Mergers, Managerial Incentives, and Efficiencies

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Abstract

We analyze the effects of synergies from horizontal mergers in a Cournot oligopoly where principals provide their agents with incentives to cut marginal costs prior to choosing output. We stress that synergies come at a cost which possibly leads to a countervailing incentive effect: The merged firm’s principal may be induced to stifle managerial incentives in order to reduce her agency costs. Whenever this incentive effect dominates the well-known direct synergy effect, synergies actually reduce consumer surplus which opposes the use of an efficiency defense in merger control.

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1 Introduction

The existing literature on efficiencies from horizontal mergers largely relies on the presumption that a merger automatically generates efficiencies within the merged firm. Precisely, it is presumed that efficiencies come at no cost and, in particular, do not affect the firms’ behavior other than the output decisions and pricing decisions, respectively.\(^1\) In addition, these efficiencies are implicitly assumed to be merger specific, so that (at least according to the US and the EU merger guidelines) they would be deemed acceptable and thus relevant for the competitive appraisal when they additionally benefit the consumers.\(^2\) Then, it follows that the challenge for an antitrust authority is to examine whether the extent to which efficiencies are realized is sufficient to offset the anticompetitive effects stemming from (unilaterally) increased market power. This trade-off was first stressed by Williamson (1968), where the maximization of aggregate surplus is assumed to be the antitrust authority’s goal. A more formal and complete treatment was later offered by Farrell and Shapiro (1990): Using a general Cournot setup, they additionally present (necessary and sufficient) conditions for a merger to be consumer surplus increasing. Thus, they provide a useful analytical framework for antitrust authorities using a consumer standard, which can be (at least broadly) confirmed for the US and the EU.

However, this standard treatment of efficiencies in merger review relies on two suppositions. First, as previously mentioned, efficiencies are deemed merger specific by antitrust authorities and thus qualify as real synergies. Unless otherwise stated, these efficiencies will be called synergies throughout the paper. Second, and more importantly, synergies unambiguously decrease the merged firm’s marginal cost, so that consumer surplus is (monotonically) increasing in synergies.

\(^1\) With respect to the former supposition, Banal-Estanol et al. (2008) and Bettignies and Ross (2013) present exceptions: They specify efficiencies as stemming from relationship-specific investments by managers and managerial effort to cut marginal costs, respectively.

\(^2\) For example, the US merger guidelines define merger-specific efficiencies as “those efficiencies likely to be accomplished with the proposed merger and unlikely to be accomplished in the absence of either the proposed merger or another means having comparable anticompetitive effects.” Note that according to both the US and the EU merger guidelines efficiencies have to be classified as verifiable (what we assume to be the case), in addition to being merger specific and beneficial to consumers. If these three criteria are cumulatively met, then a claimed efficiency will be accepted.
gies.\(^3\) Then, it is well known that synergies have to be sufficiently large for a merger to decrease price and benefit consumers, respectively.

In this paper, we partially depart from this standard treatment by considering firms which engage in cost reducing activities prior to their product market decisions. More precisely, firms compete in Cournot fashion and are characterized by an agency relationship, such that their productive efficiency is a result of managerial effort.\(^4\) Following Hermalin (1994) we suppose for convenience that each firm consists of one principal, who chooses managerial incentives and output, and one agent, who may exert effort to cut marginal costs. When firms merge, efficiency gains may be additionally generated through synergies. As is standard in the literature, we stipulate that these synergies are per se merger specific and thus contrast efficiency gains from managerial effort, which can also be realized without a merger. However, we specify that synergies are not automatically generated following a merger. Rather, we follow Farrell and Shapiro (2001) and explicitly require the merging firms to combine their core hard-to-trade assets, which refer to managerial skills in our framework, in order to realize synergies.\(^5\) In other words, we specify a merger to be a necessary prerequisite, while employing and ‘combining’, respectively, several agents is sufficient for synergies to be realized.

With such a setting at hand, we highlight that, in addition to their well-known direct effect on the merged firm’s marginal cost, synergies have an incentive effect on both the merged firm and the rival (non-merging) firms. In particular, we show that this incentive effect may be sufficiently negative, so that synergies will even have a negative impact on the merged firm’s productive efficiency and thus decrease consumer surplus in the post-merger case. This result relies on two immediate wage raising effects. First, there is a direct cost for the merged firm, as its principal must employ two agents (rather than one) in order to realize synergies. Second, and more

\(^3\)See, e.g., Besanko and Spulber (1993) and, for more recent papers, Nocke and Whinston (2010, 2013) who make use of this monotonicity result which was first highlighted by Farrell and Shapiro (1990, Lemma, p. 111).

\(^4\)Notice that managerial effort in our model exhibits parallels to process innovations as analyzed by, e.g., d’Aspremont and Jacquemin (1988), Kamien et al. (1992), and, even more generally, Vives (2008).

\(^5\)This prerequisite is not new; it has not been explicitly considered but rather implicitly presumed so far. For instance, Farrell and Shapiro (1990) specify synergies as requiring the recombination of the merging firms’ assets “to improve their joint production capabilities,” but do not explicitly account for such a recombination of assets in their model.
importantly, there is an indirect cost, as the merged firm’s principal cannot disentangle synergies from managerial effort, which makes triggering agents to cut marginal costs more expensive. Whenever this indirect (agency) cost is sufficiently large, then the merged firm’s principal will stifle managerial incentives and thus substitute managerial effort by synergies. Note, however, that in this case the rival firms’ principals will not necessarily respond by inducing their agents to work harder (although managerial incentives constitute strategic substitutes). Rather, their managerial incentives will increase if and only if the merged firm’s marginal cost overall decreases due to higher synergies.

Our analysis offers two main implications for merger control. First, given the existence of agency problems inside the merged firm, we present conditions under which the common presumption that consumer surplus is (monotonically) increasing in the synergy level holds. Second, we show when the use of an efficiency defense not accounting for the effects of synergies other than those on the merged firm’s product market decision can be misleading. This is the case when synergies make the merged firm less efficient and thus harm consumers, which clearly opposes the use of an efficiency defense in merger control.

Apart from the works on horizontal mergers, our paper is closely related to the literature on the effects of competition on managerial incentives. In general, this literature builds on the works by Hart (1983), Hermalin (1992), and Schmidt (1997) who were among the first to formalize the relationship between managerial incentives and competitive pressure. Based on these papers, a merger could simply be seen as a (marginal) reduction of competitive pressure which would equally affect all firms in the market. However, this is no longer true when an increase of market concentration stems from a merger creating synergies. Thus, in addition to the (unilateral) market power effects, we explicitly take the interplay between synergies and managerial incentives into account. Note that Bettignies and Ross (2013) also show that a merger may decrease the merged firm’s overall productive efficiency. However, they do not account for synergies arising from the merger and solely focus on a merger to monopoly.

6 Further papers in this spirit are, e.g., Scharfstein (1988), Martin (1993), Hermalin (1994), Raith (2003), and Baggs and de Bettignies (2007). In contrast to previous works, Hermalin (1994), Raith (2003), and Baggs and de Bettignies (2007) explicitly take strategic interactions between the firms into account. Finally, Vives (2008) provides a model which presents a generalization of Raith (2003) and Baggs and de Bettignies (2007), but where managerial effort is treated rather as an innovative activity.
The remainder is organized as follows. We present the model in Section 2. Section 3 provides the pre- and post-merger equilibria and discusses the effects of a merger on managerial incentives. In Section 4, we present our findings on the relationship between synergies and consumer surplus. Section 5 concludes the paper.

