An Equilibrium Analysis of Efficiency Gains from Mergers

Dragan Jovanovic, Christian Wey

July 2012
The working papers published in the Series constitute work in progress circulated to stimulate discussion and critical comments. Views expressed represent exclusively the authors’ own opinions and do not necessarily reflect those of the editor.
An Equilibrium Analysis of Efficiency Gains from Mergers*

Dragan Jovanovic† Christian Wey‡

July 2012

Abstract

We analyze the efficiency defense in merger control. First, we show that the relationship between exogenous efficiency gains and social welfare can be non-monotone. Second, we consider both endogenous mergers and endogenous efficiencies and find that merger proposals are largely aligned with a proper social welfare analysis which explicitly considers the without merger counterfactual. We demonstrate that the merger specificity requirement does not help much to select socially desirable mergers; to the contrary, it may frustrate desirable mergers inducing firms not to claim efficiencies at all.

JEL-Classification: K21, L13, L41

Keywords: Horizontal Mergers, Efficiency Defense, Merger Specific Efficiencies

---

*We thank Michael Coenen, Tomaso Duso, Justus Haucap, Kai-Uwe Kühn, John E. Kwoka, Stephen Martin, Hans-Theo Normann, Armin Schmutzler, Marius Schwartz, Yossi Spiegel, Florian Szcüz, Achim Wambach, Tobias Wenzel, and seminar participants at the CRESSE 2011 and EARIE 2011 conferences for helpful comments. We gratefully acknowledge financial support by the German Science Foundation for the research project “Market Power in Vertically Related Markets” (grant WE 4228/2-1).

†Heinrich-Heine Universität Düsseldorf, Düsseldorf Institute for Competition Economics (DICE); E-mail: jovanovic@dice.hhu.de.

‡Heinrich-Heine Universität Düsseldorf, Düsseldorf Institute for Competition Economics (DICE); E-mail: wey@dice.hhu.de.
1 Introduction

Since Williamson’s (1968) contribution on the “welfare trade-offs,” the efficiency defense has been praised by economists as an essential element of merger control.\(^1\) In the US, the horizontal merger guidelines of the FTC and the DOJ, as amended in April 1997, state that “the primary benefits of mergers to the economy is their potential to generate efficiencies.” Yet, there is not much evidence that the efficiency defense has been a success story.\(^2\) Röller (2011) provides a survey of the recent EU merger decisions and concludes that “efficiencies have not played a major role in phase II EU merger evaluations since 2004.” He examines all phase II merger cases since May 2004, when the new EU merger guidelines came into force. Only in 5 out of 37 cases efficiencies were claimed.\(^3\) The Commission accepted efficiencies only in two cases, while they were never critical for the final decision.\(^4\)

A correct assessment of efficiencies should be based on a comparison of what would happen with and without a merger.\(^5\) The analysis of the without merger scenario becomes an issue when claimed efficiencies do not qualify as synergies.\(^6\) No-synergy efficiencies are by definition also realizable without a merger through so-called internal growth. Quite intuitively, one can expect a merger to be socially undesirable if claimed efficiencies are likely to be realized without the

---

\(^1\)See for the EU, for instance, Ilzkovitz and Meiklejohn (2003). Efficiencies were introduced into the US Merger Guidelines in 1997 (Section 4) and into the European Merger Guidelines in 2004 (EC Horizontal Merger Guidelines, 2004/03, Article 77).

\(^2\)See Röller, Stennek, and Verboven (2001), and Camesasca (1999) who report that the US authorities have been very reluctant to take efficiencies into account. Yet, federal courts did so, though most times the defence was either not critical or was rejected.

\(^3\)See also Veugelers (2012) who summarizes how the EU Commission has treated efficiency claims since 2004.

\(^4\)Both practical and economic reasons have been put forward for that observation (see Motta and Vasconcelos, 2005). Specifically, Röller (2011) refers to the “lawyers’ argument” that the merging parties may run into danger of signalling a “weak” (i.e., an anticompetitive) case when claiming efficiencies.

\(^5\)See Farrell and Shapiro (2001) who emphasize that the comparison of the with merger and without merger cases is critical for the assessment of efficiencies which are not synergies. A related issue arises for the falling firm defence (see Davis and Cooper, 2010).

\(^6\)According to Farrell and Shapiro (1990) synergies are the result of the joint use of merging firms’ specific assets (see also Farrell and Shapiro, 2001). As a consequence, synergies by definition cannot be realized without a merger.
merger taking place. That kind of reasoning is mirrored in competition law in the requirement that claimed efficiencies must be *merger specific*.\(^7\)\(^8\) The merger specificity rule adds a new counterfactual: in addition, to the standard pre-merger/post-merger comparison, it requires to ask whether efficiencies are more likely to be implemented with or without the merger.\(^9\)

The relevance of the question of what would happen without the merger in practical merger control was made explicit in Judge Sporkin’s statement in *FTC v. Cardinal Health* where a pair of mergers among the four leading drug wholesalers were at stake (Farrell and Shapiro, 2001, p. 688): “Judge Sporkin accepted the defendants’ assertion that the merger would lead to significant efficiencies, but went on to say: ‘However, this Court finds that evidence presented by the FTC strongly suggests that much of the savings anticipated from the merger could also be achieved through continued competition in the wholesale industry. While it must be conceded that the merger would likely yield the cost savings more immediately, the history of the industry over the past ten years demonstrates the power of competition to lower cost structures and garner efficiencies as well’.”

Our analysis highlights the unilateral effects of a merger when both the merger decision and the efficiency are endogenous. Realization of an efficiency gain depends on the adoption of an efficiency enhancing technology. Implementation requires to incur fixed adoption costs.\(^10\) We

\(^7\)For example, the US merger guidelines define merger specific efficiencies as follows: “The Agencies credit only those efficiencies likely to be accomplished with the proposed merger and unlikely to be accomplished in the absence of either the proposed merger or another means having comparable anticompetitive effects. These are termed merger-specific efficiencies.”

\(^8\)In addition to efficiencies being merger specific, they must also be classified as verifiable and beneficial to consumers (both latter requirements define so-called “cognizable” efficiencies). If these three criteria are cumulatively met, then a claimed efficiency will be accepted by antitrust authorities according to both the EU and the US merger guidelines. Since we do not focus on issues concerning verifiability, we will assume that this criterion is always met.

\(^9\)We follow Farrell and Shapiro’s (2001) view that changing market environments and technological progress make a forward looking without merger analysis necessary to take all relevant information into account. By that, we also reject Hausman and Leonard’s (1999) position which declares the without merger counterfactual as irrelevant. Their assertion depends on restricting the time frame of competition analysis such that the without merger scenario boils down to the pre-merger situation.

\(^10\)Our modelling approach mirrors the idea that scale economies are the most typical no-synergy efficiency (Farrell and Shapiro, 2001). Yet, the realization of scale economies is not “automatic.” Instead, it requires to
focus on mergers among firms which face large rival firms. We cover both catch-up mergers and mergers to dominance. Precisely, we consider an industry with four firms, two “dominant” firms and two “non-dominant” firms. The non-dominant firms decide in the first stage of the game whether or not to merge their businesses. The second stage highlights the adoption of an efficiency enhancing technology. The efficiency can be implemented both with a merger and without a merger. In the latter case, an adoption game is considered in which the merging candidates simultaneously and non-cooperatively decide about the adoption of the efficiency. We analyze both pure strategy and mixed strategy equilibria. Finally, in the third stage of the game, all firms compete à la Cournot in the product market.

We proceed in three steps. First, we treat efficiencies as exogenous; that is, a certain efficiency arises as a result of the merger while it cannot be realized without a merger. Our analysis reveals the existence of a non-monotone relationship between efficiencies and social welfare. This result critically depends on considering a catch-up merger among relatively small firms. In that case, a merger reduces social welfare whenever efficiencies are moderate, i.e., neither too small nor too large. Such a catch-up merger lowers overall productive efficiency which is not compensated by a large enough price reduction. That result stands in contrast to works which propose a monotone relationship between social welfare and efficiencies. Similarly, it contradicts the practice of competition authorities to trade off a merger’s concentration effect with its efficiencies.

---

11We assume that the sum of the pre-merger market shares of the merger candidates does not exceed 50 percent. Constraining the market share of the merging firms to 50 percent allows us to highlight new results for mergers between relatively small firms. We note that our results for the symmetric case (all firms have the same pre-merger market share) also carry over (qualitatively) to the case where the merging firms have larger pre-merger market shares than their competitors.

12In the former case, the initially largest firms remain larger than the merged firm. In the latter case, it is the merged firm which becomes larger than each of the rival firms.

13Note that exogenous efficiencies are equivalent to synergies.

14For mergers between relatively large firms, we obtain the standard finding that efficiencies must be large enough for social welfare to increase.

15See, for instance, Williamson, (1968), Farrell and Shapiro (1990), and Besanko and Spulber (1993).

16For instance, the US Horizontal Merger Guidelines (2010, Section 10) state: “In the Agencies’ experience, efficiencies are most likely to make a difference in merger analysis when the likely adverse competitive effects,
In the second step of our analysis, we solve the entire three-stage game where the adoption of the efficiency is possible both with and without the merger (“endogenous efficiencies”). Our main concern is whether or not the self-selection process behind endogenous merger proposals is aligned with a social welfare analysis which explicitly accounts for the counterfactual in the no-merger case. The assessment critically depends on the equilibrium of the adoption game in the no-merger case which can be in pure and mixed strategies. In the former case, any (endogenous) catch-up merger turns out to increase social welfare. Considering mergers to dominance, the assessment depends on the number of firms adopting the efficiency in the without merger counterfactual. If only a single firm would adopt the efficiency without a merger, then efficiencies must be sufficiently large to make a merger proposal socially desirable. If, however, both firms adopt, then -to the contrary- efficiencies have to be sufficiently small. This result mirrors the intuition that a merger should reduce social welfare if the firms adopt the efficiency anyway. Turning to the mixed strategy equilibrium, we again obtain that a merger proposal is socially desirable when the efficiency gain is not too large. However, a merger may also increase social welfare for larger efficiency gains because of the duplication of fixed adoption costs in the no-merger case.

In the third step of our analysis, we focus on the specificity requirement to examine the question whether it helps to select those merger proposals which increase social welfare. We define an efficiency as strongly merger specific if it is adopted in the merger case, but no single firm finds it profitable to adopt the efficiency without a merger. Efficiencies are defined as weakly merger specific if the merged firm’s incentive to adopt the efficiency does not fall short of the adoption incentives in the without merger case. Our analysis shows that the strong specificity requirement holds in a parameter region where firms never find it optimal to merge in the first place. Inspection of the weak specificity requirement reveals that a significant share of socially desirable merger proposals involves efficiencies that fail to qualify as weakly merger specific. When competition for the adoption of the efficiency is strong (i.e., only one firm adopts the efficiency), then efficiencies are never weakly merger specific. Yet, in that area endogenous

absent the efficiencies, are not great.”

For a given efficiency gain, competition for technology adoption is the stronger the larger the fixed adoption costs become. Conversely, for a given level of adoption costs, competition for technology adoption increases when the efficiency gain decreases.
mergers are likely to be welfare increasing.\textsuperscript{18} If competition in the adoption game is weak, so that both firms profitably adopt the efficiency, then efficiencies are always weakly merger specific. In that area, endogenous mergers increase social welfare if the efficiency gain is not too large and otherwise reduce social welfare. Hence, the specificity test also fails to single out welfare decreasing proposals in that area.\textsuperscript{19}

In an extension, we analyze the “substantial plus specific” test which requires efficiencies to be large enough so that prices do not increase and accepts only merger specific efficiencies. Again, such a test mainly fails to mirror a proper social welfare analysis because of the specificity requirement. We conclude that a pure laissez-faire regime is likely to perform better (from a social welfare perspective) when compared with a “substantial plus specific” test. Even better results can be achieved with an exclusive “substantial” requirement which becomes more restrictive when the merging parties’ market share increases.

Another implication is that firms may refuse to claim efficiencies all together when a specificity requirement is in place. Assuming that the antitrust authority then ignores any possible efficiencies, we are back in a standard pre-merger/post-merger setting which is often more favorable for the merging parties. Overall, our analysis, therefore, offers an explanation why the high expectations in the efficiency defense have been frustrated so far.

Finally, we examine how market growth affects the specificity requirement and firms’ merger incentives. Market growth can make an efficiency less merger specific, but the opposite may also hold depending on the adoption game outcome. Similarly, we obtain ambiguous effects with regard to firms’ merger incentives.

Our model contributes to the analysis of efficiencies in horizontal mergers (for surveys, see Röller, Stennek, and Verboven, 2001; Motta, 2004; Whinston, 2007). That literature supposed efficiencies in the form of synergies which are by definition merger specific.\textsuperscript{20} Our analysis of

\textsuperscript{18}Only if the merging firms’ pre-merger market shares become large, an increasing parameter region emerges in which endogenous merger proposals are welfare decreasing. This observation is supportive for a minimal efficiency requirement which becomes more strict the larger the merging firms’ joint pre-merger market shares become.

\textsuperscript{19}However, as the specificity requirement is always fulfilled in that area it does not provoke type-I errors as it is the case in the former region, where the requirement is never met.

\textsuperscript{20}Besides other things that literature identified critical efficiency levels that should be passed to make a merger beneficial for consumers and/or welfare; either in a static setting (Farrell and Shapiro, 1990; McAfee and Williams,
exogenous efficiencies is related to Cheung (1992) who shows by example that a merger of relatively inefficient firms can reduce social welfare. Banal-Estanol, Macho-Stadler, and Seldeslachts (2008) analyze endogenous synergies, but they assume complementary resources that give rise to additional merger specific benefits. Lagerlöf and Heidhues (2005) study how costly information acquisition by the merging parties affects the costs and benefits associated with an efficiency defense. Amir, Diamantoudi, and Xue (2009) analyze how asymmetric information about efficiencies between the merging parties and rival firms affects merger incentives and their welfare effects.

The remainder is organized as follows. In Section 2, we present the model. In Section 3, we provide the merger analysis when efficiencies are exogenous. In Section 4, we introduce endogenous efficiencies and endogenous mergers and analyze the entire three-stage game. Section 5 presents our analysis of the merger specificity requirement. In Section 6, we discuss the “substantial plus specific” test and analyze the effect of a growing market. Finally, Section 7 concludes the paper.

2 The Model

We use a (linear) Cournot oligopoly model with homogeneous products which is characterized by the following elements: i) A fixed number of firms indexed by $i = 1, \ldots, N$, ii) a linear (inverse) demand schedule $p(Q) = A - Q$, and iii) constant marginal costs $MC_i \geq 0$. Firms compete in Cournot style, i.e., they set their output levels $q_i \geq 0$, with $Q := \sum_i q_i$, non-cooperatively and simultaneously.

Given that all firms are active, equilibrium quantities are given by

$$q_i^* = \frac{A - N \cdot MC_i + \sum_{j \neq i} MC_j}{N + 1}. \quad (1)$$

1992; Nocke and Whinston, 2012) or in a dynamic merger setting where the without-merger counterfactual may involve new merger proposals (Nilsson and Sorgard, 1998; Motta and Vasconceles, 2005; Nocke and Whinston, 2010). Monotonicity is also used in empirical works to identify the competitive effects of mergers (see Duso, Neven, and Röller, 2007; Duso, Gugler, and Yurtoglu, 2011).

21The Cournot model can be interpreted as a reduced form of a two-stage game where firms first choose capacities and subsequently compete in prices (Kreps and Scheinkman, 1983). It is, therefore, adequate when long-run merger effects (i.e., issues of “capacity” and/or “technology” adjustments) are relevant.
In the following, we specify $N = 4$ and consider a market structure with two large ("dominant") firms and two small ("non-dominant") firms indexed by $d = 1, 2$ and $n = 3, 4$, respectively. Hence, total quantity can be rewritten as $Q := \sum_d q_d + \sum_n q_n$.

