consider $1-\rho \leq \mu < 1$ and focus on the equilibrium in *Case 2b*. In this equilibrium the weak inequalities in Conditions (37), (38) and (40) hold as equalities. Together with Equation (16) they imply that $a_1^*(\zeta, \mu, q) = (1-\mu)(3+4\Delta)/\left[8+4\mu(2-\mu)\right]$, $p_1^{u*}(\zeta, \mu, q) = \mu(3+4\Delta)/\left[8+4\mu(2-\mu)\right]$, $p_2^{u*}(\zeta, \mu, q) = \left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right]$, $n_1^*(\zeta, \mu, q) = (4\Delta+3)/\left[4+2\mu(2-\mu)\right]$ and $n_2^*(\zeta, \mu, q) = 2\left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right]$. It holds that $a_1^*(\cdot), p_1^{u*}(\cdot), n_1^*(\cdot) > 0$, while $p_2^{u*}(\cdot), n_2^*(\cdot) > 0$ if $\Delta < \overline{\Delta}_{p^u}$. By plugging $n_2^*(\cdot)$ and $p_2^{u*}(\cdot)$ into the inequality of Condition (39) we obtain $2(1-\mu-\rho)\left(\overline{\Delta}_{p^u} - \Delta\right)/\left[2+\mu(2-\mu)\right] \leq 0$, which holds if $\Delta < \overline{\Delta}_{p^u}$. It is straightforward to check that if $\Delta \geq \overline{\Delta}_{p^u}$, then only the superior platform is active on both sides of the market and we have the equilibrium of *Case 2a*, with $p_1^{u*}(\zeta,\mu,q) = (4\Delta-1)/\left[4(2-\mu)\right]$, $a_1^*(\zeta,\mu,q) = (4\Delta-1)(1-\mu)/\left[4\mu(2-\mu)\right]$ and $n_1^* = 1$.

We finally turn to $\mu < 1-\rho$ and focus on *Case 3b*. In this equilibrium the weak inequalities in all the FOCs hold as equalities. From the FOCs and Equation (16) we get the equilibrium values

$$\begin{aligned}
p_1^{u*}(\rho,\zeta,\mu,q) &= \frac{2\mu\left(\Delta+\overline{\Delta}_{p^u}\right)-\mu\rho\left[\rho+2(\mu-1)\right]}{2\left[1+2\mu(2-\mu)\right]-2\rho\left[\rho+2(\mu-1)\right]}, \\
a_1^*(\rho,\zeta,\mu,q) &= \frac{(1-\mu)\left[2\left(\Delta+\overline{\Delta}_{p^u}\right)-\rho(\rho+2(\mu-1))\right]}{2\left[1+2\mu(2-\mu)\right]-2\rho\left[\rho+2(\mu-1)\right]}, \\
p_2^{u*}(\rho,\zeta,\mu,q) &= \frac{\left[\rho(2-\rho)+\mu(1-\rho)\right]\left(\overline{\Delta}_{p^u}-\Delta\right)}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]}, \\
a_2^*(\rho,\zeta,\mu,q) &= \frac{(1-\rho-\mu)\left(\overline{\Delta}_{p^u}-\Delta\right)}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]}.
\end{aligned} \tag{41}$$

Note that $\rho+2(\mu-1)<0$ provided $\mu<1-\rho$. Hence, $p_1^{u,*}(\cdot), a_1^*(\cdot)>0$. Moreover, $p_2^{u,*}(\cdot), a_2^*(\cdot)>0$ if $0\le\Delta<\overline{\Delta}_{p^u}$. We next compute platforms' equilibrium market shares among users. Plugging equilibrium values into Equations (16) and (17) yields

$$\begin{aligned}
n_1^*(\rho,\zeta,\mu,q) &= \frac{2\left(\Delta+\overline{\Delta}_{p^u}\right)-\rho\left[\rho+2(\mu-1)\right]}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]}, \\
n_2^*(\rho,\zeta,\mu,q) &= \frac{2\left(\overline{\Delta}_{p^u}-\Delta\right)}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]}.
\end{aligned} \tag{42}$$

$n_1^*(\cdot)>0$, while $n_2^*(\cdot)>0$ if $0\le\Delta<\overline{\Delta}_{p^u}$. Hence, if $0\le\Delta<\overline{\Delta}_{p^u}$, then both platforms are active on both sides of the market. Platforms realize profits $\pi_i^*(\rho,\zeta,\mu,q)$:

$$\begin{aligned}
\pi_1^*(\cdot) &= \frac{\left[2\left(\Delta+\overline{\Delta}_{p^u}\right)-\rho(\rho+2(\mu-1))\right]^2\left[1+\mu(2-\mu)\right]}{4\left[2-(1-\rho-\mu)^2+\mu(2-\mu)\right]^2}, \\
\pi_2^*(\cdot) &= \frac{\left(\Delta-\overline{\Delta}_{p^u}\right)^2\left[2-(1-\rho-\mu)^2\right]}{\left[2-(1-\rho-\mu)^2+\mu(2-\mu)\right]^2}.
\end{aligned}$$