2 The Model

Pre-merger case. Consider a homogeneous Cournot oligopoly where \( n \geq 3 \) firms compete in quantities \( q_i \), with \( i = 1, \ldots, n \). Each firm consists of a risk-neutral principal (she) and a risk-neutral agent (he) who is protected by limited liability.\(^7\) We analyze the following game. In the contracting stage, principals offer their respective agents contracts to induce marginal cost reductions, and then agents choose effort. Subsequently, in the market stage, production costs become common knowledge and principals compete in quantities.

Firms face an inverse demand function \( p(Q) \), with \( Q := \sum_i q_i \) denoting total output, and have constant marginal costs \( c_i > 0 \). We invoke the following standard assumptions that guarantee existence and stability of a unique Cournot-Nash equilibrium: 

1. \( p'(Q) < 0 \), i.e., demand is downward sloping,
2. \( Qp''(Q) + p'(Q) < 0 \), i.e., quantities are strategic substitutes implying strict concavity of firms’ profits, and
3. \( \lim_{Q \to \infty} p(Q) = 0 \), i.e., total output is bounded in equilibrium.\(^8\) As each principal is the residual claimant of her firm, her payoff coincides with the profit which is given by

\[
\pi_i = p(Q)q_i - c_i q_i - w_i, \tag{1}
\]

where \( c_i := c - e_i - \epsilon_i \). Marginal costs \( c_i \) comprise a constant cost parameter \( c \), agent \( i \)’s effort level \( e_i \), and an idiosyncratic cost shock \( \epsilon_i \) which is assumed to be uniformly and independently distributed with support \( [-\theta, \theta] \), where \( \theta > 0 \).\(^9\) Let \( F(\epsilon_i) \) denote the respective cumulative distribution function.

\(^7\)Notice that presuming risk-neutral agents who are protected by limited liability is an economically feasible alternative to risk-aversion (see, e.g., Laффont and Martimort (2002, Ch. 4)).

\(^8\)See, e.g., Shapiro (1989).

\(^9\)To avoid a finite support, we could have assumed that cost shocks follow a normal distribution as in, e.g., Raith (2003). However, in that case we would have to restrict the variance and the confidence level, respectively, to be able to ignore too large random cost differences. Nevertheless, our results hold under both specifications.
Principals use linear incentive schemes \( w_i := d_i + b_i (c - c_i) \) to reward their agents, which consist of a (fixed) salary \( d_i \) and a variable component \( b_i (c - c_i) \). In contrast to managerial effort, efficiency gains \( (c - c_i) \) are verifiable and can thus be contracted upon. Note that the limited liability model implies \( d_i = 0 \). The piece rate \( b_i \) represents the incentive which principal \( i \) gives her agent to cut marginal costs termed managerial incentive throughout the paper.

The agent can accept or reject the contract which is a take-it-or-leave-it offer. If agent \( i \) rejects the contract, then he realizes his reservation utility which is normalized to zero. In contrast, if agent \( i \) accepts the offer, then he receives \( w_i \) and incurs convex private costs given by \( K(e_i) = ke_i^2/2 \), with \( k \geq 1 \). Agent \( i \)'s expected utility is thus strictly concave in \( e_i \) and given by

\[
E(u_i) = \int_0^{\theta} w_i dF(e_i) - K(e_i).
\]

To ensure that there is a unique and stable solution at the contractual stage when principals optimally choose managerial incentives, we stipulate that \( \partial^2 \Pi_i/\partial b_i^2 < 0 \) holds, where (with a slight abuse of notation) we use \( \Pi_i \) to denote principal \( i \)'s expected profit.

**Post-merger case.** Suppose now that two firms merge. We specify the merger as being an acquisition, in which one firm entirely acquires the other. Thereby, the acquirer gets full control over the acquiree. It follows that the merged firm’s principal has two available agents at hand. Let \( M \) and \( N = 1, \ldots, n - 2 \) indicate the merged firm and the rival (non-merging) firms, respectively. We denote the number of employed agents within \( M \) by \( h \), with \( h \in (1, 2) \). Assume that depending on \( h \) a merger may give rise to synergies \( s \geq 0 \), such that \( M \)'s expected marginal cost is given by

\[
E(c_M) = \begin{cases} 
  c - s - \sum_r e_r & \text{if } h = 2, \\
  c - e_r & \text{otherwise},
\end{cases}
\]

with \( r \in h \). That is, we postulate that a merger is a necessary prerequisite, while employing

\[10\] Though restrictive, limiting our model to linear contracts is standard and motivated by their common use in practice.

\[11\] This follows from neither the participation constraint nor the wealth constraint being binding in models with limited liability. For an application which also analyzes linear contracts (but different performance measures) see, e.g., Raith (2008).

\[12\] Alternatively, we could have considered synergies such that \( E(c_M) = c - s \sum_r e_r \) if \( h = 2 \). However, this would not affect our results qualitatively.
more than one agent is sufficient for synergies to be realized. The chosen specification in (3) implies that, all other things held constant, synergies make both agents more productive per se which may stem from economies of scope realized through the ‘cooperation’ of multiple agents within one firm. It also captures Farrell and Shapiro’s (2001) notion that realizing synergies (from mergers) requires the combination of core hard-to-trade assets, which refer to managerial effort in our setting.

Agent \( r \) receives an expected wage of \( E(w_r) := b_r(c - E(c_M)) \). That is, \( M \)'s principal is only able to reward her agents based on actual costs rather than some counterfactual cost level, which is not realized and thus hard or even impossible to prove in court. The merged firm’s total wage cost is \( E(w_M) := \sum_r E(w_r) \). We do not endogenize \( M \)'s decision on how many agents to employ and whether or not to merge. Instead, provided that two firms have merged, we distinguish between mergers with synergies and mergers without synergies and analyze both cases with respect to their effects on managerial incentives and consumer surplus. Finally, we stipulate that \( s \) and \( \theta \) are not too large, so that all firms are active in equilibrium.

3 Horizontal Mergers and Managerial Incentives

3.1 Equilibrium Analysis

We begin by presenting the equilibria for the pre-merger case and the post-merger case. An asterisk indicates equilibrium values in both cases.

**Pre-merger case.** In the market stage, each principal learns her own and her rivals’ costs perfectly, so that marginal costs become common knowledge. As is well known, equilibrium output per firm is then implicitly defined by

\[
q^*_i = -\frac{p(Q^*) - c_i}{p'(Q^*)},
\]

\(^{13}\)Notice that there may be other means of realizing synergies, such as the centralization and standardization, respectively, of particular tasks within a given firm (Dessein et. al., 2010). However, we abstract from those and solely focus on horizontal mergers.