We analyze a merger between the non-dominant firms $n = 3, 4$. A merger may lead to efficiencies. We focus on efficiencies which directly impact on competition among firms. Efficiencies come as marginal cost reductions, parameterized by $s > 0$. A necessary prerequisite for realizing efficiencies is the implementation of a more efficient technology which comes at fixed cost $F > 0$. Hence, an efficiency is characterized by two parameters: first, the efficiency gain, $s$, and, second, the (fixed) adoption costs, $F$.

We analyze a three-stage game: In the first stage (merger stage), the non-dominant firms decide whether or not to merge. In the second stage (adoption stage), either the merged firm (if firms 3 and 4 have merged in stage one) or firms 3 and 4 independently (if they have not merged in stage one) decide whether or not to adopt an efficiency enhancing technology which reduces marginal costs by $s$. In the third stage (competition stage), all firms observe the decisions in the previous periods and compete à la Cournot.

It is convenient to assume $MC_d = a$ and $MC_n = a + c$, with $0 < a < A$, and to set $A - a = 1$.\footnote{In an extension below, we drop the assumption $A - a = 1$ to analyze how market growth or technical progress affect the assessment whether claimed efficiencies are merger specific or not.}

Moreover, we invoke the following assumption which guarantees that the non-dominant firms are active in the market.

**Assumption 1.** The non-dominant firms’ marginal costs, $c$, satisfy $0 \leq c < 1/3$.

Using formula (1), the dominant firms’ joint equilibrium market share is (weakly) larger than 50 percent for any $0 \leq c < 1/3$.\footnote{The parameter constraint $c < 1/3$ implies that the non-dominant firms produce strictly positive outputs.} It is instructive to note that the joint market share of firms 3 and 4 is strictly decreasing in $c$. Hence, $c$ is an inverse measure of firms’ market shares before the merger takes place.\footnote{Note that pre-merger market shares (and concentration measures derived from them) are an important indicator of market power (see, for instance, the use of the Herfindahl-Hirschmann Index in the US Horizontal Merger Guidelines).}

In the case of adoption, either the non-dominant firms’ or the merged firm’s marginal costs are given by $a + c - s \geq 0$. Since we allow for both catch-up mergers and mergers to dominance,
efficiencies may exceed $c$. To ensure that the initially dominant firms always remain active in the market, we make the following assumption.

**Assumption 2.** Efficiencies, $s$, fulfill $0 \leq s < 1/2$.

In addition, we rule out any corner solutions in the no-merger subgame when only a single firm adopts the efficiency. The following assumption ensures that the quantity of the non-adopting (non-dominant) firm stays strictly positive.\(^{25}\)

**Assumption 3.** Efficiencies, $s$, fulfill $0 \leq s < 1 - 3c$.

Given our assumptions, subgame perfect strategies in the last stage follow from applying formula (1). In the following, we first analyze the case of exogenous efficiencies. We proceed with the case of endogenous efficiencies where we derive the equilibrium outcome of the entire three stage game and examine its welfare implications.

### 3 Merger Analysis with Exogenous Efficiencies

Assuming that a merger between the non-dominant firms directly leads to efficiency gains $s$, we compare the pre-merger equilibrium with the post-merger equilibrium.\(^{26}\) Before the merger, the dominant and the non-dominant firms maximize their profits $\pi_d = p(Q)q_d$ and $\pi_n = (p(Q) - c)q_n$, respectively. Using (1), we obtain the firms’ pre-merger equilibrium quantities $q_d^* = (1 + 2c)/5$ and $q_n^* = (1 - 3c)/5$, where a single asterisk indicates equilibrium values in the pre-merger case. Assumption 1 implies that, before the merger, the non-dominant firms’ joint market share is not larger than the dominant firms’ joint market share.

When firms 3 and 4 merge, they realize efficiency gains denoted by $s$. We use the subscript “$m$” to refer to the merged firm. The merged firm’s profit function is then given by $\pi_m = p(Q)q_m - (c - s)q_m$. Proceeding as before, we obtain the equilibrium quantities $q_d^{**} = (1 + c - s)/4$ and $q_n^{**} = (1 - 3c - s)/5$.

\(^{25}\) Using formula (1), the equilibrium output of a non-adopting firm $n$ in the no-merger subgame when the other non-dominant firm $n'$ ($n' \neq n$) adopts is given by $q_n = (1 - 3c - s)/5$.

\(^{26}\) In this section, we abstract from any costs of implementing the efficiencies. This will be an issue below when we analyze endogenous efficiencies. Ignoring any adoption costs is reasonable when efficiencies are exogenous and, therefore, qualify as synergies. As we mentioned above, synergies are by definition an “automatic” result from merging the firms assets.
and \( q_{m}^{**} = \frac{1 - 3(c - s)}{4} \), where two asterisks indicate the equilibrium values in the post-merger case. It immediately follows that efficiencies reduce the dominant firms’ output levels, whereas they increase the merged firm’s output.

Given firms’ quantities, we obtain the equilibrium values of firms’ profits, consumer surplus, \( CS \), and social welfare, \( SW \), both before and after the merger (social welfare is defined as the sum of firms’ profits and consumer surplus). The change of the merging firms’ profits, the dominant firms’ profits, consumer surplus, and social welfare due to the merger is defined by \( \Delta \pi_m := \pi_m^{**} - 2\pi_n^{*} \), \( \Delta \pi_d := \pi_d^{**} - \pi_d^{*} \), \( \Delta CS := CS^{**} - CS^{*} \), and \( \Delta SW := SW^{**} - SW^{*} \), respectively.

The following proposition shows how the merger’s profitability, its external effect on the competitors, and its impact on consumer surplus depend on the realized efficiencies.\(^{27}\)

**Proposition 1.** Depending on the efficiency gain, \( s \), there exist unique critical values \( 0 < s_D(c) < s_B(c) < c \) such that the following ordering hold:

i) If \( s < s_D(c) \), then \( \Delta \pi_m < 0 \), \( \Delta \pi_d > 0 \), and \( \Delta CS < 0 \).

ii) If \( s_D(c) < s < s_B(c) \), then \( \Delta \pi_m > 0 \), \( \Delta \pi_d > 0 \), and \( \Delta CS < 0 \).

iii) If \( s_B(c) < s \), then \( \Delta \pi_m > 0 \), \( \Delta \pi_d < 0 \), and \( \Delta CS > 0 \).

Moreover, \( s_B(c) \) and \( s_D(c) \) are both monotonically decreasing.

Proposition 1 states that consumers and the merged entity are better off if efficiencies are substantial which corresponds to the case where \( s > s_B(c) \). Only in those instances the dominant firms’ profits decrease. For intermediate efficiencies, \( s_D(c) < s < s_B(c) \), both the dominant firms and the merged firm benefit, but consumer surplus is reduced. Finally, the dominant firms realize higher profits, while consumers and the merged entity are harmed due to the merger if the efficiency level is sufficiently small, i.e., \( s < s_D(c) \).

Proposition 1 mirrors the observation that a merger is generally more likely to be approved the smaller the merging parties’ market shares and the larger the efficiency gains.\(^{28}\) For instance, the relationship between efficiencies and the likelihood of approval is stated explicitly in US Horizontal Merger Guidelines: “The greater the potential adverse competitive effect of a merger,

\(^{27}\) All proofs are provided in the Appendix.

\(^{28}\) A common observation in empirical studies is that the probability of a phase II investigation and of a prohibition of the merger increases with the parties’ market shares (see, for instance, Bergman et al., 2005).
the greater must be the cognizable efficiencies, and the more they must be passed through to customers, for the Agencies to conclude that the merger will not have an anticompetitive effect in the relevant market.”

We next address the question how social welfare changes when the two non-dominant firms merge.

Proposition 2. A merger with efficiencies gain, $s$, affects social welfare as follows:

i) If $c < 9/107$, then there exists a unique critical value $s_H(c)$, such that $\Delta SW > 0$ ($\Delta SW < 0$) for $s > s_H(c)$ ($s < s_H(c)$). Moreover, $s_H(c)$ is monotonically decreasing.

ii) If $9/107 < c < (5\sqrt{23} + 69)/322$, a merger always increases social welfare.

iii) If $c > (5\sqrt{23} + 69)/322$, then there exist two critical values $\underline{s}(c)$ and $\overline{s}(c)$, with $\underline{s}(c) < \overline{s}(c)$, such that $\Delta SW < 0$ for $s \in (\underline{s}(c), \overline{s}(c))$, while the opposite is true for $s \in (\overline{s}(c), \underline{s}(c))$ or $s > \overline{s}(c)$. Moreover, $\underline{s}(c)$ is monotonically decreasing, and $\overline{s}(c)$ is monotonically increasing.

Proposition 2 shows that the non-dominant firms’ pre-merger market shares are important for assessing the welfare effects. Case i) mirrors the more standard result that relatively large pre-merger market shares (which follows from $c$ being relatively small) raise the bar for the efficiency level. In that case, social welfare can only increase if the efficiencies are sufficiently large. Otherwise, the negative impact on consumer surplus and the merged entity’s profit outweighs the positive external effect on the rival dominant firms’ profits.

However, this reasoning is not valid anymore when we consider mergers of smaller firms, i.e., $c > 9/107$ starts to hold. Case ii) shows that there is a region of intermediate pre-merger market shares in which any merger is socially desirable. In that area, the efficiency gain is either sufficiently small, so that the dominant firms’ gain outweighs the loss in consumer surplus, or the efficiency gain is large enough, so that the increase in consumer surplus outweighs the dominant firms’ losses.

Yet, case iii) highlights the surprising insight that mergers among relatively small firms are much more complex. It reveals that a non-monotone relationship is also possible when catch-up mergers are considered. If pre-merger market shares are small, then efficiency gains must be sufficiently low or high, so that a catch-up merger becomes welfare improving. In that parameter region, moderate efficiency levels are indicative of a welfare reducing merger. When efficiencies are small, a catch-up merger has only little influence on the dominant firms’ profit levels and
consumer surplus. Hence, for small efficiencies the merger is likely to be welfare improving as it increases the merging firms’ efficiency. If the efficiency is sufficiently large, i.e., \( s > \bar{s}(c) \) holds, then the increase in consumer surplus and in the merging firm’s profit outweigh the loss incurred by the dominant competitors.

For the case of moderate efficiencies, \( s(c) < s < \bar{s}(c) \), both consumer surplus and the merging firms’ joint profit increase. However, both effects together do not suffice to compensate for the dominant firm’s relatively large profit reduction. Social welfare decreases if efficiency gains are in that area because of the merger’s negative effect on overall productive efficiency. Our results in Proposition 2 are illustrated by Figure 1 which presents the considered parameter region and the threshold values. The grey areas indicate mergers which are welfare decreasing, while the white area represents all mergers which increase social welfare. Note also that case \( iii \) of Proposition 2 lies to the left of the 45°-degree line (where \( c = s \) holds); i.e., in the catch-up merger area.

Finally, comparing Propositions 1 and 2 shows that an increase of consumer surplus does not necessarily imply an increase in social welfare, whenever catch-up mergers between relatively small firms are prevalent. We, therefore, qualify the presumption that the monotonicity of \( \Delta CS \) with regard to \( s \) also holds for social welfare, \( \Delta SW \), as asserted, e.g., in Farrell and Shapiro.
(1990) or Besanko and Spulber (1993). When the relationship between efficiencies and social welfare is not monotone (case iii of Proposition 2), a simple minimum requirement for the level of efficiencies must fail to approximate the socially efficient rule.

4 Merger Analysis with Endogenous Efficiencies

We now solve the entire three-stage game. We first analyze the adoption stage ("endogenous efficiencies"). Then, we analyze the merger decision which allows us to examine the subgame-perfect equilibrium of the entire game. Finally, we derive the social welfare effects of a merger. That allows us to answer the question whether or not the equilibrium outcome of our game is aligned with social welfare maximization.

4.1 Endogenous Efficiencies

Depending on the non-dominant firms’ decision whether or not to merge in the first stage of the game, we have to consider two subgames: the merger subgame and the no-merger subgame. In each subgame, the non-dominant firms decide about the implementation of a new technology which reduces marginal cost by $s$ and comes at a cost of $F$. We denote the strategies “adopt” and “not adopt” by $A$ and $NA$, respectively.

Adoption incentives in the merger subgame. The merged entity implements the new technology if the profit increase does not fall short of the adoption costs, $F$. Applying (1) we obtain the merged firm’s equilibrium profit $\pi^*_m(A) = ([1 - 3(c - s)]/4)^2$ in case of adoption.

If the merged firm abstains from implementing the efficiencies, then its equilibrium profit is $\pi^*_m(NA) = ([1 - 3c]/4)^2$. Clearly, the merged firm implements the efficiency if

$$\phi_m := \pi^*_m(A) - \pi^*_m(NA) \geq F,$$

where $\phi_m$ measures the adoption incentive of the merged firm. Straightforward calculations show that (2) implies an upper bound on $F$.

Lemma 1. The merged firm adopts the technology if and only if $F \leq \phi_m$ holds, with $\phi_m := 3s[2 + 3(s - 2c)]/16$. Moreover, $\lim_{s \to 0} \phi_m = 0$, while $\phi_m > 0$, $\partial \phi_m / \partial s > 0$, and $\partial \phi_m / \partial c < 0$ hold everywhere.
Lemma 1 shows that the merged firm’s incentive to implement the efficiency is increasing in the efficiency level, \( s \), but decreasing in its initial cost level, \( c \). The latter observation implies that -ceteris paribus- larger firms have a stronger incentive to implement an efficiency that reduces marginal costs by \( s \).\(^{29}\)

In the following, we assume that the merged firm always has a strong incentive to adopt the efficiency; i.e., we restrict the analysis to values of \( F \) such that \( F < \phi_m \) holds.\(^{30}\)

**Adoption incentives in the no-merger subgame.** In the no-merger subgame, firms 3 and 4 independently decide about the adoption of the efficiency, \( s \). Again, each firm has two pure strategies “adopt” and “not adopt.”

A firm’s equilibrium output depends on both its own adoption decision, \( k \), as well as the other non-dominant firm’s adoption decision, \( k' \), with \( k, k' \in \{A, NA, r\} \), where \( r \in [0, 1] \) stands for the adoption probability. We write firm \( n \)’s output, \( q_n(k, k') \), and profit, \( \pi_n(k, k') \), as a function of its own adoption decision, \( k \), and the rival non-dominant firm’s adoption decision, \( k' \).\(^{31}\) We consider both pure strategies and mixed strategies. In the latter case, a firm selects a probability distribution \((A, NA; r, 1 - r)\), where \( r \) is the probability of adoption and \( 1 - r \) is the counter probability of no adoption. To proceed in a parsimonious way, we use \( r \) to indicate both the probability of adoption and the entire probability distribution. Below, we also use the pair \((r, r)\) as representing the (symmetric) mixed strategy equilibrium, where both firms adopt with probability \( r \).

<table>
<thead>
<tr>
<th>Firm ( n ) \ Firm ( n' )</th>
<th><strong>A</strong></th>
<th><strong>NA</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( \pi^<em>_n(A, A) - F, \pi^</em>_n(A, A) - F )</td>
<td>( \pi^<em>_n(A, NA) - F, \pi^</em>_n(NA, A) )</td>
</tr>
<tr>
<td>( NA )</td>
<td>( \pi^<em>_n(NA, A), \pi^</em>_n(A, NA) - F )</td>
<td>( \pi^<em>_n(NA, NA), \pi^</em>_n(NA, NA) )</td>
</tr>
</tbody>
</table>

\(^{29}\)We note that this property reflects the idea of efficiencies resulting from “scale economies” as emphasized in Farrell and Shapiro (2001).

\(^{30}\)This assumption simplifies our analysis because it allows us to abstract from parameter constellations \((s, c, F)\) under which the efficiency is not adopted by the merged firm, but by, at least, one non-dominant firm in the no-merger subgame.