It is straightforward to check that if $\Delta\ge\overline{\Delta}_{p^u}$, then the equilibrium of *Case 3a* emerges, with $p_1^{u*}(\zeta,\mu,q) = (4\Delta-1)/\left[4(2-\mu)\right]$, $a_1^*(\zeta,\mu,q) = (4\Delta-1)(1-\mu)/\left[4\mu(2-\mu)\right]$ and $n_1^*=1$. *Q.E.D.*

Proof of Lemma 5. Note first that if $0\le\rho_0<1-\mu$, then $1-\mu-\rho>0$ holds for any $\rho\le\rho_0$. Also, if $0\le\Delta_0<\overline{\Delta}_{p^u}$, then any $\zeta>\zeta_0$ also fulfills $\Delta<\overline{\Delta}_{p^u}$. i) We start with the superior platform. Taking derivatives of $p_1^{u*}(\cdot)$, $a_1^*(\cdot)$ and $n_1^*(\cdot)$ stated in (41) and (42) in the Proof of

Proposition 7 with respect to ρ yields

$$\frac{\partial p_1^{u*}(\cdot)}{\partial\rho} = \frac{2\mu(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0, \tag{43}$$
$$\frac{\partial a_1^{*}(\cdot)}{\partial\rho} = \frac{2(1-\mu)(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0,$$
$$\frac{\partial n_1^{*}(\cdot)}{\partial\rho} = \frac{4(1-\mu-\rho)(\overline{\Delta}_{p^u}-\Delta)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0.$$

By plugging $n_1^*(\cdot)$ and $a_1^*(\cdot)$ into Expression (2) we get the equilibrium slot price

$$p_1^{a*}(\rho,\zeta,\mu,q) = \frac{(\mu+1)\left[2(\Delta+\overline{\Delta}_{p^u})-\rho(\rho+2(\mu-1))\right]}{2\left[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))\right]}.$$

We take derivative of $p_1^{a*}(\cdot)$ with respect to ρ to obtain

$$\frac{\partial p_1^{a*}(\cdot)}{\partial\rho} = \frac{2\,(\mu+1)\,(1-\mu-\rho)\left(\overline{\Delta}_{p^u}-\Delta\right)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} > 0.$$

From $\partial p_1^{u*}(\cdot)/\partial\rho, \partial n_1^*(\cdot)/\partial\rho, \partial p_1^{a*}(\cdot)/\partial\rho, \partial a_1^*(\cdot)/\partial\rho > 0$ it is immediate that $\partial\pi_1^*(\cdot)/\partial\rho > 0$.

We next turn to the inferior platform. Taking derivative of $a_2^*(\cdot)$ in (41) with respect to ρ yields

$$\frac{\partial a_2^{*}(\cdot)}{\partial\rho} = -\frac{\left[2(\mu\rho+1)+(1-\rho)^2\right]\left(\overline{\Delta}_{p^u}-\Delta\right)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} < 0.$$

As the market is covered, $\partial n_1^*(\cdot)/\partial\rho > 0$ implies $\partial n_2^*(\cdot)/\partial\rho < 0$. To derive the change in $p_2^{u*}(\cdot)$ we consider the derivative of Expression (17) evaluated at equilibrium values with respect to ρ:

$$\frac{\partial n_2^{*}(\cdot)}{\partial\rho} = 2\frac{\partial p_1^{u*}(\cdot)}{\partial\rho} - 2\frac{\partial p_2^{u*}(\cdot)}{\partial\rho} + 2\mu\frac{\partial a_1^{*}(\cdot)}{\partial\rho} - 2\mu\frac{\partial a_2^{*}(\cdot)}{\partial\rho}.$$

As $\partial p_1^{u*}(\cdot)/\partial\rho > 0$, $\partial a_1^*(\cdot)/\partial\rho > 0$, $a_2^*(\cdot)/\partial\rho < 0$ and $\partial n_2^*(\cdot)/\partial\rho < 0$, it must hold that $\partial p_2^{u*}(\cdot)/\partial\rho > 0$. Plugging $n_2^*(\cdot)$ and $a_2^*(\cdot)$ into Expression (2) yields the price of an advertisement slot at platform 2:

$$p_2^{a*}(\rho,\zeta,\mu,q) = \frac{(1+\mu-\rho)\left(\overline{\Delta}_{p^u}-\Delta\right)}{1+2\mu(2-\mu)-\rho\left[\rho+2(\mu-1)\right]} > 0.$$

The derivative of $p_2^{a*}(\cdot)$ with respect to ρ is

$$\frac{\partial p_2^{a*}(\cdot)}{\partial\rho} = -\frac{\left(\overline{\Delta}_{p^u}-\Delta\right)\left[4\mu(1-\mu)+2(1-\mu\rho)+(1-\rho)^2\right]}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^2} < 0.$$

We also obtain

$$\frac{\partial \pi_2^{*}(\cdot)}{\partial\rho} = -\frac{2\left(\overline{\Delta}_{p^u}-\Delta\right)^2 l(\mu,\rho)}{[1+2\mu(2-\mu)-\rho(\rho+2(\mu-1))]^3},$$

where $l(\mu,\rho) := \rho^3 - 3\rho^2(1-\mu) + \rho(2\mu^2 - 4\mu + 1) + 1 - \mu$. We next show that $l(\cdot) > 0$. Taking derivative of $l(\cdot)$ with respect to μ yields $\partial l(\cdot)/\partial\mu = -4\rho(1-\mu-\rho) - 1 - \rho^2 < 0$. Hence, for any μ such that $\mu < 1-\rho$ it holds that $l(\cdot) > \lim_{\mu\to 1-\rho} l(\cdot) = 0$. It follows that $\partial\pi_2^*(\cdot)/\partial\rho < 0$.