\(^{14}\)We, in fact, assume that counterfactual cost levels cannot be contracted upon. In this respect, our setting essentially corresponds to the principal-agent literature on which a single principal faces multiple agents and cannot contract upon measures reflecting each agent’s individual effort level, see, e.g., Holmstrom (1982).
where \( p'(Q) = dp(Q)/dQ \). For given incentives, agents simultaneously decide on their effort levels. Agent \( i \)'s optimal effort choice is given by \( e_i^* = b_i/k \). That is, there is a direct link between the agents’ optimal effort choice and the managerial incentive, which is a standard result of moral hazard models.\(^{15}\)

When choosing managerial incentives, each principal faces the following optimization problem\(^{16}\)

\[
\max_{b_i \in \mathbb{R}^+} \Pi_i = \int_\theta^\theta [p(Q^*)q_i^* - c_i q_i^* - w_i] dF \\
\text{s.t. } e_i^* = b_i/k, \ E(u_i) \geq 0, \text{ and } w_i \geq 0 \forall e_i.
\]

Using the envelope theorem, the first order condition of problem (4) is given by

\[
\frac{d\Pi_i}{db_i} = \frac{\partial \Pi_i}{\partial Q^*} \frac{dQ^*}{dc_i} \frac{dc_i}{db_i} + \frac{\partial \Pi_i}{dc_i} \frac{dc_i}{db_i} + \frac{\partial \Pi_i}{dw_i} \frac{dw_i}{db_i} \\
= \int_\theta^\theta \left[ \left( -p'(Q^*) \frac{\partial Q^*}{dc_i} + 1 \right) \frac{q_i^*}{k} - \frac{2b_i}{k} \right] dF = 0,
\]

where \( Q_{-i} := \sum_{j \neq i} q_j \). Let \( b_i(b_{-i}) \) denote principal \( i \)'s incentive solving (5), where \( b_{-i} \) is a vector of all firms’ managerial incentives other than \( i \). As agent \( i \)'s effort aims at reducing the respective firm’s marginal costs, it increases, all other things held constant, its output. Recall that firms’ quantities are strategic substitutes, i.e., \( dq_i/dQ_{-i} \in (-1, 0) \). It follows that principals are always inclined to induce their respective agents to work less hard in response to an increase in their rivals’ incentives. In other words, managerial incentives are strategic substitutes, i.e., \( db_i/db_j < 0 \) holds, where \( j \neq i \).\(^{17}\)

The first order condition in (5) further reveals that managerial incentives are shaped by a strategic effect and two direct effects. The former, which is represented by the first term on the right hand side of (5), induces principal \( i \) to boost her agent’s effort, as this decreases her rivals’ output. The latter have a positive and a negative effect on the optimal level of \( b_i \), as they decrease marginal costs \( (dc_i/db_i < 0) \) but increase wage costs \( (dw_i/db_i > 0) \) at the same time.

\(^{15}\)See, e.g., Levitt (1995) for the case of risk-averse agents.

\(^{16}\)For notational simplicity, we use \( \int_\theta^\theta [\cdot] dF \) instead of \( \int_\theta^\theta ... \int_\theta^\theta [\cdot] dF(\epsilon_1) ... dF(\epsilon_n) \) to calculate expected values accounting for each of the agents’ idiosyncratic cost shocks \( \epsilon_i \), with \( i = 1, ..., n \).

\(^{17}\)This is pointed out by Hermalin (1994) who, in a similar setting, shows that the marginal gain of triggering managerial effort is decreasing in the rivals’ incentives. Further, notice that \( db_i/db_j \in (-1, 0) \) immediately follows from \( dq_i/dQ_{-i} \in (-1, 0) \).
Finally, notice that, as all agents exhibit identical effort cost functions, and the idiosyncratic cost shocks follow the same distribution, principals will end up offering symmetric incentives in equilibrium, i.e., $b^*_i = b^* \forall i$.\(^{18}\) We summarize our results in the following proposition.

**Proposition 1.** In the pre-merger equilibrium, managerial incentives solve (5) and are symmetric. Moreover, $db^*/dn < 0$ always holds.

Proposition 1 also highlights that $b^*$ decreases when the level of competition, measured by the number of firms $n$, increases.\(^{19}\) This comparative static result relies on a well-known property of Cournot models: $\partial E(q^*)/\partial n < 0$.\(^{20}\)

**Post-merger case.** If $M$’s principal decides to employ only one agent, then the equilibrium of the entire game is symmetric and identical with the pre-merger case, except that the number of firms is reduced to $n - 1$. It follows that the merger solely has a market power effect on managerial incentives, so that, according to Proposition 1, we obtain $b^*_M = b^*_N > b^*$, i.e., a merger equally increases managerial incentives within all firms in case of $h = 1$.

If, however, $M$’s principal employs $h = 2$ agents, then the result changes significantly. The reason is that $M$’s principal now realizes synergies. The rival firms are, however, not able to employ several agents. Each of their principals continues to face a single agent and thus derives efficiency gains exclusively through managerial effort.

It is important to note that synergies do not directly affect the agents’ effort choice which is given by $e^*_r = b_r/k$, $r = 1, 2$, and $e^*_N = b_N/k$, respectively. However, synergies will have an effect on principal $M$’s decision at the initial stage of the game, which is reflected by the following

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\(^{18}\)Hermalin (1994) shows that with otherwise symmetric firms asymmetric equilibria may also exist. However, his result does not apply to our setting, as it is based on a different specification on how managerial effort impacts the principals’ profits.

\(^{19}\)This result is also found by Raith (2003) and Vives (2008) for an exogenous market structure.

\(^{20}\)Notice that $db^*/dn$ can be rewritten as follows $db^*/dn = [dE(q^*)/dn]/[dE(q^*)/db^*]$, with $dE(q^*)/db^* > 0$. 
first order condition

\[
\begin{align*}
\frac{d\Pi_M}{db_r} &= \frac{\partial \Pi_M}{\partial Q^*_{-M}} \frac{\partial Q^*_{-M}}{\partial c_{r}} + \frac{\partial \Pi_M}{\partial c_{M}} \frac{\partial c_{M}}{\partial b_r} + \frac{\partial \Pi_M}{\partial w_M} \frac{\partial w_M}{\partial b_r} \\
&= \int_0^\theta \left[ \left( -p'(Q^*) \frac{\partial Q^*_{-M}}{\partial c_{M}} + 1 \right) \frac{q_M^*}{k} - s - \frac{2(b_r + b_{-r})}{k} \right] dF = 0,
\end{align*}
\]

where (6) is solved for both agents simultaneously and \(-M\) and \(-r\) indicate all firms other than \(M\) and all of \(M\)'s agents other than \(r\), respectively. Note that (6) consists of a (positive) strategic effect and two direct effects as before in the pre-merger case. However, it reveals two striking features which are specific to the post-merger case. First, principal \(M\)'s best response per agent \(r\) is not only shaped by the strategic interaction between her incentive and the rivals' incentives but also by *intra-firm* strategic relations. More specifically, denote the best response solving (6) for agent \(r\) with \(b_r(b_{-r}, b_{-M})\). As \(M\)'s agents are symmetric and perform identical tasks, which implies that \(b_r^* = b_M^*\) \(\forall r\) in equilibrium, it is easily seen that the principal will decrease one agent's wage when she induces the other agent to work harder. Intuitively, pushing one agent to work harder exerts a negative externality on the other agent, because it increases \(M\)'s wage payment by more than it creates productive efficiency gains. This also implies that \(M\)'s principal offers lower incentives when the number of her agents increases. This result is stated in Lemma 1.