\(^{31}\)For instance, \( \pi_3(NA, A) \) denotes firm 3’s profit if it does not adopt, while firm 4 adopts.
Table 1 illustrates the technology adoption subgame where $\pi^*_n$ and $\pi^*_{n'}$ are the non-dominant firms’ equilibrium profits contingent on their adoption decisions. The following lemma states the subgame perfect equilibria of the adoption game depicted in Table 1.

**Lemma 2.** There exist two critical values $F := \pi^*_n(A, A) - \pi^*_n(NA, A) = 8s(1 + s - 3c)/25$ and $\overline{F} := \pi^*_n(A, NA) - \pi^*_n(NA, NA) = 8s(1 + 2s - 3c)/25$, with $0 < F < \overline{F}$, such that the equilibrium of the adoption game is as follows:

i) If $F < F$, then $(A, A)$ is the unique equilibrium.

ii) If $F \leq F \leq \overline{F}$, then there are two pure strategy equilibria, $(A, NA)$ and $(NA, A)$.

iii) If $F \leq F \leq \overline{F}$, then there exists a unique mixed strategy equilibrium $(r, r)$. The equilibrium mixed strategy, $r$, is monotonically decreasing in $F$, approaches one at the lower bound and goes to zero at the upper bound.

iv) If $\overline{F} < F$, then the unique Nash equilibrium is $(NA, NA)$.

Moreover, $\partial F/\partial c > 0$, $\partial \overline{F}/\partial s > 0$ and $\partial \overline{F}/\partial s > \partial F/\partial s > 0$, while $\lim_{s \to 0} F = \lim_{s \to 0} \overline{F} = 0$.

Lemma 2 states that the implementation of the efficiency enhancing technology becomes more attractive for larger values of the marginal cost reduction, $s$, and/or lower values of $c$ (i.e., firms’ initial market shares). Implementation is most attractive for the “first” firm adopting the technology. It can be still attractive to implement the efficiency as a “second” firm, though the profit differential is smaller than in the former case; i.e., $0 < F < \overline{F}$ holds. Note that this ordering holds for all positive efficiency gains $s$ and that the difference $\overline{F} - F$ (i.e., the area where cases ii) and iii) apply) widens as $s$ increases. In case i), both firms adopt whenever adoption costs are small ($F < F$). In case iv), fixed adoption costs are prohibitive so that none of the firms adopts ($F > \overline{F}$). In the intermediate range of cases ii) and iii), $F$ is such that only a single firm can profitably adopt the efficiency enhancing technology with probability one. This gives rise to two pure strategy equilibria and a unique mixed strategy equilibrium. In the mixed strategy equilibrium, the probability of adoption monotonically decreases when $F$ increases.

---

32 The equilibrium profits stated in Table 1 are presented in the Appendix (Proof of Lemma 2).

33 Note that the “first” adopting firm gains $\overline{F} := \pi^*_n(A, NA) - \pi^*_n(NA, NA)$, while the “second” firm gets $F := \pi^*_n(A, A) - \pi^*_n(NA, A)$.

34 The existence of a unique mixed strategy equilibrium follows from noticing that the adoption of the efficiency by one firm exerts a negative externality on the remaining non-dominant firm; i.e., $\pi^*_n(NA, NA) > \pi^*_n(NA, A)$.
Lemma 2 also shows that the adoption game (Table 1) becomes more competitive when the efficiency (characterized by parameters $s$ and $F$) becomes less attractive; i.e., either $F$ increases (while holding $s$ constant) or $s$ decreases (while fixing $F$). When $F$ surpasses the threshold value $E$, then only a single firm can profitably implement the efficiency, while for lower values of $F$ both firms will adopt the efficiency for sure. Moreover, when $F$ surpasses the upper threshold value $F$, then no firm finds it profitable to implement the efficiency.

Similarly, a higher value of the efficiency gain, $s$, reduces -ceteris paribus- the intensity of adoption competition. This follows from noticing that both threshold values $E$ and $F$ are monotonically increasing in $s$, while the latter value increases faster than the former. It follows that for a given value of the adoption costs, $F$, an outcome according to case i), where both firms adopt, becomes more likely when the efficiency gain increases.

### 4.2 Merger Incentives and Endogenous Efficiencies

We are now in a position to analyze the entire three-stage game. In the first stage, the two non-dominant firms decide whether or not to merge. To derive the subgame perfect equilibrium, we compare the net profits in the merger case, $\Pi_m(A) := \pi^{**}(A) - F$, with those in the no-merger case, $\Pi_n^{k,k'}$. The latter depends on the equilibrium of the adoption game (cases i)-iv) of Lemma 2. We say that firms have strict merger incentives if

$$\theta^{k,k'} := \Pi_m(A) - \left[ \Pi_n^{k,k'} + \Pi_n^{k',k} \right] > 0, \quad n \neq n', \; k \neq k'$$

holds; i.e., the merged firm’s net profit is larger than the sum of the non-dominant firms’ profits in the no-merger subgame. The subgame perfect equilibrium of the entire game is summarized in Proposition 3.

**Proposition 3.** Suppose $F < \phi_m$. The merger decision depends on the outcome of the adoption game as follows:

i) If $(A, A)$, then there exists a critical value $s_1(c)$, such that $\theta^{A,A} < 0$ holds for all $s < s_1(c)$. If $s > s_1(c)$, then there is a critical value $F_\alpha$, such that $\theta^{A,A} > 0 \; (\theta^{A,A} < 0)$ whenever $F > F_\alpha \; (F < F_\alpha)$. Moreover, $\partial s_1/\partial c < 0$.

always holds. In addition, the negative externality is monotonically increasing in $s$. 

16
ii) If \((\alpha; NA)\), then there exists a critical value \(s_K(c)\), such that \(\theta^{\alpha,NA} < 0\) for \(s < s_K(c)\), while \(\theta^{\alpha,NA} > 0\) holds for \(s > s_K(c)\). Moreover, \(\partial s_K/\partial c < 0\).

iii) If \((r; r)\), then there exists a critical value \(s_I(c)\), such that \(\theta^{r,r} < 0\) holds for all \(s < s_I(c)\). If \(s > s_I(c)\), then there is a critical value \(F_\beta\), such that \(\theta^{r,r} > 0\) \((\theta^{r,r} < 0)\) whenever \(F < F_\beta\) \((F > F_\beta)\). Moreover, \(\partial F_\beta/\partial s > 0\).

iv) If \((NA; NA)\), then \(\theta^{NA,NA} < 0\) always holds.

Proposition 3 shows that merger incentives critically depend on the without merger outcome. When both firms adopt in the no-merger subgame, then both the efficiency level and the fixed adoption costs must be large enough to induce a merger. While efficiencies must be large enough to make the merger profitable in the first place, firms’ merger incentive is then driven by the desire to avoid duplication of adoption costs. In Figure 2, the black area below the critical value \(s_I(c)\) indicates all combinations \((s, c)\) for which a merger is never proposed. The complementary area indicates cases where a merger occurs, whenever \(F\) is larger than \(F_\alpha\).

In case iv), firms never find it profitable to merge. The reason is that firms do not adopt the efficiency in the no-merger case, while it is implemented in case of a merger. As adoption costs are very high in that area, firms choose not to merge in order to avoid the costs of implementing the efficiency.

Turning to case ii), it is instructive to examine the decision rule for a merger which becomes

\[
\theta^{\alpha,NA} = \pi^*_m(\alpha) - [\pi^*_n(\alpha, NA) + \pi^*_n(NA, \alpha)].
\]  

Equation (3) does not include fixed adoption costs because only one firm incurs them in the no-merger case. Proposition 3 shows that the sign of \(\theta^{\alpha,NA}\) is negative for small efficiencies, while it is positive for larger values of \(s\). If efficiencies are relatively small, then the merger is not profitable because the dominant firms’ output increases in that area. When the level of efficiencies increases, then the profit of the merged firm tends to increase faster than the sum of profits in case of no merger. That relationship is depicted in Figure 3 where the black area indicates all \((s, c)\) combinations for which a merger is not profitable. The complementary area stands for cases where firms find it optimal to merge.

Case iii) can be seen as a combination of the results presented in cases i) and iv). If fixed adoption costs are large, then adoption is less likely in the mixed strategy equilibrium of the
adoption game, so that possible gains from fixed cost savings disappear. Hence, for large enough $F$, merger incentives are absent. If, however, $F$ becomes smaller, the probability of adoption increases in the no-merger subgame. Hence, fixed cost savings become important which makes a merger attractive again. Proposition 3 also states that the parameter range for a profitable merger increases when the level of the efficiency increases. In that sense, a merger becomes more likely in that area for larger efficiency levels. That relationship is presented in Figure 4, where the black area below the critical value $s_I(c)$ indicates all $(s,c)$ combinations for which a merger is never proposed. In the complementary area, efficiencies are large enough to make a merger profitable. However, for a merger to occur the fixed adoption costs have to be sufficiently small (precisely, $F < F_\beta$ must additionally hold).\textsuperscript{35}

Overall, our results show that a necessary condition for a profitable merger is that efficiencies are sufficiently large. First of all, the no-merger outcome of case iv) is less likely to occur when the efficiency gain becomes larger (note: $\partial F/\partial s > 0$). Second, cases i)-iii) highlight minimum requirements on the efficiency gain (see the threshold values $s_I(c)$ and $s_K(c)$) which are necessary conditions for a profitable merger. Moreover, the minimal efficiency level necessary to make the merger profitable always increases the larger the merging firms pre-merger market shares.

Our analysis also reveals that the level of the adoption costs impacts differently on merger incentives depending on the outcome of the adoption game. If the efficiency is quite attractive, such that both firms will adopt it without the merger, then a higher level of the adoption cost affects merger incentives positively because of larger fixed cost savings. If, however, the mixed strategy equilibrium applies, then lower levels of the adoption costs tend to make the merger more profitable. Otherwise, adoption incentives in the no-merger case would become too low which reduces the expected fixed cost savings through a merger.

### 4.3 Endogenous Efficiencies and Social Welfare

We analyze how a merger affects social welfare when efficiencies are endogenous. This allows us to examine the socially efficient decision an antitrust authority should apply for a merger.\textsuperscript{36}

\textsuperscript{35}The probability of adoption decreases overproportionally when $F$ increases. Hence, the incentive to save on the duplication of fixed adoption costs through a merger becomes the more pronounced the smaller $F$.

\textsuperscript{36}It might be questioned whether competition authorities follow a social welfare standard because many countries appear to apply something close to a consumer standard (see Whinston, 2007). Yet, Neven and Röller (2005)
Given a social welfare rule, a merger is only approved if social welfare is larger after the merger when compared with the equilibrium emerging without a merger. For such an analysis it is critical to foresee the outcome of the adoption game as stated in Lemma 2. The comparison, therefore, depends on the cases i)-iv). Accordingly, we define $\Delta SW^{k,k'} := SW^{**} - F - SW^{k,k'}$, where $SW^{**}$ stands for social welfare in the merger case, and $SW^{k,k'}$ denotes social welfare (net of fixed adoption costs) in the no-merger case depending on the equilibria of the adoption game. Hence, a welfare maximizing antitrust authority should approve a merger only if $\Delta SW^{k,k'} > 0$.

Our results are as follows.

**Proposition 4.** Suppose $F < \phi_m$. The welfare effects of a merger depend on the outcome of the adoption game as follows:

i) If $(A,A)$, then there exists a critical value $s_N(c,F)$, such that $\Delta SW^{A,A} > 0$ ($\Delta SW^{A,A} < 0$) whenever $s < s_N(c,F)$ ($s > s_N(c,F)$). Moreover, $s_N(c,F)$ is monotonically increasing in $c$ and $F$.

ii) If $(A,NA)$, then there exists a critical value $s_P(c)$, such that $\Delta SW^{A,NA} > 0$ holds for $s > s_P(c)$, while $\Delta SW^{A,NA} < 0$ holds for $s < s_P(c)$. Moreover $s_P(c)$ is monotonically decreasing.

iii) If $(r,r)$, two cases depending on $c$ can be distinguished:

a) Suppose $c < 9/107$. Then two critical values $s_R(c) < s_U(c)$ exist, such that $\Delta SW^{r,r} < 0$ for $s < s_R(c)$, while $\Delta SW^{r,r} > 0$ holds for $s > s_U(c)$. If $s_R(c) < s < s_U(c)$, then there exists a threshold value $F_\gamma$, such that $\Delta SW^{r,r} > 0$ ($\Delta SW^{r,r} < 0$) whenever $F < F_\gamma$ ($F > F_\gamma$).

b) Suppose $c \geq 9/107$. Then there exists a critical value $s_V$, such that $\Delta SW^{r,r} > 0$ if $s < s_V$. For $s > s_V$, there exists a threshold value $F_\gamma$, such that $\Delta SW^{r,r} > 0$ ($\Delta SW^{r,r} < 0$) whenever $F < F_\gamma$ ($F > F_\gamma$).

iv) If $(NA,NA)$, then the welfare effects of a merger correspond to our findings in Proposition 2 under the constraint that $s < s_W(c)$.

Given that $(A,A)$ is the adoption game outcome, a merger increases social welfare only if show that if firms can lobby efficiently, then an authority with a consumer standard will end up maximizing social welfare (i.e., the sum of firms’ profits and consumer surplus). Finally, we note that the debate is not fully settled yet. For instance, Farrell and Katz (2006) and Rosch (2006) discuss the pros and cons of a “total welfare” standard. Relatedly, Renckens (2007) argues that a total welfare standard is better suited than the consumer surplus standard for merger control which allows for an efficiency defence.
the efficiency level is sufficiently small, i.e., $s < s_N(c, F)$ holds. Intuitively, a merger tends to be socially undesirable if both merger candidates implement the efficiency also without the merger. The critical value, $s_N(c, F)$, is increasing in $c$ which reflects the dominant firms’ response to a merger; the smaller the merging firms’ pre-merger market shares, the larger the output expansion of the dominant firms in case of a merger. That effect tends to increase overall productive efficiency resulting in the positive slope of $s_N(c, F)$ which is depicted in Figure 2. Figure 2 also shows that $s_N(c, F)$ becomes less binding when $F$ increases (in Figure 2, $F' < F''$ holds) reflecting the adverse welfare effects of fixed cost duplication.

When only one non-dominant firm adopts the technology in the no-merger subgame, then social welfare is increased if the efficiency is larger than the threshold value $s_P(c)$. The slope of this threshold value is negative, so that the minimal efficiency for a welfare increasing merger becomes stricter when the merging firms’ pre-merger market shares increase. It is also interesting that the critical value $s_P(c)$ gives rise to a maximum pre-merger market share below which any proposed merger (even with negligible efficiencies) is welfare improving (mirroring case ii) of Proposition 2).

When firms 3 and 4 play mixed strategies, we distinguish between relatively large and relatively small non-dominant firms. Proposition 4 shows that initially large non-dominant firms should only be approved if they exhibit large efficiency gains or small fixed adoption costs. This finding reflects the intuition that the anticompetitive effect of a merger can only be compensated by sufficiently high efficiency gains or rather small adoption cost levels accompanied by moderate efficiency gains. The upper bound on the fixed adoption costs follows from noticing that the adoption probability decreases overproportionally when $F$ increases. Hence, fixed cost duplication is only an issue for small enough $F$.

When the non-dominant firms are relatively small ($c \geq 9/107$), the result is partially reversed. While for moderate and large efficiencies adoption cost levels continue to be critical (again, $F$ must be sufficiently small for welfare to increase), a merger always exhibits a positive welfare effect when efficiency levels are small. The latter result follows (partially) from the business stealing effect being relatively small when $s$ is small, so that the overall productive efficiency increases and offsets the per se anticompetitive effects of a merger.

Finally, in case iv), where both non-dominant firms choose not to adopt in the no-merger
subgame, a merger’s effect on social welfare corresponds to Proposition 2 except that we have to account for the restriction $s < s_W(c)$. By considering rather low efficiency levels, we ensure that adoption costs are not too large to leave the merger itself profitable when adopting $s$.