We now turn to the effect of AP technology licensing on joint profits. The derivative of joint profits with respect to ρ is

$$\frac{\partial\pi_1^*(\cdot)}{\partial\rho} + \frac{\partial\pi_2^*(\cdot)}{\partial\rho} = \frac{(1-\mu-\rho)\left(\overline{\Delta}_{p^u} - \Delta\right)h(\mu,\rho)}{2\left[2-(1-\rho-\mu)^2 + \mu(2-\mu)\right]^3},$$

where $h(\mu,\rho) := 4\Delta\left[3 + 2\mu(2-\mu) + \rho(1-\rho-\mu) + \rho(1-\mu)\right] + k(\mu,\rho)$ and $k(\mu,\rho) := 4\mu^4 + 4\mu^3\rho - 16\mu^3 + 2\mu^2\rho^2 - 12\mu^2\rho + 12\mu^2 - 4\mu\rho^2 + 2\mu\rho + 8\mu - 3\rho^2 + 6\rho + 1$. Taking derivative of $k(\cdot)$ with respect to ρ yields $\partial k(\cdot)/\partial\rho = 2(1-\mu-\rho)\left[3 + 2\mu(2-\mu)\right] > 0$. Hence, for any $\mu < 1-\rho$ and ρ it holds that $k(\cdot) > \lim_{\rho\to 0} k(\cdot) = 4\mu^4 + 12\mu^2(1-\mu) + 4\mu(2-\mu^2) + 1 > 0$. It follows that $h(\cdot) > 0$. Consequently, $\sum_i \partial\pi_i^*(\cdot)/\partial\rho > 0$.

ii) It is straightforward that $\partial p_i^{u*}(\cdot)/\partial\zeta$, $\partial a_i^*(\cdot)/\partial\zeta$, $\partial p_i^{a*}(\cdot)/\partial\zeta$, $\partial n_i^*(\cdot)/\partial\zeta$ are negative (positive) for $i = 1$ ($i = 2$). This implies that $\partial\pi_1^*(\cdot)/\partial\zeta < 0$ and $\partial\pi_2^*(\cdot)/\partial\zeta > 0$. We now consider how platforms' joint profits change:

$$\frac{\partial\pi_1^*(\cdot)}{\partial\zeta} + \frac{\partial\pi_2^*(\cdot)}{\partial\zeta} = -\left[\frac{4\Delta\left[(2-\rho^2) + 2\mu(2-\mu) + 2\rho(1-\mu)\right] + \rho\left[(1-\rho-\mu) + (1-\mu)\right]}{2\left(1 + 2\mu(2-\mu) - \rho(\rho + 2(\mu-1))\right)^2}\right] q \leq 0,$$

holding with equality if $\Delta = \rho = 0$. As ζ cannot be further increased if $\Delta = 0$, we have $\sum_i \partial\pi_i^*(\cdot)/\partial\zeta < 0$. *Q.E.D.*

References

Anderson, S. P., S. Coate, 2005, "Market Provision of Broadcasting: A Welfare Analysis," *Review of Economic Studies*, 72, 947-972.

Armstrong, M., 2006, "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 668-691.

Crampes, C., C. Haritchabalet and B. Jullien, 2009, "Advertising, Competition and Entry in Media Industries," *Journal of Industrial Economics*, 57, 7-31.

DoJ, 2008, "Yahoo! Inc. and Google Inc. Abandon Their Advertising Agreement," Press Release, November 5, 2008. http://www.usdoj.gov/opa/pr/2008/November/08-at-981.html

Evans, D. S., 2008, "The Economics of the Online Advertising Industry," *Review of Network Economics*, 7, 359-391.

Farrell, J., N. T. Gallini, 1988, "Second-Sourcing as a Commitment: Monopoly Incentives to Attract Competition," *The Quarterly Journal of Economics*, 103, 673-694.

Gal-Or, E., A. Dukes, 2003, "Minimum Differentiation in Commercial Media Markets," *Journal of Economics & Management Strategy*, 12, 291-325.

Peitz, M., T. Valletti, 2008, "Advertising and Content in Media: Pay-TV versus Free-to-Air," *International Journal of Industrial Organization,* 26, 949-965.

Rochet, J.-C., J. Tirole, 2003, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1, 990-1029.

Shepard, A., 1987, "Licensing to Enhance Demand for New Technologies," *The RAND Journal of Economics*, 18, 360-368.

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