**Lemma 1.** Managerial incentives within the merged firm are strategic substitutes, i.e., \(db_r/db_{-r} < 0\), and decrease when \(M\) employs more agents.

**Proof.** See the Appendix.

Second, (6) shows that, in addition to the direct cost resulting from the necessity of paying two agents rather than one, synergies come at an *indirect* (agency) cost. Since \(M\)'s principal cannot use performance measures, which are independent of the synergy level, synergies give rise to an additional wage cost component. That is, \(M\)'s principal faces a trade-off when synergies become larger: All other things held constant, synergies directly decrease \(c_M\), but they also make managerial effort more costly for \(M\). This trade-off stands in sharp contrast to the works that neglect agency issues when analyzing the effect of synergies. It follows that \(M\)'s principal

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\(21\) Again, for notational simplicity, we use \(\int_0^\theta \left[ \right] dF\) to calculate expected values accounting for each of \(M\)'s agents' and the rival firms' agents' idiosyncratic cost shocks \(c_r\) and \(c_N\), with \(r = 1, 2\) and \(N = 1, \ldots, n - 2\), respectively.
may substitute managerial effort by synergies whenever the latter’s wage raising effect is strong.

We will show that this is actually the case if and only if wage costs are sufficiently sensitive to incentivizing agents to cut marginal costs, i.e., $k$ is sufficiently large.

Finally, note that synergies do not directly affect the rivals’ profits. We can thus infer that principal $N$’s first order condition, which implicitly defines $b_N^*$, essentially resembles the one in the pre-merger case and is given by

$$
\frac{d\Pi_N}{db_N} = \frac{\partial \Pi_N}{\partial Q_{-N}^*} \frac{\partial Q_{-N}^*}{\partial c_N} \frac{\partial c_N}{\partial b_N} + \frac{\partial \Pi_N}{\partial c_N} \frac{\partial c_N}{\partial b_N} + \frac{\partial \Pi_N}{\partial w_N} \frac{\partial w_N}{\partial b_N} 
$$

$$
= \int_\theta \left[ \left( -p'(Q^*) \frac{\partial Q_{-N}^*}{\partial c_N} \frac{dc_N}{db_N^*} + 1 \right) \frac{q_N^*}{k} - \frac{2b_N}{k} \right] dF = 0,
$$

with the rivals’ principals offering symmetric incentives in equilibrium. However, synergies enter the rivals’ optimization problem via $c_M$ through two channels: directly and indirectly via $b_M^*$. While the direct effect of $s$ on $c_M$ always stifles $b_N^*$ (notice that $\partial^2 \Pi_N/\partial b_N \partial s < 0$), the indirect effect of $s$ on $c_M$ via $b_M^*$ is ambiguous. Hence, the latter must be positive, i.e., $db_M^*/ds < 0$ holds, and sufficiently large for $b_N^*$ to ultimately increase. Otherwise, larger synergies will always induce the rivals’ principals to offer weaker incentives. The conditions for that to happen as well as the characterization of the equilibrium incentives in the post-merger case are offered in Proposition 2.

**Proposition 2.** In the post-merger equilibrium, the merged firm’s and the rival firms’ managerial incentives solve (6) and (8), respectively, where $db_M^*/ds < 0$ ($db_M^*/ds \geq 0$) holds if $k > k_{\tilde{k}}$ ($k < \tilde{k}$) and $db_N^*/ds < 0$ ($db_N^*/ds \geq 0$) holds if $k < \tilde{k}$ ($k \geq \tilde{k}$), with $\tilde{k} > k$.

**Proof.** See the Appendix.

A key implication of Proposition 2 is that synergies have an incentive effect on both the merged firm and the rival firms. While synergies exert a negative incentive effect only on the merged firm (rivals) when they are sufficiently costly (cheap), i.e., $k > \tilde{k}$ ($k < \tilde{k}$) holds, they stifle managerial incentives within all firms whenever they imply moderate additional wage costs for $M$, i.e., $k \in [\tilde{k}, \hat{k}]$ holds. Notice that in the latter case the rivals’ incentives decrease, although $M$’s incentives decrease as well. This simply follows from the direct synergy effect dominating the indirect effect via $b_M^*$, so that $|\partial c_M/\partial s| > |(\partial c_M/\partial b_M^*) (db_M^*/ds)|$ holds for $k \in [\tilde{k}, \hat{k}]$, where $db_M^*/ds < 0$. 

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Based on Proposition 2, we can immediately compare equilibrium incentives across firms. Therefore, it is important to recall that $M$’s incentives are strictly lower than those of the rival firms when $s = 0$. This follows from Lemma 1 which shows that employing several agents (rather than one) has a strictly negative impact on $b_M^*$. Then, for $M$’s principal to offer stronger incentives than her rivals, synergies become important provided that $k < \bar{k}$ holds.

**Corollary 1.** In the post-merger equilibrium, the merged firm’s managerial incentives are larger than those of the rival firms if $k < \bar{k}$ and $s \geq \bar{s}$ hold. Otherwise, the rival firms always offer stronger incentives in equilibrium.

**Proof.** See the Appendix. □

### 3.2 The Effects of a Merger on Managerial Incentives

Now we are in the position to analyze how a horizontal merger affects managerial incentives. For this purpose, we compare the equilibrium incentives in the pre-merger case with those in the post-merger case when synergies prevail. It is instructive to recall that the merger itself exerts a positive (unilateral) market power effect on managerial incentives (see Proposition 1), as the number of competing firms is reduced. Therefore, we have previously concluded that a merger would equally increase managerial incentives within all firms, i.e., $b_M^* = b_N^* > b^*$ holds, if $M$’s principal employs only one agent and thus does not realize any synergies.

However, when $M$’s principal employs $h > 1$ agents and realizes synergies, there are two additional effects, which were stressed in the previous section. First, the synergy comes at an indirect cost: Synergies raise $M$’s wage payment, as individual effort levels cannot be exclusively used for rewarding the agents, so that providing incentives to cut marginal costs becomes more costly. This (indirect) wage raising effect represents an additional agency cost faced by $M$’s principal. Second, as $M$’s principal employs two agents, she needs to incentivize and reward, respectively, both of them which, all other things held constant, increases her wage cost as well and reduces managerial incentives (see Lemma 1). For convenience, we suppose that the latter

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22 It is well known that a firm’s degree of market power can be measured by $\frac{p(Q^*) - c_i}{p(Q^*)} = -\frac{s_i}{\varepsilon_p}$, where the term on the left-hand side is the Lerner index, $s_i$ denotes firm $i$’s market share, and $\varepsilon_p$ is the price elasticity of demand. Then, it is straightforward to check that the Lerner index increases, and thereby a firm’s market power when the number of firms in the market decreases.
(negative) effect is sufficiently large so as to offset the positive impact of (unilaterally) increased market power on $b^*_M$.\textsuperscript{23} Taken altogether, we obtain the following result:

**Proposition 3.** Suppose that $b^*_M < b^*$ holds at $s = 0$. Then, a merger leading to synergies $s$ has the following impact on managerial incentives:

i) If $k < \bar{k}$, then $b^*_M > b^*$ ($b^*_M \leq b^*$) holds whenever $s > s_M$ ($s \leq s_M$). Otherwise, $b^*_M < b^*$ always holds.

ii) If $k < \bar{k}$, then $b^*_N \geq b^*$ ($b^*_N < b^*$) holds whenever $s \leq s_N$ ($s > s_N$). Otherwise, $b^*_N > b^*$ always holds.