Given our results in Propositions 3 and 4, we can analyze the effects of endogenous mergers on social welfare. Note that case iv) becomes irrelevant, since in this case the non-dominant firms never decide to merge in equilibrium. In contrast to our previous reasoning, we explicitly distinguish between catch-up mergers and mergers to dominance which are characterized by $s < c$ and $s \geq c$, respectively.

First, we concentrate on cases i) and ii) given that proposed mergers are catch-up mergers. Our main result is the following.

**Corollary 1.** Endogenous catch-up mergers always increase social welfare in both case i) and case ii).

Corollary 1 highlights that an antitrust authority should approve every catch-up merger in cases i) and ii). In other words, every proposed merger with $s < c$ is aligned with an increase in social welfare. That result is depicted in Figures 2 and 3. In both figures the admissible parameter region to the left of the 45°-degree line (where $s = c$ holds) is either black or white. Note that the black area indicates mergers which are never profitable, while the white area stands for endogenous merger proposals which always increase social welfare.

![Figure 2: Welfare Effects of Proposed Mergers in Case i)](image-url)
If we shift our focus to mergers to dominance, then the welfare effect of a proposed merger becomes ambiguous in cases i) and ii). Our findings are summarized in the next corollary.

**Corollary 2.** The welfare effects of endogenous mergers to dominance depend on \( c, s, \) and \( F \) as follows:

i) If \((A, A)\), then \( \Delta SW^{A,A}_A > 0 \) (\( \Delta SW^{A,A}_A < 0 \)), whenever \( s < s_N(c, F) \) (\( s > s_N(c, F) \)).

ii) If \((A, NA)\), then \( \Delta SW^{A,NA}_A > 0 \) (\( \Delta SW^{A,NA}_A < 0 \)), whenever \( s > s_P(c) \) (\( s < s_P(c) \)).

Our findings from Corollary 2 are illustrated in Figure 2 and Figure 3, respectively. Note that the grey areas indicate endogenous merger proposals which always decrease social welfare.

![Figure 3: Welfare Effects of Proposed Mergers in Case ii)](image)

When proposed mergers go along with \( s \geq c \), then the decision to merge is not perfectly aligned with an increase in social welfare. In case i), a merger to dominance should only be approved if the efficiency is not too large, i.e., \( s < s_N(c, F) \). Moreover, due to fixed cost duplication an approval becomes more attractive the higher \( F \). In Figure 2, the effect of fixed cost duplication is represented by a shift from \( s_N(c, F = F') \) to \( s_N(c, F = F'') \), with \( F' < F'' \), which would obviously reduce the area where \( \Delta SW^{A,A}_A < 0 \) holds.

In case ii), negative welfare effects from proposed mergers to dominance can only emerge if the non-dominant firms are initially large, i.e., \( c < 1/50 \). Then, sufficiently small efficiencies \( (s < s_P(c)) \) lead to a decrease in social welfare. However, Figure 3 also shows that rather a
small part of all possible proposed mergers to dominance leading to $\Delta SW^{A,NA} < 0$ is affected. The much larger area of proposed mergers in case $ii)$ leads to an increase in social welfare.

Finally, we analyze case $iii)$ where the mixed strategy equilibrium solves the adoption game in the no-merger subgame. Corollary 3 presents our results.

**Corollary 3.** If $(r,r)$, then an endogenous merger always increases social welfare whenever $s < s_V$. For $s > s_V$, an endogenous merger leads to $\Delta SW^{r,r} < 0$ ($\Delta SW^{r,r} > 0$) if $F > F_\gamma$ ($F < F_\gamma$).

In contrast to cases $i)$ and $ii)$, proposed catch-up mergers should not per se be approved by a welfare maximizing antitrust authority. They may induce negative welfare effects whenever efficiencies and adoption costs are too large. The same holds for proposed mergers to dominance. Our findings are illustrated in Figure 4. Note that the grey area in Figure 4 represents those merger proposals which only reduce social welfare if the adoption costs are relatively large (i.e., whenever $F > F_\gamma$ holds). Hence, the grey area reflects a necessary condition rather than a sufficient condition for $\Delta SW^{r,r} < 0$ to hold.

![Figure 4: Welfare Effects of Proposed Mergers in Case iii)](image)

Overall, our findings of Proposition 2 are mirrored in Proposition 4. However, our results show that an appropriate assessment, which foresees the adoption game outcome in the without merger counterfactual, is much more complicated as the adoption costs become critical for both the outcome of the adoption game in the no-merger case and for the overall welfare assessment.
5 The Specificity Requirement

Whether or not a certain efficiency claim is merger specific depends on a comparison of firms' incentives to implement the efficiency with and without a merger. We propose to consider an efficiency claim as merger specific if and only if the merger increases the incentives to realize the efficiency. We distinguish two definitions of merger specificity which we will analyze formally below within the realm of our model.

Definition 1. (weak merger specificity). A claimed efficiency is weakly merger specific if the merged firm has a strictly larger incentive to implement the efficiency, $s$, than any non-dominant firm without the merger.

The assessment of whether or not claimed efficiencies are weakly merger specific is based on a comparison of the merged firm’s adoption incentives with firms’ individual incentives in the no-merger case. It might be too difficult in practice to calculate adoption incentives due to unavailability of data and other practical constraints. We, therefore, provide an alternative definition which is less informational demanding.

Definition 2. (strong merger specificity). A claimed efficiency is strongly merger specific if the efficiency, $s$, is only adopted after the merger. That is, each non-dominant firm does not find it profitable to implement $s$ individua. 

The adoption incentives of the merged firm are summarized in Lemma 1. To conclude whether a claimed efficiency is weakly merger specific or not, we compare the merged firm’s incentives with the adoption incentives in the no-merger case. In the no-merger case, we focus on equilibrium incentives; i.e., we examine a firm’s unilateral incentive given that the other firm plays a best response in the adoption game. Our incentive measure in the no-merger case is then given by the difference between the equilibrium profit level and the hypothetical profit level in case of committing not to adopt the efficiency.

We obtain the following adoption incentives, $\phi_{n,n}^{k,k'}$, depending on the cases i)-iii) stated in Lemma 2.\footnote{If case iv) applies, then the fixed cost of technology adoption, $F$, is such that in equilibrium each of the non-dominant firms does not have an incentive to implement the efficiency. In that area, adoption incentives are given by $\phi_{n,n}^{N,A} := \pi_n^*(A,NA) - \pi_n^*(NA,NA)$ which is exactly the value of the upper bound $F$.} In case i), the adoption equilibrium is $(A,A)$. Hence, the adopting firm obtains the

...
equilibrium profit $\pi^*_n(A, A)$. If a firm commits not to adopt, it obtains $\pi^*_n(NA, A)$ because the other firm still adopts the efficiency in equilibrium. Adoption incentives are, therefore, given by

$$\phi^{A,A}_n := \pi^*_n(A, A) - \pi^*_n(NA, A).$$

(4)

In case $ii$, only one firm adopts in equilibrium.\(^{38}\) The equilibrium profit of the adopting firm is $\pi^*_n(A, NA)$. If that firm commits not to adopt, then its profit becomes $\pi^*_n(NA, A)$ because the other firm’s best response is to adopt. Hence, the incentive measure becomes

$$\phi^{A,NA}_n := \pi^*_n(A, NA) - \pi^*_n(NA, A).$$

(5)

In case $iii$, both firms adopt with some probability $r \in [0, 1]$. Hence, a firm realizes the expected (equilibrium) profit $\pi^*_n(r, r)$ which must be equal to $\pi^*_n(A, r)$.\(^{39}\) Note that $\pi^*_n(A, r)$ includes the fixed adoption costs fully, so that gross equilibrium profits are $\pi^*_n(A, r) + F$. If a firm commits not to adopt, then the other firm plays a best response which is to adopt for sure; that is, the hypothetical profit in case of choosing not to adopt is $\pi^*_n(NA, A)$.\(^{40}\) Taking that together, adoption incentives in case $iii$ are given by

$$\phi^{r,r}_n := \pi^*_n(A, r) + F - \pi^*_n(NA, A).$$

(6)

Comparison of the incentive measures (4)-(6) reveals that adoption incentives are largest in case $ii$ where only one firm adopts for sure in equilibrium. This follows from the ordering $\pi^*_n(A, NA) > \pi^*_n(A, A)$, while the term $\pi^*_n(A, r) + F$ must lie in between both values.\(^{41}\)

We first focus on strong merger specificity which is only possible if an efficiency is only adopted in case of a merger. We then examine weak merger specificity where we assume that $F < \min\{F, \phi_m\}$ holds, so that the considered efficiencies are adopted both in case of a merger and in the absence of the merger.

\(^{38}\)The equilibrium outcome in case $ii$ of Lemma 2 is either $(A, NA)$ or $(NA, A)$. Because of symmetry, we use the former, $(A, NA)$, to denote that case.

\(^{39}\)The mixed strategy equilibrium requires that a player is indifferent between his pure strategies. Hence, $\pi^*_n(A, r) = \pi^*_n(NA, r) = \pi^*_n(r', r)$ always holds given the other firm plays the equilibrium mixed strategy.

\(^{40}\)Recall that the parameter regions of case $ii$ and case $iii$ in Lemma 2 are identical. That is, if the mixed strategy equilibrium exists, then the pure strategy equilibria, $(A, NA)$ and $(NA, A)$, also exist (and vice versa).

\(^{41}\)Note that the last term on the right-hand side of equations (4)-(6) is always the same. Hence, the comparison only depends on the remaining terms.
**Strongly merger specific efficiencies.** Case iv) of Lemma 2 represents the only candidate for strong merger specificity to emerge. The existence of such a constellation depends on whether or not values of $s$ and $F$ are feasible, such that technology adoption is profitable in the merger case, but not otherwise; i.e., $\bar{F} < F \leq \phi_m$ must hold.\(^{42}\) The next proposition shows that such constellations are possible.

**Proposition 5.** There exists a critical value $s_W(c)$, with $s_W(c) := 22(1 - 3c)/31$, such that $\bar{F} < \phi_m$ holds whenever $s < s_W(c)$. It follows that efficiencies are strongly merger specific if and only if $s < s_W(c)$ and $F \in [\bar{F}, \phi_m]$. Otherwise, efficiencies are never strongly merger specific. Moreover, $\partial s_W / \partial c < 0$ and $\lim_{c \to 1/3} s_W(c) = 0$.

Proposition 5 shows that strong merger specificity can only occur if the adoption costs are sufficiently large (so that $F > \bar{F}$ holds), while the efficiency gain is not too large; i.e., $s < s_W(c)$ holds. Only if both conditions are met, then the efficiency is adopted in the merger case, but not in the no-merger case. If, otherwise, efficiencies exceed $s_W(c)$, then they are adopted also without the merger which makes those efficiencies not strongly merger specific. Similarly, if we fix the value of $s$, then adoption costs $F$ must be sufficiently large (i.e., $F > \bar{F}$) to obtain strong merger specificity.\(^{43}\) Hence, a necessary condition for strong merger specificity is that a claimed efficiency is sufficiently *unattractive* in the adoption game. This is more likely to be the case the larger the adoption costs, $F$, and/or the smaller the efficiency gain, $s$.

Proposition 5 confirms the assertion that an efficiency defense may run into danger of being *two-edged*.\(^{44}\) If the merging parties try to make a case for substantial efficiencies to be realized with a merger, then it is doubtful that they will qualify as merger specific; simply because, if the efficiency gain becomes too large, then it will also be implemented in the absence of the merger.

In addition, strong merger specificity is also less likely to occur, the smaller the merging firms’ pre-merger market shares. For a given efficiency level, $s$, a smaller pre-merger market share lowers the post-merger market shares which negatively affects adoption incentives. According to Proposition 5, this logic goes so far that almost any claimed efficiency cannot be strongly merger specific whenever firms’ pre-merger market shares become very small (i.e., approaches

\(^{42}\)Note that the upper bound $\bar{F}$ is exactly equal to the “first” firm’s adoption incentives $\phi_{n,A}^A$.

\(^{43}\)Of course, adoption costs must not exceed the merged firm’s adoption incentive, $\phi_m$.

\(^{44}\)That point is nicely worked out in Farrell and Shapiro (2001).
Weakly merger specific efficiencies. We turn to weak merger specificity by comparing the merged firm’s incentives to adopt the efficiency with the incentives without the merger. Hence, we must distinguish the cases i)-iii) as stated in Lemma 2. According to Definition 2, we take efficiencies as weakly merger specific whenever adoption incentives are larger in the merger case, \( \phi_m \), than in the no-merger case, \( \phi_n^{k,k'} \); i.e., inequality

\[
\Psi^{k,k'} := \phi_m - \phi_n^{k,k'} > 0
\]

holds, where \( \Psi^{k,k'} \) stands for the difference of incentives depending on the adoption game outcome. If the sign of \( \Psi^{k,k'} \) is negative, then incentives are larger without a merger, so that efficiencies do not qualify as weakly merger specific. Our results are presented in the following proposition.\(^{45}\)

**Proposition 6.** Suppose \( F < \min\{F, \phi_m\} \). Whether or not efficiencies are weakly merger specific depends on the outcome of the adoption game as follows:

i) If \((A, A)\), then \( \Psi^{A,A} > 0 \) always holds.

ii) If \((A, NA)\), then \( \Psi^{A,NA} < 0 \) always holds.

iii) If \((r, r)\), then there exists a critical value \( F_\delta \), such that \( \Psi^{r,r} > 0 \) (\( \Psi^{r,r} < 0 \)) holds for \( F < F_\delta \) (\( F > F_\delta \)), with \( \partial F_\delta / \partial s > 0 \).

Finally, \( \Psi^{k,k'} = 0 \) holds at \( F = \underline{F}, \overline{F} \) and, in case iii), also at \( F = F_\delta \).

Proposition 6 shows that a certain efficiency is more likely to be weakly merger specific when it becomes more attractive. In other words, if the intensity of competition for technology adoption in the no-merger subgame is relatively low. Fixing the level of \( F \), case i) is more likely to hold the larger the efficiency gain \( s \), while case ii) applies for lower values of \( s \). This feature is also present in case iii) where the mixed strategy equilibrium holds in the adoption game. Again, fixing \( s \), efficiencies are weakly merger specific for small values of \( F \), whereas the opposite holds for values of \( F \) which surpass the critical value \( F_\delta \).

\(^{45}\)To understand the ordering in Proposition 6, note that \( \phi_m > \underline{F} \) is always true. The sign of \( \phi_m - \overline{F} \) follows from Proposition 5. By assuming \( F \leq \min\{F, \phi_m\} \), we do not consider cases where \( \phi_m \leq F < \overline{F} \). In those instances the efficiency is only adopted without a merger.
Similarly, fixing \( F \), we obtain that relatively low values of the efficiency gain, \( s \), make it less likely that a claimed efficiency gain is weakly merger specific. A smaller value of \( s \) increases the intensity of competition for the efficiency gain in the adoption game, so that incentives tend to be larger without a merger than with a merger. This relationship is also present in case \( iii \) of Proposition 6, where \( F \) is the upper bound of the fixed adoption costs for which an efficiency is weakly merger specific. That critical value is increasing in the efficiency level, \( s \), so that weak merger specificity is more likely to hold the larger the claimed efficiency level, \( s \).

We note that our results concerning the weak merger specificity criterion do not confirm the assertion that claiming efficiencies is two-edged. While this is an issue for strong merger specificity, a contradiction between the attractiveness of the claimed efficiency (in terms of parameters \( s \) and/or \( F \)) and the weak specificity requirement is largely absent. A high value of \( s \) and/or a low value of \( F \) make it more likely that a claimed efficiency is weakly merger specific. The opposite holds for the strong merger specificity criterion. In that case, a more attractive efficiency (with a higher value of \( s \) and/or a lower value of \( F \)) is less likely to be strongly merger specific.

We next turn to the question whether the specificity requirement helps to select welfare improving merger proposals. In Table 2, we summarize our results concerning endogenous merger proposals, merger specificity, and the social welfare effects of a merger.