**Proof.** See the Appendix. \hfill \blacksquare

Our results in Proposition 3 are illustrated in Figure 1, where the solid lines depict the cases $k > \bar{k}$ and $k > \hat{k}$, while the dotted lines reflect the cases $k < \bar{k}$ and $k < \hat{k}$. The critical synergy levels $s_M$, $\bar{s}$, and $s_N$ are represented by the dashed lines.\textsuperscript{24}

![Figure 1: The Impact of a Merger on Managerial Incentives](image)

Notice that a merger may induce both $M$ and the rivals to offer managerial incentives which are lower than those in the pre-merger case. That is the case when $M$’s wage cost is moderately sensitive to synergies ($k \in [\bar{k}, \hat{k}]$) and synergies are sufficiently large ($s > s_N$). As a result, $M$’s

\textsuperscript{23}Notice that this holds for, e.g., the linear Cournot model.

\textsuperscript{24}In Figure 1, we have assumed that the ordering $s_M < \bar{s} < s_N$ holds.
principal is forced to substitute relatively costly managerial effort by synergies. However, the rival firms’ principals will not push their agents to work harder in response to that. Rather, synergies exert a negative effect on their managerial incentives as well, as the negative direct effect of synergies on \( b_N^* \) exceeds the positive indirect effect of synergies relying on \( d b_M^* / ds < 0 \).

4 Synergies, Managerial Incentives, and Consumer Surplus

In this section, we ask how synergies affect (expected) consumer surplus in the post-merger case. Focusing on consumer surplus is motivated by the supposition that many antitrust authorities seem to apply a consumer standard rather than a welfare standard (see, e.g., Whinston, 2007, and Röller, 2011).\(^{25}\) Recent papers on horizontal mergers, such as Nocke and Whinston (2010, 2013), also build on this view. In addition to using a consumer standard, many antitrust authorities allow for an efficiency defense which relies on the common belief that consumer surplus is (monotonically) increasing in the synergy level, so that for sufficiently large synergy levels a merger will be to the benefit of consumers.\(^{26,27}\) A necessary condition for this to hold is that \( M \)'s marginal cost is decreasing in the synergy level, i.e., \( dc_M / ds < 0 \) holds. Then, as Farrell and Shapiro (1990, Lemma, p. 111) have shown, the merged firm will expand its output when synergies get larger and, as a result, total output will increase, as the non-merging firms' output reduction is not sufficient to offset the merged firm’s output expansion.

However, we will show that in the presence of endogenous efficiencies resulting from managerial effort the relationship between (expected) consumer surplus and synergies is ambiguous, and may even be negative whenever \( M \)'s wage cost is sufficiently sensitive to further triggering agents to work harder. This result obviously relies on the incentive effect of synergies which was

\(^{25}\) One reason for a consumer surplus standard involves the firms’ possibility to lobby efficiently (see Neven and Röller, 2005).

\(^{26}\) A sufficient condition for a merger to increase total output and consumer surplus, respectively, is offered by Farrell and Shapiro (1990). It states that total output will increase only if \( M \)'s markup is higher (at the pre-merger output level) than the sum of the merging firms’ pre-merger markups, i.e., \( p(Q^*) - c_M > \sum_{i \in M} [p(Q^*) - c_i] \). Notice that this condition can be easily rearranged so as to define a critical synergy level \( s_{CS} \) (provided that \( c_M = c - s \)):

\[
s > s_{CS} := (m - c) - \sum_{i \in M} c_i / p(Q^*),
\]

where \( m \) is the number of merging firms.

\(^{27}\) Notice that this monotonicity result is also used in empirical works to identify the competitive effects of mergers (see Duso et al., 2007; and Duso et al., 2011).
stressed in the previous section. In the following, we will show how the incentive effect shapes consumer surplus in the post-merger case.

Let (with a slight abuse of notation) \( CS^* \) denote expected consumer surplus, with \( CS^* := \int_0^Q [p(z) - p(Q^*)]dz \). It is well known that \( dCS^*/dQ^* = -p'Q^* > 0 \). When signing the effect of an exogenous change in \( s \) on \( CS^* \), we can thus focus on changes in (expected) total output:

\[
\frac{dQ^*}{ds} = \frac{\partial Q^*}{\partial s} + \frac{\partial Q^*}{\partial b^*_M} \frac{db^*_M}{ds} + \sum_N \frac{\partial Q^*}{\partial b^*_N} \frac{db^*_N}{ds} . \tag{10}
\]

The first term on the right-hand side of (10) reflects the (direct) synergy effect and mirrors the standard reasoning when endogenous efficiencies from managerial effort are absent. An increase in \( s \) raises \( M \)'s output. The non-merging firms will respond by reducing their output, but by less, so that total output increases. The second and the third term on the right-hand side of (10) represent the incentive effect via \( i) \) \( M \)'s managerial incentives and \( ii) \) the rivals' managerial incentives. As laid down in Proposition 2, the incentive effect is ambiguous depending on the sensitivity of \( M \)'s wage cost toward \( s \) that is reflected by \( k \). In order to check whether the overall incentive effect, provided that it is negative, can offset the positive (direct) synergy effect, we make use of the fact that \( Q^* \) moves in the same direction as \( M \)'s output \( q^*_M \). Hence, expected total output and thus \( CS^* \) will decrease if and only if synergies increase \( c_M \) and, as a result, exert a negative impact on \( q^*_M \). This is the case if and only if the following condition holds

\[
\frac{dc_M}{ds} = \frac{\partial c_M}{\partial s} + \frac{\partial c_M}{\partial b^*_M} \frac{db^*_M}{ds} > 0 \tag{11}
\]

given that \( \frac{db^*_M}{ds} < 0 \) (i.e., \( k > \tilde{k} \)) holds. Condition (11) shows that for \( dCS^*/ds < 0 \) to hold, it is not sufficient that \( M \)'s managerial incentives are negatively affected by a higher synergy level. In addition, \( b^*_M \) must decrease by more than \( 2/k \).

However, condition (11) can be simplified to \( k > \tilde{k} \). This follows from quantities being strategic substitutes, so that \( M \)'s rivals will expand their output in response to \( dc_M/ds > 0 \). As this output expansion relies on rivals' incentives being increased due to a higher synergy level, i.e., \( \frac{db^*_N}{ds} > 0 \) holds, we can immediately infer from Proposition 2 that \( k > \tilde{k} \) must hold for consumer surplus to be negatively affected by synergies. Proposition 4 summarizes our results.
Proposition 4. A higher synergy level decreases consumer surplus if and only if $M$’s wage cost is sufficiently sensitive to inducing agents to work harder, i.e., $k > \hat{k}$ holds. Otherwise, synergies positively affect consumer surplus.

Proof. See the Appendix.

Proposition 4 stresses that there exist mergers which reduce consumer surplus even more when synergies get larger so that an efficiency defense would necessarily fail to benefit consumers. The reason is that for sufficiently sensitive wage costs, i.e., $k > \hat{k}$, the negative incentive effect cannot be offset by the positive synergy effect, so that $M$’s marginal cost increases in $s$. Whenever $M$’s wage cost is not too sensitive to synergies ($k < \hat{k}$), however, the opposite is true. In that case, the standard reasoning applies: Consumer surplus is increasing in $s$ and there may thus exist a critical synergy level $s_{CS}$, such that a merger increases consumer surplus whenever $s \geq s_{CS}$ holds. It should be noted, however, that this critical synergy level is increasing in the sensitivity of $M$’s wage cost to synergies, i.e., $ds_{CS}/dk > 0$ holds, making it harder for a merger to be beneficial for consumers.

We illustrate our finding in Figure 2, where (expected) consumer surplus in the post-merger case (pre-merger case) is depicted by the black lines (grey lines).

![Figure 2: The Relationship Between Synergies and Consumer Surplus](image-url)
5 Conclusion

In a first step, we have analyzed the effects of synergies from horizontal mergers on managerial incentives to cut marginal costs. We have shown that synergies may induce the merged firm to stifle managerial incentives. In that case, the merged firm’s principal simply substitutes cost reductions from managerial effort by synergies. This result relies on the fact that synergies come at a cost: in particular, synergies make incentivizing the agents to cut marginal costs more expensive for the merged firm’s principal, which corresponds to an increase of the merged firm’s agency cost. If, however, this wage raising effect is sufficiently low, the merged firm’s principal will provide stronger incentives so that synergies and managerial effort are aligned. Although the impact of synergies on the rival firms’ managerial incentives is reversed, what seems straightforward, as managerial incentives are strategic substitutes, it is remarkable that for moderately sensitive wage costs, both the merged firm and its rivals offer weaker incentives in response to a higher synergy level. This follows from the direct synergy effect being stronger than the negative indirect effect via the merged firm’s managerial incentives.

In a second step, we have taken the previous results to lay down the implications for a merger’s effect on consumer surplus. Thereby, we have highlighted that, in addition to the well-known (direct) synergy effect, the incentive effect of synergies takes on a significant role in shaping the relationship between synergies and consumer surplus. Precisely, whenever the overall incentive effect is negative, mergers are accompanied by a trade-off which has been neglected in the literature on horizontal mergers so far: synergies decrease the merged firm’s marginal cost directly but stifle managerial incentives at the same time. Whenever the latter effect dominates, we identify a negative relationship between consumer surplus and synergies.

We draw two main implications for merger control. First, our paper offers conditions under which the common presumption that consumer surplus is (monotonically) increasing in the synergy level holds. Second, our results stress in the converse case that the efficiency defense in merger control could be misleading. This follows from the fact that, instead of decreasing price and benefiting consumers, respectively, synergies may harm consumers and even more so when they are large. Hence, synergies do not always constitute a countervailing factor toward a merger’s negative effect of (unilaterally) increased market power and may thus oppose the use of an efficiency defense in merger control.
Appendix

In this Appendix we provide the omitted proofs.

Proof of Lemma 1. First, we sign the slope of \( b_r(b_{-r}) \), which is given by

\[
\frac{db_r}{db_{-r}} = -\frac{\partial^2 \Pi_M / \partial b_r \partial b_{-r}}{\partial^2 \Pi_M / \partial b_r^2},
\]

where

\[
\frac{\partial^2 \Pi_M}{\partial b_r^2} = \left[ \frac{\partial^2 \Pi_M}{\partial Q^*_{-M} \partial q^*_M} \frac{\partial c_M}{\partial b_r} + \frac{\partial^2 \Pi_M}{\partial c_M \partial q^*_M} \right] \frac{\partial q^*_M \partial c_M}{\partial c_M} \frac{\partial b_r}{db_r} + \frac{\partial^2 \Pi_M}{\partial w^2_M} \frac{\partial w^2_M}{\partial b_r^2} < 0
\]

holds by assumption (strict concavity). Hence, we only need to examine

\[
\frac{\partial^2 \Pi_M}{\partial b_r \partial b_{-r}} = \left[ \frac{\partial^2 \Pi_M}{\partial Q^*_{-M} \partial q^*_M} \frac{\partial c_M}{\partial b_r} + \frac{\partial^2 \Pi_M}{\partial c_M \partial q^*_M} \right] \frac{\partial q^*_M \partial c_M}{\partial c_M} \frac{\partial b_r}{db_r} + \frac{\partial^2 \Pi_M}{\partial w^2_M} \frac{\partial w^2_M}{\partial b_r \partial b_{-r}}
\]

in order to determine \( \text{sgn}(db_r/db_{-r}) \). Note that \( \partial c_M / \partial b_r = \partial c_M / \partial b_{-r} = -1/k \), \( \partial^2 c_M / \partial b_r \partial b_{-r} = \partial^2 c_M / \partial b_r^2 = 0 \), and \( w_M = \frac{1}{k}(b_r + b_{-r})(b_r + k \epsilon_r + b_{-r} + k s + k \epsilon_{-r}) \), where \( \partial w_M / \partial b_r = s + 2(b_r + b_{-r})/k + \epsilon_r + \epsilon_{-r} \) and \( \partial^2 w_M / \partial b_r \partial b_{-r} = 2/k \), so that (13) can be written as

\[
\frac{\partial^2 \Pi_M}{\partial b_r \partial b_{-r}} = \int_{\theta}^{\theta} \left[ p'(Q) \frac{\partial Q^*_{-M}}{\partial c_M} \frac{\partial q^*_M}{\partial c_M} 1 \frac{1}{k^2} - \frac{\partial q^*_M}{\partial c_M} 1 \frac{1}{k^2} - \frac{2}{k} \right] dF.
\]

It is immediately checked that (14) is always negative, as

\[
k > -\frac{1}{2} \int_{\theta}^{\theta} \left[ \frac{\partial q^*_M}{\partial c_M} \gamma_M \right] dF,
\]

is implied by (12), where \( \gamma_M := 1 - p'(Q^*) \partial Q^*_{-M} / \partial c_M > 0 \) is a measure of the rivals' sensitivity toward changes in \( c_M \).

Finally, we demonstrate that \( M \)'s principal reduces each of her agents' incentives when she employs more agents, i.e., \( h \) increases. Therefore, we treat \( h \) as a continuous variable and allow \( h > 2 \) to hold. Further, we assume (without loss of generality) for the case where \( h > 2 \) that all agents other than \( r \) are symmetric, so that \( w_M = b_r(c - c_M) + (h - 1)b_j(c - c_M) \) if \( h > 2 \), where \( j \neq r \) and \( c_M = c - s - b_r/k - (h - 1)(b_j/k) \). Proceeding as before, we thus obtain that \( \text{sgn}(db_r/dh) = \text{sgn}(\partial^2 \Pi_M / \partial b_r \partial h) \) holds. Inspecting \( \partial^2 \Pi_M / \partial b_r \partial h \), which is given by

\[
\frac{\partial^2 \Pi_M}{\partial b_r \partial h} = \int_{\theta}^{\theta} \left[ p'(Q) \frac{\partial Q^*_{-M}}{\partial c_M} \frac{\partial q^*_M}{\partial c_M} 1 \frac{1}{k^2} - \frac{\partial q^*_M}{\partial c_M} 1 \frac{1}{k^2} - \frac{2}{k} \right] dF,
\]

18
it is straightforward to show that $db_r/dh < 0$ always holds, as $\partial^2 \Pi_M / \partial b_r \partial h < 0$ is always implied by (12). Q.E.D.