If both firms adopt \( s \) in the no-merger subgame, efficiencies are always merger specific, but the merger is only socially desirable for sufficiently small efficiencies or sufficiently large adoption costs. In this case, a welfare improving merger is always accompanied by merger specific efficiencies. Note, however, that a merger is only profitable whenever efficiencies are sufficiently large which does not necessarily coincide with an increase of social welfare when mergers to dominance are proposed (see Corollary 2). From a social welfare perspective the specificity requirement does not help to make better merger control decisions in that area.
Table 2. Merger Decision, Merger Specificity, and Social Welfare

<table>
<thead>
<tr>
<th>Adoption Outcome</th>
<th>$\rho^{k,k'} &gt; 0$</th>
<th>$\Psi^{k,k'} &gt; 0$</th>
<th>$\Delta SW^{k,k'} &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no-merger case: $k, k'$</td>
<td>merger proposal</td>
<td>merger specific</td>
<td>social welfare</td>
</tr>
<tr>
<td>$(A, A)$</td>
<td>if $s$ and $F$ large</td>
<td>always*</td>
<td>if $s$ small</td>
</tr>
<tr>
<td>$(A, NA)$</td>
<td>if $s$ large</td>
<td>never*</td>
<td>if $s$ large</td>
</tr>
<tr>
<td>$(r, r)$</td>
<td>if $s$ large and $F$ small</td>
<td>if $s$ large*</td>
<td>if $s$ large or $s$ and $F$ small</td>
</tr>
<tr>
<td>$(NA, NA)$</td>
<td>never</td>
<td>always**</td>
<td>see Proposition 2</td>
</tr>
</tbody>
</table>

* refers to weak merger specificity
** refers to strong merger specificity

Turning to case ii), claimed efficiencies are never merger specific, but all catch-up merger proposals and a large majority of merger to dominance proposals are socially desirable as we have shown in Corollaries 1 and 2. We conclude that the specificity requirement may more often than not lead to a rejection of welfare increasing mergers in this area.

When considering the mixed strategy equilibrium, $(r, r)$, proposed mergers are not always accompanied by merger specific efficiencies. It, again, follows that not every welfare enhancing merger exhibits efficiencies which meet the criterion of merger specificity. Finally, for $(NA, NA)$ no mergers are proposed, so that the efficiency defense does not apply. Interestingly, only those efficiencies, which are never adopted in the no-merger case, qualify as strongly merger specific. Our model then predicts that those instances are completely irrelevant, since firms never find it optimal to submit a merger proposal when efficiencies are strongly merger specific.

It follows that the specificity requirement may only be helpful when the mixed strategy equilibrium solves the adoption game in the no-merger subgame or case ii) of Lemma 2 applies where only one firm adopt for sure in the no-merger case. In the former case, the specificity requirement may not be met for endogenous mergers which would reduce social welfare. In the latter case, there exists an area where mergers among large firms reduce social welfare if the efficiency gain $s$ is too small. In all other instances, the specificity requirement is either superfluous or excessively restrictive leading to the blockage of merger proposals which would enhance social welfare.

The fact that the specificity requirement largely fails to select those endogenous merger proposals which are welfare enhancing, but rather leads to significant type-I errors is a strong
argument for a *laissez-faire* regime. The main reason why a *laissez-faire* regime is advisable is that the self selection process tends to favor merger proposals with relatively large efficiencies. Allowing for some qualifications (in particular, case $(A, A)$ in Table 2), this observation tends to be more in line with a social welfare assessment when compared with the specificity requirement.

Another implication of the incongruence of the specificity requirement and the social welfare objective should be that merging firms completely abstain from submitting an efficiency defense. Assume that the antitrust authority then evaluates the merger on the ground that no efficiencies will be realized (i.e., it expects $s \to 0$).\footnote{It is reasonable that the efficiency level is private information of the merging parties. As the merging firms carry the burden of proof to make use of the efficiency defence, we can safely suppose that the antitrust authority disregards any efficiencies all together when not submitted.} In those instances, a welfare maximizing authority would refer to the measure $\Delta SW$ as stated in Proposition 2. From Proposition 2, we know that for $s \to 0$, any relatively small merger (with $c > 9/107$) is welfare improving. In cases $(A, A)$ and $(A, NA)$ (and to some extent also in case $(r, r)$), not claiming efficiencies should be the only way to get a merger accepted by a welfare maximizing antitrust authority which requires efficiencies to be merger specific. This observation may explain that the efficiency defense has not become a success story.

### 6 Discussion and Extensions

In this section, we examine how the substantial plus specific test -which is prominent in current merger control- relates to the socially optimal assessment of merger proposals. Furthermore, we analyze the relationship between market growth and the specificity requirement.

**Substantial plus specific test.** One test which is possibly closest to current practice is to require that efficiencies are both substantial and merger specific. We call that approach the substantial plus specific test (SST). Suppose substantial means that prices do not increase relative to the pre-merger equilibrium.\footnote{This seems to be in line with both US and EC merger guidelines where it is postulated that consumers are (at least) not worse off as a result of the merger.} In the parlance of our model, efficiencies are substantial whenever consumer surplus is larger after the merger, $\text{CS}^{**}(A)$, when compared with the consumer surplus before the merger, $\text{CS}^{*}(NA, NA)$, i.e., $\Delta \text{CS} > 0$ holds (see Proposition 1).

It is reasonable that the efficiency level is private information of the merging parties. As the merging firms carry the burden of proof to make use of the efficiency defence, we can safely suppose that the antitrust authority disregards any efficiencies all together when not submitted.

This seems to be in line with both US and EC merger guidelines where it is postulated that consumers are (at least) not worse off as a result of the merger.
The following table illustrates the conditions under which efficiencies are substantial and merger specific for all possible adoption equilibria in the no-merger case. In addition, it presents our findings on the welfare effects of a merger from Proposition 4.

Table 3. Substantial Plus Specific Test and Social Welfare

<table>
<thead>
<tr>
<th>Adoption Outcome</th>
<th>$\Delta CS &gt; 0$</th>
<th>$\Psi^{k,k'} &gt; 0$</th>
<th>$\Delta SW^{k,k'} &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no-merger case: $k, k'$</td>
<td>substantial</td>
<td>merger specific</td>
<td>social welfare</td>
</tr>
<tr>
<td>$(A, A)$</td>
<td>if $s$ large</td>
<td>always*</td>
<td>if $s$ small</td>
</tr>
<tr>
<td>$(A, NA)$</td>
<td>if $s$ large</td>
<td>never*</td>
<td>if $s$ large</td>
</tr>
<tr>
<td>$(r, r)$</td>
<td>if $s$ large</td>
<td>if $s$ large*</td>
<td>if $s$ large or $s$ and $F$ small</td>
</tr>
<tr>
<td>$(NA, NA)$</td>
<td>if $s$ large</td>
<td>always**</td>
<td>see Proposition 2</td>
</tr>
</tbody>
</table>

* refers to weak merger specificity  
** refers to strong merger specificity

When the antitrust authority applies a SST rule, it runs into danger to block socially desirable mergers or to allow too many mergers. As we noted above, the merger specificity requirement is not aligned with the social welfare evaluation of an efficiency creating merger. While the specificity requirement completely fails to help in the selection process under $(A, A)$, there can be instances that the specificity requirement is not met under some mergers which involve substantial efficiencies in cases $(A, NA)$ and $(r, r)$.

Nevertheless, the results are far from clear cut, so that we conclude that a SST rule is at least questionable, as a simpler “substantial-only” requirement is likely to do not much worse. Anyway, a substantial-only requirement would avoid an additional rejection of welfare increasing mergers due to an excessively restrictive specificity requirement.

Finally, and as noted before, firms may be well advised to drop an efficiency claim altogether in which case the antitrust authority would evaluate the merger based on $s \to 0$. While this strategy can only be successful when the merging firms’ market share is small enough (or, $c$ sufficiently large), it appears, in many instances, the only way to avoid the pitfalls associated with an efficiency defense.

**Growing and declining markets.** We now examine the argument that an efficiency claim is less likely to be merger specific in a growing market (see Farrell and Shapiro, 2001). The
reasoning is that firms’ will adopt efficiencies by internal growth when market size is increasing. The opposite should then be true in a declining market. We drop our initial parameter restriction $A - a \equiv 1$ and instead define the parameter $\lambda := A - a$, with $\lambda \geq 1$. An increase (decrease) of $\lambda$ is indicative of market growth (decline). We ask how our merger specificity measure, $\Psi_{k,k'}$ (see (7)), reacts to a marginal change of $\lambda$; i.e., we calculate $\partial \Psi_{k,k'}/\partial \lambda$. If that derivative is positive (negative), then market growth tends to make a certain efficiency level $s$ more likely to be merger specific. In addition, we are interested how $\lambda$ affects firms’ incentive to merge (measured by $\theta_{k,k'}$) in the first place. Our results are as follows.

**Proposition 7.** Suppose $F < \phi_m$. A change in $\lambda$ has the following effects on merger specificity, $\Psi_{k,k'}$, and firms’ merger incentives, $\theta_{k,k'}$:

i) $(F < F)$: If $(A, A)$, then $\partial \Psi_{A,A}/\partial \lambda > 0$ and $\partial \theta_{A,A}/\partial \lambda < 0$.

ii) $(F \leq F \leq F)$: If $(A, NA)$, then $\partial \Psi_{A,NA}/\partial \lambda < 0$, while there exists a critical value $s_Y$, such that $\partial \theta_{A,NA}/\partial \lambda > 0$ ($\partial \theta_{A,NA}/\partial \lambda < 0$) holds for $s > s_Y$ ($s < s_Y$).

iii) $(F \leq F \leq F)$: If $(r, r)$, then $\partial \Psi_{r,r}/\partial \lambda > 0$ and $\partial \theta_{r,r}/\partial \lambda > 0$.

iv) $(F > F)$: If $(NA, NA)$, then there exists a critical value $s_Z$, such that $\partial \theta_{NA,NA}/\partial \lambda > 0$ ($\partial \theta_{NA,NA}/\partial \lambda < 0$) holds for $s > s_Z$ ($s < s_Z$), while the specificity measure does not change.

Given that both non-dominant firms adopt the technology, i.e., case i) applies, market growth always makes efficiencies more merger specific. Interestingly, the merger incentives tend to decline with market growth in that area, so that this constellation appears to become less relevant. If $(A, NA)$ is the adoption game outcome in the no-merger case, efficiencies are initially never merger specific. A growing market even increases the extent to which efficiencies are merger “unspecific”, while merger incentives are increasing when the efficiency gain is large enough. Hence, we expect that region to become more relevant under market growth and large enough efficiencies. In the mixed strategy equilibrium, market growth has a positive effect on merger specificity. We conclude that our hypothesis that in growing markets efficiencies are less convincing can only be supported for $(A, NA)$. For $(A, A)$ and $(r, r)$, we find that the opposite holds: a growing market makes efficiencies more merger specific.

Turning to the non-dominant firms’ incentive to merge, $\theta_{k,k'}$, we also find ambiguous effects of market growth. If $(A, A)$ is the adoption equilibrium, then non-dominant firms are less inclined to merge when markets are growing, i.e., $\partial \theta_{A,A}/\partial \lambda < 0$ always holds. The opposite
holds for \((r, r)\), i.e., the non-dominant firms’ incentives to merge are strictly increased. In the asymmetric equilibrium, \((A, NA)\), the marginal effect of market growth depends on \(c\) and \(s\). We find that for sufficiently high (low) efficiency levels, \(s\), non-dominant firms have stronger (weaker) incentives to merge. Qualitatively, the same relationship is true for \((NA, NA)\). We conclude, that a growing market does not necessarily make a merger less attractive. Rather, the effect of market growth crucially depends on the adoption game equilibrium in the no-merger subgame as well as on the level of efficiency gains, \(s\).

7 Conclusion

In the first part of our paper, we analyzed the social welfare effect of a merger when efficiencies are exogenous. For mergers between relatively small firms a non-monotone relationship between efficiency gains and social welfare exists. Hence, small firm mergers may be not so innocuous as is often presumed; in contrast, a small firm merger with moderate efficiency gains can be more harmful to society than a merger between much larger firms generating no efficiency gains at all.

In the second part, we extended the analysis by introducing endogenous efficiencies which are also realizable in the absence of a merger. We introduced an adoption stage in which firms can choose whether or not to implement an efficiency enhancing technology with a merger and without a merger. Implementation of the technology reduces unit costs and requires to incur fixed adoption costs. As larger firms always have larger incentive to adopt the technology, our specification also mirrors the idea that no-synergy efficiencies are closely related to economies of scale.

In addition, we endogenized the merger decision to analyze the relationship between the self-selection process behind merger proposals and their social welfare consequences. Our results crucially depend on the outcome of the adoption game in the no-merger case; i.e., the without merger counterfactual. We show that any proposed merger increases social welfare, if it is a catch-up merger. However, proposed mergers to dominance reveal some potential for negative welfare effects. If only a single firm would adopt the efficiency without a merger, then efficiencies must be sufficiently large to make a merger proposal socially desirable. If both firms adopt, then the converse is true: efficiencies have to be sufficiently small for social welfare to increase. This
result mirrors the intuition that mergers are socially not desirable if all firms adopt the efficiency anyway. Turning to the mixed strategy equilibrium, we again obtain that a merger proposal is welfare enhancing when efficiencies are small. However, at the same time a merger may also increase social welfare for larger efficiency gains because of the duplication of fixed adoption costs in the no-merger case.

In the third part, we examine whether the merger specificity requirement in merger control helps to select socially desirable merger proposals. For this purpose, it is important to note that by endogenizing efficiency gains we explicitly account for the without merger counterfactual. We distinguish between strong and weak merger specificity. The strong specificity case allows us to single out most clearly the two-edge character of an efficiency defense: efficiencies are only merger specific whenever they are sufficiently small given relatively large adoption cost levels. However, for the weak merger specificity case, the two-edge character cannot be confirmed. More precisely, we find that efficiencies tend to be less likely to be specific when the adoption costs increase or the efficiency level decreases. We finally show that the specificity rule fails to mirror a correct social welfare assessment. When both firms adopt the efficiency in the no-merger case, then all endogenously proposed mergers meet the specificity requirement, while only those proposals are socially desirable which involve relatively small efficiency gains. If only a single firm adopts the efficiency in the no-merger case, then the specificity requirement is never fulfilled while endogenously proposed mergers are always welfare enhancing when the efficiency gain is large enough.

We interpret our results as supportive to a laissez-faire approach which is more often than not better aligned with a consumer surplus or a social welfare standard. The main reason behind this observation is that endogenously proposed mergers tend to exhibit efficiency gains which are increasing in the merging firms’ pre-merger market shares. For many constellations, therefore, the selection process behind firms’ merger decisions tends to favor mergers with relatively large efficiencies which is often aligned with a social welfare objective. Another important implication is that the specificity requirement should induce firms not to claim efficiencies at all. In several instances, such a strategy is the only possible way to convince the authority to confirm the merger. This result may explain why the efficiency defense has played a minor role in phase II mergers at the EU level so far.
We finally show that market growth has an ambiguous effect on both the specificity of efficiencies and merger incentives. We conclude that a proper merger analysis should fully take care of the \textit{with} and the \textit{without} merger counterfactuals. Condensing the entire without merger analysis into a “specificity” requirement for claimed efficiencies must lead to inconsistencies and false merger decisions.
Appendix

In this Appendix we provide the omitted proofs.

Proof of Proposition 1. We derive the critical values stated in the proposition and their properties. It is straightforward to calculate \( \Delta \pi_d = (q_d^{**})^2 - (q_d^*)^2 \), \( \Delta \pi_m = (q_m^{**})^2 - 2(q_m^*)^2 \), and \( \Delta CS = [(2(q_d^{**} + q_m^{**}))^2]/2 - 2(q_d^* + q_m^*)^2 \) (the values of firms’ equilibrium quantities are stated in Section 3). Solving \( \Delta \pi_d = 0 \) yields two zeros \( c_A = (5s - 9)/13 \) and \( c_B = (1 - 5s)/3 \). Obviously, the first root is not feasible. Rewriting the second root gives \( s_B(c) := (1 - 3c)/5 \). Note that \( \partial s_B(c)/\partial c < 0 \). It is easily checked that \( \Delta \pi_d < 0 \) holds if \( s > s_B(c) \), while the opposite is true for \( s < s_B(c) \).