**Proof of Proposition 2.** In this proof, we demonstrate that $db^*_M/ds < 0$ ($db^*_M/ds \geq 0$) if $k > \tilde{k}$ ($k \leq \tilde{k}$) and $db^*_N/ds < 0$ ($db^*_N/ds \geq 0$) if $k < \tilde{k}$ ($k \geq \tilde{k}$) hold. Totally differentiating (6) and (8) yields

$$
\frac{d}{ds} \left[ \frac{d \Pi_M}{db^*_M} \right] = \frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} \frac{db^*_M}{ds} + \frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M} \frac{db^*_N}{ds} + \frac{\partial^2 \Pi_M}{\partial b^*_M} \partial h = 0
$$

and

$$
\frac{d}{ds} \left[ \frac{d \Pi_N}{db^*_N} \right] = \frac{\partial^2 \Pi_N}{\partial b^*_N \partial b^*_M} \frac{db^*_N}{ds} + \frac{\partial^2 \Pi_N}{\partial b^*_M \partial b^*_N} \frac{db^*_M}{ds} + \frac{\partial^2 \Pi_N}{\partial b^*_N} = 0,
$$

which can be combined to

$$
\left( \begin{array}{c}
\frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} \\
\frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M}
\end{array} \right) \left[ \begin{array}{c}
\frac{db^*_M}{ds} \\
\frac{db^*_N}{ds}
\end{array} \right] = \left[ \begin{array}{c}
-\frac{\partial^2 \Pi_M}{\partial b^*_M \partial h} \\
-\frac{\partial^2 \Pi_N}{\partial b^*_N \partial h}
\end{array} \right].
$$

Using Cramer’s rule, we obtain

$$
\frac{db^*_M}{ds} = \frac{\frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} \frac{\partial^2 \Pi_N}{\partial b^*_M} \partial h - \frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M} \frac{\partial^2 \Pi_N}{\partial b^*_M} \partial h}{\frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} - \frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M}} \quad (15)
$$

and

$$
\frac{db^*_N}{ds} = \frac{\frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} \frac{\partial^2 \Pi_N}{\partial b^*_N} \partial h - \frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M} \frac{\partial^2 \Pi_N}{\partial b^*_N} \partial h}{\frac{\partial^2 \Pi_M}{\partial b^*_M \partial b^*_N} - \frac{\partial^2 \Pi_M}{\partial b^*_N \partial b^*_M}} \quad (16)
$$

Note that the denominator in both (15) and (16) is clearly positive, which shifts the focus to the numerators in order to examine (15) and (16). Therefore, consider

$$
\frac{\partial^2 \Pi_N}{\partial b^*_M} \partial h = \left[ \frac{\partial^2 \Pi_N}{\partial Q^*_M} \partial q^*_N \partial c_N + \frac{\partial^2 \Pi_N}{\partial B^*_M} \partial q^*_N \partial b^*_M \partial h \right] \frac{\partial q^*_N}{\partial c_M} \partial c_M
$$

$$
= \int_\theta^\theta \left[ p''(Q^*) \frac{\partial^2 Q^*_N}{\partial c_N \partial c_M} \frac{\partial q^*_N}{\partial c_M} + p'(Q^*) \frac{\partial^2 Q^*_N}{\partial c_N^2} \frac{\partial q^*_N}{\partial c_N} \frac{1}{k} - \frac{\partial q^*_N}{\partial c_N} \frac{1}{k} \right] dF,
$$

where we have made use of $\partial c_M / \partial s = -1$ and $\partial c_N / \partial b^*_N = -1/k$. It is straightforward to verify that $\partial^2 \Pi_N / \partial b^*_N \partial h = k[\partial^2 \Pi_N / \partial b^*_N \partial b^*_M]$ (which implies $|\partial^2 \Pi_N / \partial b^*_N \partial h| > |\partial^2 \Pi_N / \partial b^*_N \partial b^*_M|$), so that $\partial^2 \Pi_N / \partial b^*_N \partial h < 0$ holds, as managerial incentives are strategic substitutes. Further, notice that

$$
\frac{\partial^2 \Pi_M}{\partial b^*_M} \partial h = \left[ \frac{\partial^2 \Pi_M}{\partial Q^*_M \partial q^*_M} \partial q^*_M \partial c_M + \frac{\partial^2 \Pi_M}{\partial M \partial q^*_M \partial b^*_M} \partial q^*_M \partial c_M \partial h \right] \frac{\partial q^*_M}{\partial c_M} \partial c_M
$$

$$
= \int_\theta^\theta \left[ p'(Q^*) \frac{\partial^2 Q^*_M}{\partial c_M} \frac{\partial q^*_M}{\partial c_M} \frac{1}{k} - \frac{\partial q^*_M}{\partial c_M} \frac{1}{k} \right] dF,
$$

19
as $\partial c_M/\partial s = -1$, $\partial c_M/\partial b_M^* = -1/k$, $\partial^2 c_M/\partial b_M^* \partial s = 0$, and $\partial^2 w_M/\partial b_M^* \partial s = 1$. It is easily checked that $\partial^2 \Pi_M/\partial b_M^* \partial s < 0 \ (\geq 0)$ holds only if $k > \bar{k} \ (k \leq \bar{k})$, where

$$\bar{k} := -\int_\theta^\beta \left[ \frac{\partial q_M^*}{\partial c_M} \gamma_M \right] dF$$

and $\gamma_M := 1 - p'(Q^*) \partial Q^*_{-M}/\partial c_M > 0$. Note that neither $k > \bar{k}$ nor $k \leq \bar{k}$ is implied by (12), which rather postulates that $k > -\int_\theta^\beta \left[ (\partial q_M^* / \partial c_M) \gamma_M / 2 \right] dF$. We can immediately infer that $db_M^*/ds \geq 0$ holds when $k \leq \bar{k}$, as $\partial^2 \Pi_M/\partial b_M^* \partial b_N^* < 0$, $\partial^2 \Pi_N/\partial b_N^* \partial b_M^* < 0$, and (by assumption) $\partial^2 \Pi_N/\partial b_N^2 < 0$ implying that the numerator in (15) is assuredly non-negative. However, in case of $k > \bar{k}$ we obtain that $db_M^*/ds \geq 0$ ($db_M^*/ds < 0$) only if $k \leq \tilde{k} \ (k > \bar{k})$, where

$$\tilde{k} := \frac{\bar{k}}{1 + (\bar{z}/\bar{\alpha})}.$$  

with $\tilde{z} := (\partial^2 \Pi_M/\partial b_M^* \partial b_N^*)(\partial^2 \Pi_N/\partial b_N^* \partial s) > 0$ and $\tilde{\alpha} := \partial^2 \Pi_N/\partial b_N^2 < 0$, so that $1 + (\bar{z}/\bar{\alpha}) < 1$ implying $\tilde{k} > \bar{k}$. We can thus summarize that $db_M^*/ds \geq 0$ ($db_M^*/ds < 0$) only if $k \leq \tilde{k} \ (k > \bar{k})$.