Inspecting next \( \Delta \pi_m = 0 \), we get two roots \( c_C = 1/3 - 5s(5 - 4\sqrt{2})/7 \) and \( c_D = 1/3 - 5s(5 + 4\sqrt{2})/7 \). Again, the first root is never feasible. Rewriting the second root gives \( s_D(c) := 7(1 - 3c)/[15(5 + 4\sqrt{2})] \). Note that \( \partial s_D(c)/\partial c < 0 \). It follows that \( \Delta \pi_m < 0 \) if \( s < s_D(c) \), while the opposite holds for \( s > s_D(c) \).

The ordering \( s_D(c) < s_B(c) \) follows from noticing that \( \lim_{s \to 0} c_B = \lim_{s \to 0} c_D = 1/3 \) together with \( |\partial c_B/\partial s| = 5/3 < 5(5 + 4\sqrt{2})/7 = |\partial c_D/\partial s| \).

Finally, examining \( \Delta CS = 0 \) we get two zeros \( c_B = (1 - 5s)/3 \) and \( c_E = (31 + 5s)/13 \). Obviously, the second one is not feasible. The first zero gives \( s_B(c) := (1 - 3c)/5 \). It is easily checked that \( \Delta CS > 0 \) holds, if \( s > s_B(c) \), while the opposite is true for \( s < s_B(c) \).

Proof of Proposition 2. Calculating \( \Delta SW = 2\Delta \pi_d + \Delta \pi_m + \Delta CS = 0 \) we get two roots

\[
\begin{align*}
c_G(s) &= \left( \frac{67 - 575s + 40\sqrt{322s^2 + 2s + 1}}{321} \right) \quad (8) \\
c_H(s) &= \left( \frac{67 - 575s - 40\sqrt{322s^2 + 2s + 1}}{321} \right) \quad (9)
\end{align*}
\]

Note that \( \lim_{s \to 0} c_G = 1/3 \) and \( \lim_{s \to 0} c_H = 9/107 \). Note also that \( \partial c_G/\partial s|_{\lim s \to 0} = -5/3 < 0 \). Hence, both roots cut through the feasible set. Moreover, \( \partial c_G/\partial s < 0 \) is always true. Hence, for all \( c < 9/107 \) there exists a unique critical value \( s_H(c) \), with \( s_H(c) := [c_H(s)]^{-1} \), for which \( \partial s_H(c)/\partial c < 0 \) holds. It is easily checked that \( \Delta SW > 0 \) if \( s > s_H(c) \), while the opposite is true for \( s < s_H(c) \).

Turning to \( c_G(s) \), we obtain \( \partial c_G/\partial s = 0 \) at \( s' = (5\sqrt{23} - 1)/322 \), while \( \partial^2 c_G/\partial s^2 > 0 \) holds everywhere. Evaluating \( c_G(s) \) at \( s = s' \) gives \( c_G(s') = (5\sqrt{23} + 69)/322 < (323 - 5\sqrt{23})/966, \)
so that \( c_G \) reaches its (global) minimum in the feasible set.\(^{48}\) Hence, \( c_G(s) \) exhibits a strictly negative slope over the interval \( s = (0, s') \) and a strictly positive slope over the interval \( s = (s', 1 - 3c) \). The inverse of \( c_G(s) \) is thus a correspondence which assigns to all \( c > c_G(s') \) exactly two values \( \underline{s}(c) \) and \( \overline{s}(c) \), with \( \underline{s}(c) < \overline{s}(c) \). From the strict convexity of \( c_G(s) \) it follows that \( \partial \underline{s}(c)/\partial c < 0 \) and \( \partial \overline{s}(c)/\partial c > 0 \), for all \( c > c_G(s') \). It is easily checked that \( \Delta SW < 0 \) if \( s \in (\underline{s}(c), \overline{s}(c)) \), while the opposite is true for \( s \in (0, \underline{s}(c)) \cup (\overline{s}(c), 1/2) \).

Furthermore, note that solving \( c_G(s) = 1/3 \) for \( s \) gives two thresholds: \( s = 0 \) and \( s = 16/69 \). That is, \( c_G(s) \), and thus \( \underline{s}(c) \) and \( \overline{s}(c) \) are only relevant for catch-up mergers.

Finally, the intervals stated in the proposition follow from noting that \( c_G(s') > \lim_{s \to 0} c_H(s) \). Hence, we can distinguish three different intervals depending on \( c \) as stated in the proposition.

**Proof of Lemma 1.** The merged firm’s incentive, \( \phi_m = (q_m^* (A))^2 - (q_m^* (NA))^2 \) can be rewritten as \( \phi_m = 3s [2(1 - 3c) + 3s]/16 \) (the equilibrium outputs of the merged firm are stated in the main text). Hence, the efficiency is (strictly) implemented if and only if \( F < \phi_m = 3s [2(1 - 3c) + 3s]/16 \). The properties \( \phi_m > 0 \), \( \partial \phi_m/\partial s > 0 \), and \( \partial \phi_m/\partial c < 0 \) follow immediately.

**Proof of Lemma 2.** Using (1) we can directly calculate the non-dominant firms’ \((n = 3, 4)\) equilibrium outputs depending on their adoption decisions (the first argument of \( q_n(\cdot, \cdot) \) stands for firm \( n \)’s and the second argument for firm \( n' \)’s, \( n \neq n' \), adoption decision): \( q_n^*(NA, NA) = (1 - 3c)/5 \), \( q_n^*(A, NA) = (1 - 4(c-s) + c)/5 \), \( q_n^*(NA, A) = (1 - 3c - s)/5 \), and \( q_n^*(A, A) = (1 - 3c - s)/5 \). Accordingly, firms’ equilibrium profit levels are given by \( \pi_n^*(k, k') = [q_n^*(k, k')]^2 \). Calculating the profit differentials \( F := \pi_n^*(A, A) - \pi_n^*(NA, A) \) and \( \overline{F} := \pi_n^*(A, NA) - \pi_n^*(NA, NA) \), we obtain the ordering \( 0 < F < \overline{F} \). The first inequality is obvious, and the second inequality follows from

\[
\pi_n^*(A, A) - \pi_n^*(NA, A) - [\pi_n^*(A, NA) - \pi_n^*(NA, NA)] = -8s^2/25 < 0.
\]

Given that ordering, it is obvious that \((A, A)\) is the only Nash equilibrium if \( F < \overline{F} \), while \((NA, NA)\) must be the only Nash equilibrium whenever \( F > \overline{F} \). If \( F \) lies in between both values \((\underline{F} < F < \overline{F})\), then two pure strategy Nash equilibria exist where one firm adopts and the other firm abstains from adopting. Moreover, in that interval there is a unique Nash equilibrium in

\(^{48}\)Note that Assumption 3 requires that \( c < (1 - s)/3 \) holds. Inserting \( s' = (5\sqrt{23} - 1)/322 \), we obtain that \( c < (323 - 5\sqrt{23})/966 \) must hold.
mixed strategies where both firms choose the same probability distribution \( r \). The equilibrium probability, \( r \), with which each firm chooses to adopt follows from an indifference condition; for instance, \( \pi_n(A, r) = \pi_n(NA, r) \), where \( \pi_n(\cdot) \) stands now for the expected value of firm \( n \)'s profit. Solving that indifference condition for \( r \) yields

\[
r = \frac{\pi_n^*(A, NA) - \pi_n^*(NA, NA) - F}{[\pi_n^*(A, NA) - \pi_n^*(A, A) + \pi_n^*(NA, A) - \pi_n^*(NA, NA)]}.
\]  

(10)

In the assumed interval, \( F < F < F \), both the numerator and the denominator are always positive. Moreover, \( r \) (as given by (10)) is monotonically decreasing in \( F \). It approaches zero, if \( F \to 0 \), whereas it approaches one, if \( F \to F \). Because of the symmetry of the adoption game, (10) characterizes the unique mixed strategy equilibrium, \((r, r)\).

Finally, differentiating \( F \) and \( F \) with respect to \( s \) gives

\[
\frac{\partial F}{\partial s} = 8s (1 - 3c + s) / 25 \quad \text{and} \quad \frac{\partial F}{\partial s} = 8s (1 - 3c + 2s) / 25,
\]

(11)

(12)

from which \( \partial F / \partial s > \partial F / \partial s \) follows. Moreover, inspecting the right-hand expression of (11) and (12) it is immediate that both values \( F \) and \( F \) approach zero when \( s \) goes to zero.

Proof of Proposition 3. The proof follows from calculating the sign of \( \theta^{k,k'} \) for all cases i)-iv) stated in Lemma 2.

Case i). When both firms adopt in the no-merger subgame, then \( \theta^{A,A} = \pi_n^*(A) - 2\pi_n^*(A, A) + F \). Obviously, \( \theta^{A,A} \) increases in \( F \). Note that \( \theta^{A,A}(F = 0) < 0 \). Setting \( \theta^{A,A} = 0 \), we obtain the threshold value \( F_\alpha \) stated in the proposition; namely, \( F_\alpha := -[\pi_n^*(A) - 2\pi_n^*(A, A)] = 7(1 - 3(c - s))^2 / 400 \). Hence, \( \theta^{A,A} < 0 \) holds for \( F < F_\alpha \). Moreover, \( F_\alpha \) lies in the feasible set, if \( F_\alpha < F \). We obtain for the difference \( F - F_\alpha \) the expression \( 8s (1 - 3c + s) / 25 - 7(1 - 3(c - s))^2 / 400 \) which has two roots \( c_l(s) = (1 - 13s) / 3 \) and \( c_f(s) = (7 + 5s) / 21 \). The latter one is not feasible. The former root, \( c_f(s) \), is feasible and monotonically decreasing in \( s \). It is then easily checked that \( F_\alpha < F \) holds for \( s > s_l(c) := (1 - 3c) / 13 \). If, otherwise, \( s < s_l(c) \), then \( \theta^{A,A} < 0 \) is always true where \( s_l(c) = [c_l(s)]^{-1} \).

Case ii). In both pure strategy equilibria of this interval only one firm adopts. Hence, \( \theta^{A,NA} = \pi_n^*(A) - F - [\pi_n^*(A, NA) - F + \pi_n^*(NA, A)] \). Clearly, \( F \) cancels out so that the sign of \( \theta^{A,NA} \) only depends on \( c \) and \( s \). Substituting the profit levels, we obtain two roots
$c_K(s) = (7 - 47s)/21$ and $c_L(s) = (1 - s)/3$. The latter root is not feasible as $c < (1 - s)/3$ must hold (Assumption 3). Considering the former critical value, we find that $\lim_{s \to 0} c_K = 1/3$, with $\partial c_K/\partial s = -47/21$. Hence, $c_K(s)$ cuts through the feasible set. Note also that $\lim_{s \to 0} \theta^{A,NA} < 0$ for all $c < 1/3$. Define $s_K := [c_K(s)]^{-1}$. It is then easily checked that $\theta^{A,NA} < 0$ if $s < s_K(c)$, while $\theta^{A,NA} > 0$ holds for $s > s_K(c)$.

Case iii). If the mixed strategy equilibrium is played in the adoption game, then $\theta^{r,r} = \pi_m^*(A) - F - 2E\pi_n^*(NA, r)$. Using (17) and (18), we can rewrite this equality as

$$\theta^{r,r} = \pi_m^*(A) + F - 2 \left[ \pi_n^*(A, NA) - \frac{\phi_n^{NA,NA} - F}{\phi_n^{NA,NA} - \phi_n^{AA}} \right].$$

(13)

We obtain

$$\frac{\partial \theta^{r,r}}{\partial F} = - \frac{\pi_n^*(NA, NA) - \pi_n^*(NA, A)}{\phi_n^{NA,NA} - \phi_n^{AA}} < 0.$$ 

Hence, merger incentives are monotonically decreasing in $F$. Using (13) to solve for $F = F_\beta$ such that $\theta^{r,r}(F_\beta) = 0$ we get

$$F_\beta = \left( \phi_n^{NA,NA} - \phi_n^{AA} \right) \left( \pi_m^*(A) - 2\pi_n^*(A, NA) \right) + 2\phi_n^{NA,NA} \left[ \pi_n^*(A, NA) - \pi_n^*(A, A) \right] \over 2 \left[ \pi_n^*(A, NA) - \pi_n^*(A, A) \right] - \left( \phi_n^{NA,NA} - \phi_n^{AA} \right).$$

(14)

Note that the denominator is always positive which follows from

$$2 \left[ \pi_n^*(A, NA) - \pi_n^*(A, A) \right] > \left( \phi_n^{NA,NA} - \phi_n^{AA} \right) \Rightarrow \pi_n^*(A, NA) - \pi_n^*(A, A) > - \left[ \pi_n^*(NA, NA) - \pi_n^*(NA, A) \right].$$

We next turn to feasibility of $F_\beta$. Inspecting the lower bound of the interval and using (14), we get that $F_\beta > F$ holds if

$$\pi_m^*(A) - \pi_n^*(A, A) - \pi_n^*(NA, A) > 0.$$ 

(15)

Inserting the profit levels into the left-hand side of (15), we obtain the expression

$$
- \frac{63}{400} c^2 - \frac{129}{200} cs + \frac{21}{200} c + \frac{13}{80} s^2 + \frac{43}{200} s - \frac{7}{400}
$$

which has two roots: $c_1(s) = (1 - 13s)/3$ and $c_M(s) = (3 + 5s)/21$. Only the former is feasible. Define $s_I(c) := [c_I(s)]^{-1} = (1 - 3c)/13$. It is then easily checked that $F_\beta > F$ holds for all $s > s_I(c)$. If, otherwise, $s < s_I(c)$, then $\theta^{r,r} < 0$ is always true.

We turn to the upper bound. We obtain that $F_\beta < F$ holds if

$$\pi_m^*(A) - \pi_n^*(A, NA) - \pi_n^*(NA, NA) < 0.$$ 

(16)
Substituting the profit levels, we get that inequality (16) holds for all feasible \( c \) and \( s \). Hence, when \( F \) approaches the upper bound of the considered interval, then \( \theta^{r,r} < 0 \) is always true. We finally, calculate \( \partial F_\beta / \partial s \) and obtain the expression

\[
-513c^3 + 1476c^2s + 513c^2 - 852cs^2 - 984cs - 171c + 161s^3 + 284s^2 + 164s + 19 \\
50(3s - 6c + 2)^2 / 3
\]

The denominator of that expression is always positive, so the sign of \( \partial F_\beta / \partial s \) depends on the sign of the numerator which we define by \( \eta(c, s) \). We show that \( \eta(c, s) > 0 \) holds, so that \( \partial F_\beta / \partial s > 0 \) follows.

**Ancillary Claim.** \( \eta(c, s) > 0 \) holds everywhere.

**Proof.** We successively differentiate \( \eta(c, s) \) with respect to \( s \). This yields \( \eta'(c, s) = 1476c^2 - 1704cs - 984c + 483s^2 + 568s + 164 \), \( \eta''(c, s) = 966s - 1704c + 568 \), and \( \eta'''(c, s) = 966 \). Hence, \( \eta''(c, s) \) is increasing in \( s \). Evaluating \( \eta''(c, s) \) at \( s = 0 \), we get \( \eta''(c, s = 0) = -1704c + 568 \).

Clearly, \( \eta''(c, s = 0) > 0 \) for all \( 0 \leq c < 1/3 \). Hence, \( \eta''(c, s) > 0 \) holds everywhere. Evaluating \( \eta'(c, s) \) at \( s = 0 \), we get \( \eta'(c, s = 0) = 1476c^2 - 984c + 164 \). We get that \( \partial \eta'(c, s = 0) / \partial c = 0 \) at \( c = 1/3 \). As \( \partial^2 \eta'(c, s = 0) / \partial c^2 > 0 \) holds, \( \eta'(c, s = 0) \) reaches a global minimum at \( c = 1/3 \).

Evaluating \( \eta'(c, s) \) at \( s = 0 \) and \( c = 1/3 \) we get \( \eta'(c, s) = 0 \). Hence, \( \eta'(c, s) > 0 \) holds everywhere. Then, we get \( \eta(c, s = 0) = 19(1 - 3c)^3 \) which, of course, is strictly positive for all \( 0 \leq c < 1/3 \).

This proves the claim.

**Case iv).** When both firms do not adopt the efficiency in the no-merger subgame, then \( \theta^{NA,NA} = \pi_m^*(A) - F - 2\pi_m^*(NA, NA) \). Note that \( \theta^{NA,NA} \) is monotonically decreasing in \( F \).

Evaluating \( \theta^{NA,NA} \) at the lower bound \( F = \overline{F} \) we get the expression

\[
-(7 - 42c + 63c^2 + 66sc - 22s + 31s^2) / 400
\]

which has no real root. It is then easily checked that \( \theta^{NA,NA} < 0 \) holds for all considered \( c \) and \( s \).

**Proof of Proposition 4.** The proof follows from calculating the sign of \( \Delta SW^{k,k'} \) for the cases (i)-(iv) stated in Lemma 2. Consumer surplus in the no-merger case, \( CS^*(k, k') = |Q(k, k')|^2 / 2 \), depends on the non-dominant firms’ adoption decisions \( k, k' = A, NA, r \). Note that \( Q^*(k, k') = 2q^*_d(k, k') + q^*_n(k, k') + q^*_n(k', k) \), \( n \neq n' \). We stated the values of \( q^*_n(k, k') \) in the proof of Lemma 2. Using (1), we obtain for the dominant firms’ equilibrium outputs \( q^*_d(A, A) = (1 + 2(c - s)) / 5 \), \( q^*_d(A, NA) = (1 + 2c - s) / 5 \), and \( q^*_d(NA, NA) = (1 + 2c) / 5 \).
Case i). If \((A, A)\), then \(\Delta SW^{A, A} = SW^{**} - F - SW^{A, A}\), with \(SW^{A, A} = 2\pi_d^{A, A} + 2\pi_n^{A, A} + CS^{A, A} - 2F\). Using the value of \(SW^{**}\) from the proof of Proposition 2, we obtain \(\Delta SW^{A, A} = (-321c^2 + 642cs + 134c - 321s^2 - 134s - 9)/800 + F\) which has two roots \(c_N(s, F) = s + (67 - 20\sqrt{2}/321F + 2)/321\) and \(c_O(s, F) = s + (67 + 20\sqrt{2}/321F + 2)/321\). Note that \(\partial c_O(s, F)/\partial F > 0\) holds. Evaluating \(c_O(s, F)\) at \(F = 0\), we get \(c_O(s, 0) = 1/3\), so that this threshold value is never feasible. Turning to the first root, \(c_N(s, F)\), we get that \(c_N(s, 0) = s + 9/107\). Hence, \(c_N(s, F)\) cuts through the feasible set. It is easily checked that \(\Delta SW^{A, A} > 0\) for \(c > c_N(c, F)\). Because of \(\partial c_N(s, F)/\partial F < 0\), the constraint becomes less binding when \(F\) increases. Taking the inverse of \(c_N(s, F)\), we get the critical value \(s_N(c, F) = c + (20\sqrt{2}/321F + 2 - 67)/321\) which is obviously increasing in \(F\).

We note that \(s_N(c, F)\) is not binding for large \(F\). The smallest adoption cost value, \(F^+\), such that \(s_N(c, F)\) becomes not feasible is calculated from \(s_N(0, F^+) = 1/2\) which gives \(F^+ = 25/128\). Note that \(F^+\) is feasible; i.e., \(F^+ = 25/128 < 12/25 = F(c = 0, s = 1/2)\).

Case ii). If \((A, NA)\), then \(\Delta SW^{A, NA} = SW^{**} - F - SW^{A, NA}\), where \(SW^{A, NA} = 2\pi_d^{A, NA} + \pi_n^{A, NA} - 2\pi_{n'}^{A, NA} + CS^{A, NA} - F\), with \(n \neq n'\). We then get

\[
\Delta SW^{A, NA} = (-321c^2 - 254cs + 134c - 49s^2 + 58s - 9)/800
\]

which has two roots \(c_L(s) = (1 - s)/3\) and \(c_P(s) = (9 - 49s)/107\). The first root is not feasible as Assumption 3 requires \(c < (1 - s)/3\). The second root, \(c_P(s)\), cuts through the feasible set with negative slope. It is easily checked that \(\Delta SW^{A, NA} > 0\) if \(c > c_P(s)\), while the opposite is true for \(c < c_P(s)\). Taking the inverse gives the critical value \(s_P(c) = (9 - 107c)/49\), and the result stated in the proposition follows.

Case iii). If \((r, r)\), then \(\Delta SW^{r, r} = SW^{**} - F - SW^{r, r}\), where \(SW^{r, r} = 2\pi_d^{r, r} + 2\pi_n^{r, r} + CS^{r, r}\) is the expected social welfare in the no-merger case. Firms' expected profits and expected consumer surplus are given by

\[
\pi_d^{r, r} = r^2\pi_d^*(A, A) + 2r(1 - r)\pi_d^*(A, NA) + (1 - r)^2\pi_d^*(NA, NA),
\]

\[
\pi_n^{r, r} = r^2\pi_n^*(A, A) + (1 - r)r\pi_n^*(A, NA) + (1 - r)\pi_n^*(NA, A) + (1 - r)^2\pi_n^*(NA, NA),
\]

\[
CS^{r, r} = \frac{r^2[Q^*(A, A)]^2 + 2(1 - r)r[Q^*(A, NA)]^2 + (1 - r)^2[Q^*(NA, NA)]^2}{2}.
\]
The change in social welfare is then calculated as

\[
\Delta SW^{r,r} = \frac{5}{16} s - \frac{5}{16} c - F - \frac{23}{16} cs + \frac{23}{32} c^2 + \frac{23}{32} s^2 + \frac{15}{32} \frac{\varphi}{800s^2},
\]

with \( \varphi = 1875F^2 + 4400Fcs - 3000F^2s^2 - 800Fs + 3392c^2s^2 - 3392cs^3 - 1408cs^2 + 1152cs + 704s^4 + 448s^2 \).

To show how \( \Delta SW^{r,r} \) depends on \( c, s, \) and \( F \), we first evaluate \( \Delta SW^{r,r} \) at the lower bound, \( F \), and examine how \( \Delta SW^{r,r} \) behaves when \( F \) is marginally varied at \( F = F \). Finally, we analyze the sign of \( \Delta SW^{r,r} \) at the upper bound, \( F \). This strategy enables us to identify critical values for \( c \) and \( s \), respectively, for which the initial sign of \( \Delta SW^{r,r} \) can be determined and ii) for \( F \) such that the sign of \( \Delta SW^{r,r} \) can be specified when \( F \) increases given that \( F \leq F \).

It is immediately verified that \( \Delta SW^{r,r} \) at \( F = F \) yields \((-321c^2 - 126cs + 134c - 65s^2 + 122s - 9)/800 \). We obtain two zeros

\[
c_R(s) = \frac{67}{321} - \frac{400}{321} \sqrt{-\frac{66}{625}s^2 + \frac{24}{125}s + \frac{1}{10} - \frac{21}{107}}, \quad \text{and} \quad c_S(s) = \frac{400}{321} \sqrt{-\frac{66}{625}s^2 + \frac{24}{125}s + \frac{1}{10} - \frac{21}{107}},
\]

where the latter is not feasible. This follows from \( c_S(s = 0) = 1/3 \) and

\[
\frac{\partial c_S(s)}{\partial s} = -\frac{1}{107} \frac{21\sqrt{-264s^2 + 480s + 25} - 640 + 704s}{\sqrt{-264s^2 + 480s + 25}} > 0.
\]

The first root, \( c_R(s) \), is monotonically decreasing in \( s \); i.e.,

\[
\frac{\partial c_R(s)}{\partial s} = -\frac{1}{107} \frac{21\sqrt{-264s^2 + 480s + 25} - 704s + 640}{\sqrt{-264s^2 + 480s + 25}} < 0,
\]

and \( c_R(s = 0) = 9/107 \). Hence, \( c_R(s) \) cuts through the feasible set with negative slope. Evaluating \( \Delta SW^{r,r}(s = 0) \) at \( F = F \) yields \(-321c^2 + 67/800 c - 9/800 \). It is straightforward to check that \( \Delta SW^{r,r} > 0 \) holds if \( c > c_R(s) \), while the opposite holds for \( c < c_R(s) \). Define \( s_R(c) := [c_R(s)]^{-1} \). Then, we obtain that \( \Delta SW^{r,r}(F) > 0 \) (\( \Delta SW^{r,r}(F) < 0 \)) holds if \( s > s_R(c) \) (\( s < s_R(c) \)).

Moreover, we find that \( \Delta SW^{r,r}(F) \) is either strictly decreasing or strictly concave at \( F = F \).

More specifically, we obtain

\[
\frac{\partial \Delta SW^{r,r}(F)}{\partial F} \bigg|_{F = F} = \frac{1}{4s} (-4c + 5s - 2),
\]

\[
\frac{\partial^2 \Delta SW^{r,r}(F)}{\partial F^2} = -\frac{75}{16s^2}.
\]
It is easily checked that the first derivative is positive (negative) if \( s > s^+(c) \) \((s < s^+(c))\), where \( s^+(c) = (2 + 4c)/5 \), whereas the second derivative is always negative. Thus, \( \Delta SW^{r,r}(F) \) is monotonically decreasing if \( s < s^+(c) \) and \( \Delta SW^{r,r}(F) \) is strictly concave if \( s > s^+(c) \), where we also obtain that \( F_{\text{max}} = \arg \max_{F \geq 0} \Delta SW^{r,r}(F) \) given by \( F_{\text{max}} = (16s - 88cs + 44s^2)/75 \) is always in the feasible region; i.e., \( F \leq F_{\text{max}} \leq F \). It follows that \( \Delta SW^{r,r}(F) \) being strictly concave at \( F = F \) always implies that \( \Delta SW^{r,r}(F) > 0 \), since \( s^+(c) > s_R(c) \) strictly holds.

Finally, we set \( \Delta SW^{r,r} = 0 \) at \( F = F \) which gives the following two zeros

\[
ct(s) = \frac{67}{321} - \frac{8}{321} \sqrt{886s^2 - 430s + 25} - \frac{191}{321}s \quad \text{and} \\
cu(s) = \frac{67}{321} + \frac{8}{321} \sqrt{886s^2 - 430s + 25} - \frac{191}{321}s,
\]

where \( ct(s = 0) = 9/107 \) and \( cu(s = 0) = 1/3 \), so that \( ct(s) \) is only feasible for \( s < (215 - 15\sqrt{107})/886 \), whereas \( cu(s) \) is feasible whenever \( s < (215 - 15\sqrt{107})/886 \) or \( s > (215 + 15\sqrt{107})/886 \). It can be immediately checked that \( ct(s) \) is increasing and convex in \( s \). However, \( cu(s) \) is either decreasing or increasing in \( s \). More precisely, \( \partial cu(s)/\partial s < 0 \) holds for \( s < (215 - 15\sqrt{107})/886 \) and \( \partial cu(s)/\partial s > 0 \) holds for \( s > (215 + 15\sqrt{107})/886 \). If we use the inverses, \( st(c) := [ct(s)]^{-1} \) and \( su(c) := [cu(s)]^{-1} \), we find the following ordering which depends on the non-dominant firms’ pre-merger market shares, and thus on \( c \). Given \( 0 < c < 9/107 \), \( \Delta SW^{r,r}(F) > 0 \) \((\Delta SW^{r,r}(F) < 0)\) holds whenever \( s > s_U(c) \) \((s < s_U(c)) \). If, however, the non-dominant firms are relatively small, i.e., \( 9/107 < c < 1/3 \), then \( \Delta SW^{r,r}(F) > 0 \) only holds for \( s < st(c) \cup su(c) = s_V \). Otherwise \( \Delta SW^{r,r}(F) < 0 \) holds. The first derivative of \( \Delta SW^{r,r}(F) \) at \( F = F \) always satisfies \( (\partial \Delta SW^{r,r}(F)/\partial F)|_{F=F} < 0 \).

Furthermore, it is instructive to note that a merger’s impact on social welfare gives rise to a critical adoption cost level, which we denote with \( F_\gamma \), whenever \( \Delta SW^{r,r}(F) > 0 \) holds at \( F = F \) and \( \Delta SW^{r,r}(F) < 0 \) holds at \( F = F \). Thereby, \( \Delta SW^{r,r} > 0 \) \((\Delta SW^{r,r} < 0)\) if \( F < F_\gamma \) \((F > F_\gamma) \).

Altogether, a merger’s effect on social welfare is summarized by distinguishing two cases: a) \( c < 9/107 \) and b) \( c > 9/107 \).

a) If \( s < s_R(c) \), then \( \Delta SW^{r,r}(F) < 0 \) always holds, whereas \( \Delta SW^{r,r}(F) > 0 \) always holds if \( s > s_U(c) \). Intermediate efficiency levels, i.e., \( s_R(c) < s < s_U(c) \), give rise to a critical adoption cost level. If \( s_R(c) < s < s_U(c) \), then \( \Delta SW^{r,r}(F) > 0 \) \((\Delta SW^{r,r}(F) < 0)\) whenever \( F < F_\gamma \) \((F > F_\gamma) \).
b) If \( s < s_V \), then \( \Delta SW^{r,r}(F) > 0 \) always holds. If \( s > s_V \), then \( \Delta SW^{r,r}(F) > 0 \) \((\Delta SW^{r,r}(F) < 0)\) whenever \( F < F_r \) \((F > F_r)\).

**Case iv.** If \((NA, NA)\), then \( \Delta SW^{NA,NA} = SW^{\ast \ast} - F - SW^{NA,NA} \), with \( SW^{NA,NA} = 2\pi_a^{NA,NA} + 2\pi_n^{NA,NA} + CS^{NA,NA} \). It follows from Proposition 5 that the merged firm’s incentive, \( \phi_m \), is positive over the interval \( F \in (\pi_n^*(A, NA) - \pi_n^*(NA, NA); \phi_m) \) if \( s < s_W(c) = 22(1-3c)/31 \) holds. Obviously, \( \Delta SW^{NA,NA} \) equals the change in social welfare presented in Proposition 2 except that the cost of technology adoption has to be subtracted. We obtain

\[
\Delta SW^{NA,NA} = - \frac{321}{800} c^2 - \frac{23}{16} cs + \frac{67}{400} c + \frac{23}{32} s^2 + \frac{5}{16} s - \frac{9}{800} - F.
\]

Taking the inverse of \( s_W(c) \), yields the identical condition \( c < c_W(s) := (22 - 31s)/66 \) which proves easier to compare with the relevant threshold values \((8)\) and \((9)\) (see Proof of Proposition 2). Inspection of \( c_W(s) - c_H(s) \) yields the expression \( 3111s/2354 + 40(\sqrt{322s^2 + 2s + 1} + 1)/321 \) which is strictly positive for all \( s \). Hence, \( s_H(c) < s_W(c) \) always holds.