Correspondingly, we obtain that $db_N^*/ds \geq 0$ ($db_N^*/ds < 0$) only if $k \geq \tilde{k} \ (k < \bar{k})$, where

$$\tilde{k} := \frac{\bar{k}}{1 + (\bar{z}/\bar{\alpha})},$$

with $\tilde{z} := (\partial^2 \Pi_M/\partial b_{M}^2)(\partial^2 \Pi_N/\partial b_N^* \partial s) > 0$ and $\tilde{\alpha} := \partial^2 \Pi_N/\partial b_N^* \partial b_M^2 < 0$. Notice that $\tilde{z}/\tilde{\alpha} < \bar{z}/\bar{\alpha} < 0$, so that $\tilde{k} > \tilde{k} > \bar{k}$ holds. Q.E.D.

Proof of Corollary 1. Recall that by Lemma 1 a higher number of employed agents decreases $M$’s managerial incentives, as $\partial^2 \Pi_M/\partial b_r \partial h < 0$ holds. We can immediately infer that $b_M^* < b_N^*$ must hold for all $k$ at $s = 0$. Further, recall that $db_M^*/ds < 0$ holds if $k > \bar{k}$, so that we need to focus on the case where $k < \bar{k}$ and thus $db_M^*/ds > 0$ holds. It follows that for $k < \bar{k}$ there exists a critical synergy level $\tilde{s} := \{ s > 0 : b_M^* = b_N^* \}$ such that $b_M^* > b_N^*$ ($b_M^* \leq b_N^*$) if $s > \tilde{s}$ ($s \leq \tilde{s}$). Q.E.D.

Proof of Proposition 3. First, recall that we have assumed that $\partial^2 \Pi_M/\partial b_r \partial h < 0$ for $h \geq 1$ is sufficiently larger so that $b_M^* < b^*$ holds at $s = 0$. It follows that $b_N^* > b^*$ holds at $s = 0$ which relies on managerial incentives being strategic substitutes and the positive (unilateral) market power effect of the merger. Using Proposition 1, we can immediately infer that $b_M^* (b_N^*)$ may be larger (smaller) than $b^*$ only if $k < \bar{k}$ and thus $db_M^*/ds > 0 \ (k < \bar{k})$ and thus $db_N^*/ds < 0$ holds. This implies for the merged firm that there exists a critical synergy level $s_M = \{ s > 0 : b_M^* = b^* \}$
such that \( b_{M}^{*} > b^{*} \) (\( b_{M}^{*} \leq b^{*} \)) holds whenever \( s > s_{M} \) (\( s \leq s_{M} \)). Accordingly, for the non-merging firms we obtain that there exists a critical synergy level \( s_{N} := \{ s > 0 : b_{N}^{*} = b^{*} \} \) such that \( b_{N}^{*} \geq b^{*} \) (\( b_{N}^{*} < b^{*} \)) holds whenever \( s \leq s_{N} \) (\( s > s_{N} \)). \textbf{Q.E.D.}

**Proof of Proposition 4.** First, notice that firms’ quantities are strategic substitutes, i.e.,

\[
\frac{dq_{M}}{dQ_{-M}} = -\frac{\partial^{2} \Pi_{M}/\partial q_{M} \partial Q_{-M}}{\partial^{2} \Pi_{M}/\partial q_{M}^{2}} \in (-1, 0) \tag{17}
\]

and correspondingly for the rival firms. Second, following Farrell and Shapiro (1990, Lemma, p. 111), we show that \( Q^{*} \) and \( q_{M}^{*} \) exhibit identical signs when synergies increase. Notice that, in particular, \( |dq_{M}/ds| > |dQ/ds| > 0 \) holds, which is simply due to quantities being strategic substitutes. Define \( \beta_{M} := -(\partial^{2} \Pi_{M}/\partial q_{M} \partial Q_{-M} \partial^{2} \Pi_{M}/\partial q_{M}^{2}) \in (-1, 0) \). Then, rearranging (17) gives \( dq_{M} = \beta_{M} dQ_{-M} \). Adding \( \beta_{M} dq_{M} \) to both sides of the equation yields \( dq_{M} = [\beta_{M} / (1 + \beta_{M})] dQ = -\lambda_{M} dQ \), with \( \lambda_{M} > 0 \), which holds for any firm \( i \) in the market; i.e., \( dq_{i} = -\lambda_{i} dQ \). Summing up over \( N \), we obtain \( dQ_{-M} = -\sum_{N} \lambda_{i} dQ \) and adding \( dq_{M} \) to both sides yields \( dQ = -\sum_{N} \lambda_{i} dQ + dq_{M} \) or \( dQ = \left( 1 + \sum_{i \neq M} \lambda_{i} \right) dq_{M} \).

As \( \lambda_{i} > 0 \), it follows that \( dQ/dq_{M} \in (0, 1) \).

Third, we show that \( dq_{M}^{*} / dc_{M} < 0 \) and thus \( dq_{N}^{*} / dc_{M} > 0 \), as the overall effect of \( s \) on \( c_{M} \), which is given by

\[
\frac{dc_{M}}{ds} = \frac{\partial c_{M}}{\partial s} + \frac{\partial c_{M} \partial b_{M}^{*}}{\partial b_{M}^{*} / ds},
\]

is relevant for evaluating \( dQ^{*} / ds \). Totally differentiating \( \Pi_{M}(q_{M}^{*}, Q_{-M}^{*}, c_{M}) \) and \( \Pi_{N}(q_{N}^{*}, Q_{-N}^{*}, c_{N}) \) with respect to \( c_{M} \) and applying Cramer’s rule, we obtain

\[
\frac{dq_{M}^{*}}{dc_{M}} = -\frac{\partial^{2} \Pi_{M} / \partial q_{M}^{2} \partial \Pi_{N} / \partial q_{M}^{*}}{\partial^{2} \Pi_{M} / \partial q_{M} \partial q_{M}^{*}} < 0.
\]

It follows from strategic substitutability that \( dq_{N}^{*} / dc_{M} > 0 \) and \( |dq_{M}^{*} / dc_{M}| > |dq_{N}^{*} / dc_{M}| > 0 \), which is also immediately verified by inspecting

\[
\frac{dq_{N}^{*}}{dc_{M}} = \frac{\partial^{2} \Pi_{M} / \partial q_{M} \partial \Pi_{N} / \partial q_{M}^{*}}{\partial^{2} \Pi_{M} / \partial q_{M} \partial q_{M}^{*}} > 0.
\]

Finally, notice that for \( dCS^{*} / ds < 0 \) and thus \( dQ^{*} / ds < 0 \) to hold, it must be that \( dc_{M} / ds > 0 \) and condition (11) is met, respectively. This implies that \( dCS^{*} / ds < 0 \) only if \( dq_{N}^{*} / ds > 0 \),
which can only be true if $db_N^*/ds > 0$, as $db_N^*/ds = (dq_N^*/ds)/(dq_N^*/db_N^*)$, where $dq_N^*/db_N^* > 0$, and thus $sgn(db_N^*/ds) = sgn(dq_N^*/ds)$. Q.E.D.
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