We turn to the second critical value \((9)\). Inspection of the difference \( c_G(s) - c_W(s) \) yields the expression \( 40(\sqrt{322s^2 + 2s + 1} - 1)/321 - 3111s/2354 \) which obtains two zeros at \( s_1 = 0 \) and at \( s_2 = \frac{139040}{156373} \approx 0.092 \). It is then easily checked that \( c_G(s) - c_W(s) > 0 \) for all \( s > s_2 \), while the opposite holds for \( s < s_2 \). Recall that \( c_G(s) \) reaches its global minimum at \( s' = (5\sqrt{23} - 1)/322 \approx 0.071 \). Hence, \( c_G(s') \) lies in the feasible set which is also true for \( c_G(s_2) = c_W(s_2) \). As \( s_2 > s' \), we obtain the interval \((c_G(s'), c_W(s_2))\) for which \( \Delta SW^{NA,NA} > 0 \) holds if \( s \in (0, g) \cup (\overline{s}, s_W) \), while the opposite holds for \( s \in (g, \overline{s}) \). The presence of the constraint \( c < c_W(s) \) is therefore the only difference of the social welfare comparison between \( \Delta SW^{NA,NA} \) and \( \Delta SW \). Note that \( \partial \Delta SW^{NA,NA}/\partial F < 0 \) which implies that the space of feasible combinations of \( c \) and \( s \) gets smaller when \( F \) increases.

**Proof of Corollaries 1 to 3.** The proof follows from calculating the sign of \( \Delta SW^{k,k'} \) given that firms 3 and 4 have decided to merge at the initial stage of the game. Hence, we have to consider only cases \( i)-iii) \), since in case \( iv) \) the non-dominant firms never decide to merge (see Proposition 5).

**Case i.** We know that firms 3 and 4 only decide to merge in equilibrium if \( F > F_\alpha \) (given \( s > s_I(c) \)). The change in social welfare is given by \( \Delta SW^{A,A} = (-321c^2 + 642cs + 134c - 321s^2 - 134s - 9)/800 + F \). Thereby, \( \Delta SW^{A,A} > 0 \) if \( s < s_N(c, F) \) and \( \Delta SW^{A,A} < 0 \) otherwise, with \( \partial s_N(c, F)/\partial F > 0 \). We calculate \( s_N(c, F = F_\alpha) = c + 1/13 \), i.e., the threshold value for which
mergers just become profitable. Since \( s_N(c, F = F_\alpha) > c \), it is immediately checked that for all \( s < c \) (catch-up mergers) \( \Delta SW^{A,A} > 0 \) holds. If, however, \( s \geq c \) (mergers to dominance), we conclude that the effect on social welfare is only positive if \( s < s_N(c, F) \) given \( F \in (F_\alpha, F) \). Otherwise, i.e., \( s > s_N(c, F) \), \( \Delta SW^{A,A} < 0 \) holds.

**Case ii.** We know that the non-dominant firms choose to merge in equilibrium if the efficiency level, \( s \), satisfies \( s > s_K(c) \). In addition, it can be checked (see Proposition 4) that a merger increases social welfare whenever \( s > s_P(c) \). Hence, we have to compare \( s_P(c) \) and \( s_K(c) \) in order to check whether or not a proposed merger increases social welfare. It turns out that \( s_P(c) > s_K(c) \) holds only for \( c < 1/50 \). That is, a proposed merger leading to efficiencies which are less than \( s_P(c) \), and thereby decreases social welfare, i.e., \( \Delta SW^{A,N_A} < 0 \), is only feasible for \( c < 1/50 \). Moreover, \( s_P(c) > c \) and \( s_K(c) > c \) hold for any \( c \in [0, 1/50) \). Thus, it is again mergers to dominance which reveal a potential for negative welfare effects, whereas every proposed catch-up merger always increases social welfare.

**Case iii.** We know from Proposition 3 that the non-dominant firms only choose to merge in equilibrium if \( F < F_\beta \) holds given \( s > s_I(c) \). It is immediately checked that \( s_I(c) > s_U(c) \) always holds. That is, every proposed merger increases social welfare at \( F = F_\beta \). Evaluating the effect of a merger proposal at \( F = F_\beta \), we find that both the level of \( F \) and the efficiency level are critical, since \( s_I(c) < s_U(c) \) always holds and \( s_I(c) < s_V(c) \) holds for \( c > 12/101 \). It follows that a proposed merger always increases welfare if \( s < s_Y \). If, however, \( s > s_Y \), then proposed mergers increase social welfare whenever \( F_\beta < F_\gamma \). Otherwise, proposed mergers decrease social welfare.

**Proof of Proposition 5.** We have to compare \( \phi_m = 3s [2(1 - 3c) + 3s] / 16 \) and \( F = \pi^*_n(A, NA) - \pi^*_n(NA, NA) \). We obtain \( \pi^*_n(A, NA) - \pi^*_n(NA, NA) = 8s (1 + 2s - 3c) / 25 \). We then get that \( \phi_m - [\pi^*_n(A, NA) - \pi^*_n(NA, NA)] \) has a unique zero at \( s = s_W(c) := 22(1 - 3c)/31 \). Clearly, \( s_W(c) > 0 \) and \( \partial s_W / \partial c < 0 \). It is then easily checked that \( \phi_m > \pi^*_n(A, NA) - \pi^*_n(NA, NA) \) if and only if \( s < s_W(c) \). Given that \( s < s_W(c) \) holds, we can find values of \( F \), with \( \pi^*_n(A, NA) - \pi^*_n(NA, NA) < F < \phi_m \), such that the efficiency is implemented only if there is a merger. In other words, in those instances the efficiencies are strongly merger specific according to Definition 2.

**Proof of Proposition 6.** The proof follows from calculating the sign of \( \Psi^{k,k'} \) for cases i)-iii)
stated in Lemma 2.

Case i). With $\phi_n^{AA} := \pi_n^*(A, A) - \pi_n^*(NA, A) = 8s(1 - 3c + s)/25$, calculating $\Psi^{AA} = \phi_m - \phi_n^{AA}$, we obtain the expression $s[22(1 - 3c) + 97s]/400$ which is strictly positive for all feasible $c$ and $s$.

Case ii). With $\phi_n^{ANA} := \pi_n^*(A, NA) - \pi_n^*(NA, A) = s(2(1 - 3c) + 3s)/5$, calculating $\Psi^{ANA} = \phi_m - \phi_n^{ANA}$, we obtain $\Psi^{ANA} = -s(2(1 - 3c) + 3s)/80 < 0$.

Case iii). We have to examine $\phi_n^{r,r} := m - \phi_n^{r,r}$, with $\phi_n^{r,r} := \pi_n^*(A, r) + F - \pi_n^*(NA, A)$. In the mixed strategy equilibrium, each firm is indifferent between any pure strategy given that the other firm plays the equilibrium mixed strategy, $r$. The expected profit in the mixed strategy equilibrium can be derived from $\pi_n^*(A, r)$, which is the expected profit of firm $n$, when firm $n$ plays the pure strategy $A$ and firm $n'$ ($n' \neq n$) plays the equilibrium mixed strategy, $r$. We then get

$$\pi_n^*(A, r) = r[\pi_n^*(A, A) - F] + (1 - r)[\pi_n^*(A, NA) - F].$$  \hspace{1cm} (17)

Using the definition of $\phi_n^{AA}$ and defining $\phi_n^{NA,NA} := \pi_n^*(A, NA) - \pi_n^*(NA, NA)$, we can write the equilibrium mixed strategy as

$$r = \frac{\phi_n^{NA,NA} - F}{\phi_n^{NA,NA} - \phi_n^{AA}}. \hspace{1cm} (18)$$

Substituting (18) into (17) yields

$$\phi_n^{r,r} = \pi_n^*(A, NA) - \pi_n^*(NA, A) - \frac{\phi_n^{NA,NA} - F}{\phi_n^{NA,NA} - \phi_n^{AA}}[\pi_n^*(A, NA) - \pi_n^*(A, A)].$$

We then obtain

$$\frac{\partial\phi_n^{r,r}}{\partial F} = \frac{\pi_n^*(A, NA) - \pi_n^*(A, A)}{\phi_n^{NA,NA} - \phi_n^{AA}} > 0,$$

i.e., incentives are increasing in $F$ in the no-merger subgame. In contrast, $\phi_m$ is independent of $F$, so that $\partial\phi_m/\partial F = 0$ holds. Hence, setting $\phi_m = \phi_n^{r,r}$, and solving for $F$, we get a unique solution

$$F = F_\delta := \phi_n^{NA,NA} - \frac{(\phi_n^{NA,NA} - \phi_n^{AA})(\phi_m - \pi_n^*(A, NA) + \pi_n^*(NA, A))}{\pi_n^*(A, NA) - \pi_n^*(A, A)}. \hspace{1cm} (19)$$

We have to consider the relevant interval of case iii) which we can write as $\phi_n^{AA} < F < \phi_n^{NA,NA}$.

We now show that $F_\delta$ lies always in that interval. Inspecting $\phi_n^{AA} < F_\delta$, we get the condition $\phi_m > \pi_n^*(A, A) - \pi_n^*(NA, A) = \phi_n^{AA}$. Further, $F_\delta < \phi_n^{NA,NA}$ implies $\phi_m < \pi_n^*(A, NA) -
\( \pi^*_n(NA, A) = \phi^{A,NA}_n \). As we have shown in case \( i) \) and case \( ii) \) of this proposition, both conditions are fulfilled. Finally, calculating \( \partial F_\delta / \partial s \) by using (19) we obtain the expression \( \delta(c, s)/[25 (7 s - 6 c + 2)^2] \) with

\[
\delta(c, s) := -864 c^3 + 3348 c^2 s + 864 c^2 - 4104 cs^2 - 2232 cs - 288 c + 1673 s^3 + 1368 s^2 + 372 s + 32.
\]

We show that \( \delta(c, s) > 0 \) holds everywhere, so that \( \partial F_\delta / \partial s > 0 \) follows.

**Ancillary Claim.** \( \delta(c, s) > 0 \) holds everywhere.

**Proof.** We successively differentiate \( \delta(c, s) \) with respect to \( s \).\(^{49}\) This yields

\[
\delta'(c, s) = 3348 c^2 - 8208 cs - 2232c + 5019 s^2 + 2736 s + 372,
\]

\[
\delta''(c, s) = 10038 s - 8208 c + 2736, \text{ and } \delta'''(c) = 10038.
\]

As \( \delta''(c, s) \) is strictly increasing in \( s \), we evaluate \( \delta'(c, s) \) at \( s = 0 \) to obtain \( \delta''(c, s = 0) = -8208 c + 2736 \) which is decreasing in \( c \) and strictly for all \( 0 \leq c < 1/3 \). It follows that \( \delta''(c, s) > 0 \) holds everywhere. Evaluating \( \delta'(c, s) \) at \( s = 0 \) we get \( \delta'(c, s = 0) = 3348 c^2 - 2232c + 372 \). We find that \( \partial \delta'(c, s = 0)/ \partial c = 0 \) at \( c = 1/3 \) and \( \partial^2 \delta'(c, s = 0)/ \partial^2 c > 0 \). Hence, \( \delta'(c, s = 0) \) reaches a global minimum at \( c = 1/3 \). This gives \( \delta'(c = 1/3, s = 0) = 0 \), while \( \delta'(c, s) > 0 \) holds for all feasible \( c \) and \( s \). Finally, we get \( \delta(c, s = 0) = 32 (1 - 3c)^3 \) which is strictly positive for all \( 0 \leq c < 1/3 \). This proves the claim.

**Proof of Proposition 7.** We have to consider all cases \( i)-iv) \) of Lemma 2. We first examine how a change of \( \lambda \) impacts on merger specificity and then on firms' merger incentives.

**Case i).** Given that \( (A, A) \) is the adoption game equilibrium, then our measure of (weak) merger specificity (7) becomes \( \Psi^{A,A} = s [22(\lambda - 3c) + 97s] / 400 \) which is positive for all \( c \) and \( s \). Differentiating \( \Psi^{A,A} \) with respect to \( \lambda \) gives

\[
\frac{\partial \Psi^{A,A}}{\partial \lambda} = \frac{11}{200} s > 0.
\]

**Case ii).** If only one of the two non-dominant firms adopts the technology, i.e., \( (A, NA) \), then the specificity measure is \( \Psi^{A,NA} = -s (2(\lambda - 3c) + 3s) / 80 < 0 \) which leads us to

\[
\frac{\partial \Psi^{A,NA}}{\partial \lambda} = \frac{-1}{40} s < 0.
\]

\(^{49}\)We define \( \delta'(\cdot) := \partial \delta(\cdot)/ \partial s, \delta''(\cdot) := \partial^2 \delta(\cdot)/ \partial s^2 \) and so on.

47
Case iii). Finally, we analyze how growing markets affect merger specificity when 3 and 4 play mixed strategies in the no-merger subgame. The marginal effect of $\lambda$ on $\Psi^{r,r}$ can be calculated as

$$\frac{\partial \Psi^{r,r}}{\partial \lambda} = \frac{1}{200} (59s + 16(\lambda - 3c)),$$

which is strictly positive for all $c$ and $s$.

We turn next to our merger incentive measure $\theta^{k,k'}$. Again, we analyze all relevant cases of Lemma 2.

Case i). If $(A, A)$, then our incentive measure is $\theta^{A,A} = -7(\lambda - 3c + 3s)^2/400 + F$ which yields the derivative

$$\frac{\partial \theta^{A,A}}{\partial \lambda} = -\frac{\lambda - 3(c - s)}{200/7} < 0.$$

Case ii). Given $(A, NA)$, we obtain $\theta^{A,NA} = 63[(1/3)\lambda - c - (47/21)s] [(c + (1/3)s - (1/3)\lambda)] / 400$ from which

$$\frac{\partial \theta^{A,NA}}{\partial \lambda} = \frac{3(7c + 9s) - 7\lambda}{200} \quad (20)$$

follows. The sign of (20) is determined by the numerator. Setting $3(7c + 9s) - 7\lambda = 0$, we get the critical efficiency level $s_Y := 7(\lambda - 3c)/27$. It is then straightforward to check that for $s > s_Y$ ($s < s_Y$) the sign of (20) is positive (negative).

Case iii). If firms 3 and 4 play mixed strategies in the no-merger subgame, then growing markets strictly increase merger incentives where the marginal effect is given by

$$\frac{\partial \theta^{r,r}}{\partial \lambda} = \frac{\lambda - 3(c - s))}{8} > 0.$$

Case iv). Finally, given $(NA, NA)$, then the merger incentive becomes $\theta^{NA,NA} = (\Delta - 3(c - s))^2/16 - 2(\Delta - 3c)^2/25 - F$ from which we obtain the derivative

$$\frac{\partial \theta^{NA,NA}}{\partial \lambda} = \frac{21c + 75s - 7\lambda}{200}. \quad (21)$$

Setting the term in the numerator of (21) equal to zero, we obtain the threshold value $s_Z = 7(\lambda - 3c)/75$. It is then easily checked that $\partial \theta^{NA,NA}/\partial \lambda > 0$ ($< 0$) if $s > s_Z$ ($s < s_Z$).
References


PREVIOUS DISCUSSION PAPERS


63 Dewenter, Ralf, Jaschinski, Thomas and Kuchinke, Björn A., Hospital Market Concentration and Discrimination of Patients, July 2012.


60 Jentzsch, Nicola, Sapi, Geza and Suleymanova, Irina, Targeted Pricing and Customer Data Sharing Among Rivals, July 2012.


57 Dewenter, Ralf and Heimeshoff, Ulrich, More Ads, More Revs? Is there a Media Bias in the Likelihood to be Reviewed?, June 2012.


53 Benndorf, Volker and Rau, Holger A., Competition in the Workplace: An Experimental Investigation, May 2012.


48 Herr, Annika and Suppliet, Moritz, Pharmaceutical Prices under Regulation: Tiered Co-payments and Reference Pricing in Germany, April 2012.


Pagel, Beatrice and Wey, Christian, Unionization Structures in International Oligopoly, February 2012.


Christin, Cémence, Nicolai, Jean-Philippe and Pouyet, Jerome, The Role of Abatement Technologies for Allocating Free Allowances, October 2011.


Hauck, Achim, Neyer, Ulrike and Vieten, Thomas, Reestablishing Stability and Avoiding a Credit Crunch: Comparing Different Bad Bank Schemes, August 2011.


Balsmeier, Benjamin, Buchwald, Achim and Peters, Heiko, Outside Board Memberships of CEOs: Expertise or Entrenchment?, June 2011.


