City Age and City Size

Kristian Giesen, Jens Suedekum

November 2013
City age and city size *

Kristian Giesen† Jens Suedekum‡

November 19, 2013

Abstract

There has been vast interest in the distribution of city sizes in an economy, but this research has largely neglected that cities also differ along another fundamental dimension: age. Using novel data on the foundation dates of more than 10,000 American cities, we find that older cities in the US tend to be larger than younger ones. To take this nexus between city age and city size into account, we introduce endogenous city creation into a dynamic economic model of an urban system. The city size distribution that emerges in our economy delivers a close fit to different types of US city size data. This evidence can resolve several recent debates, and build a bridge between different views in the literature on city size distributions.

Keywords: Zipf’s law, Gibrat’s law, city size distributions, city age, DPLN distribution

JEL Classification: R11; R12

---

*We thank Xavier Gabaix, Jacques Thisse, Henry Overman, Kurt Schmidheiny, Rafael Gonzáles-Val, Hernan Makse, Diego Rybski, Gary Brand, and the audiences at various seminar presentations for very helpful comments. All errors are solely our responsibility. Parts of this paper were written while Suedekum was visiting SERC/CEP at the London School of Economics. He gratefully acknowledges the hospitality of this institution.

†Mercator School of Management, University of Duisburg-Essen. E-mail: kristian.giesen@uni-due.de.
‡corresponding author. Mercator School of Management, University of Duisburg-Essen; Ruhr Graduate School of Economics, CESifo and IZA. E-mail: jens.suedekum@uni-due.de
1 Introduction

Ever since the seminal works by Auerbach (1913) and Zipf (1949), there has been vast interest in the distribution of city sizes in an economy. This research has largely neglected, however, that cities also differ along another fundamental dimension: age. Using novel data on the foundation dates of more than 10,000 American cities, we show that age heterogeneity is a salient empirical fact. The average US city in our sample is 139 years old today, but there are strong differences. Boston was founded around 383 years ago, while places like Laguna Woods (CA) not even had their 13th birthday yet. Importantly, we find that age and size are positively correlated: Doubling the age of a city is – on average – associated with an increase of the city’s current population size by 57%. The country’s city size distribution and the city age distribution, therefore, have a systematic relationship that we explore in this paper.

In this paper, we introduce endogenous city creation and age heterogeneity into a dynamic economic model of an urban system. Our starting point is the influential approach by Gabaix (1999) and Eeckhout (2004) who consider urban systems where Gibrat’s law is satisfied, that is, where all cities grow with the same expected rate irrespective of their current size. In Eeckhout (2004) there is a fixed population that is freely mobile across a fixed number of equally old cities. City sizes then – in fact, only then – converge to a lognormal (LN) distribution, as cities face random productivity shocks and thus obey to the “pure” Gibrat’s law. The famous Zipf’s law for city sizes emerges instead of the LN when an “impurity” is added, and cities are prevented from becoming too small (Gabaix 1999).1

We assume that the country’s total population is growing. If the number of cities were fixed, this would lead to rising congestion and decreasing equilibrium utility over time, as more and more people have to be squeezed into the urban system. We hence allow for the creation of new cities, which enables the population to spread across more and leads to age differences between cities. When a new city is founded, it starts from a randomly drawn initial productivity which may reflect some deep characteristics of the city’s location (Bleakley and Lin, 2012). Given this initial draw, a new city accordingly adjusts to its equilibrium starting size through population inflows from the established cities, and entrant cities with better productivity draws start off larger. Afterwards, all cities are subject to random shocks which affect the evolution of their equilibrium sizes. Since expected city growth is positive, our model then predicts – in line with the aforementioned facts – that older cities tend to be larger than younger ones.

As for the distribution of growth rates across cities, Gibrat’s law is at work in our model as all cities grow with the same expected rate in the long run. Yet, there are also deviations: new cities (which tend to be relatively small) exhibit strong population growth rates during the transition

1Zipf’s law states that city sizes follow a Pareto distribution with tail exponent close to one. The country’s largest city is then twice as large as the second-largest, three times as large as the third-largest city, and so on.
towards spatial equilibrium, much higher than in established cities. Young cities thus initially grow faster, but revert to the economy-wide average later on. Such a pattern is consistent with recent empirical evidence on US urban growth over the last two centuries. In particular, the studies by Desmet and Rappaport (2012) and Gonzáles-Val, Sánchez-Vidal and Viladecans-Marsal (2012) find that, among young US cities, small ones initially grow faster than the rest of the economy. Among old cities, however, small and large ones tend to grow with the same rate. Our model exhibits such a pattern of urban growth. Furthermore, Michaels, Redding and Rauch (2012) report that, in a comprehensive data set comprising not only large metropolitan areas, small cities tend to grow faster. These authors do not consider city age, but bearing in mind that small cities are on average younger, our model is in line with that evidence as well.2

From this urban system model with its empirically relevant new features, we are able to derive a closed-form solution for the city size distribution (CSD) that emerges endogenously in our economy. This turns out to be the so-called double Pareto lognormal (DPLN) distribution. It is characterized by a lognormal body and power laws in the tails, which are fatter the stronger the age differences between cities are. It thus unifies the LN suggested by Eeckhout (2004), and the Pareto distribution (Zipf’s law) advocated by Gabaix (1999) and by Rozenfeld, Rybski, Gabaix and Makse (2011) in a single model for the overall CSD. As has been shown elsewhere (see Giesen, Suedekum and Zimmermann 2010), the DPLN distribution delivers a close fit to empirical city size data, both in the US and in various other countries, and it (easily) outperforms the LN, Zipf’s law and also other functional forms that have been suggested. It also does so in terms of ”adjusted fit”, that is, when penalizing the DPLN for having a more flexible functional form with more free parameters.

The main contribution of this paper is then twofold. First, we derive a micro-founded economic theory to explain why DPLN distributed city sizes may emerge.3 Second, we show with novel city age data that the main buildings blocks of our model – city age heterogeneity and the positive correlation of city age and city size – are empirically highly relevant. Ultimately, we therefore argue that our urban system model which takes the nexus of city age and city size into account, is more successful in matching contemporaneous city size data than alternative theoretical frameworks that disregard this relationship. Furthermore, it leads to a pattern of urban growth that is consistent with recent evidence, namely Gibrat’s law with stronger initial growth of young cities.

Finally, an additional contribution of this paper is that it can potentially settle a controversial issue from the recent literature, which deals with the question on how to define a city in the first

---

2Finally, our model is also roughly consistent with the sequential urban growth pattern found by Cuberes (2012) as fast-growing young cities will, over time, have a higher average rank in the urban hierarchy.

3The stochastic foundations of the DPLN distribution are discussed in Reed and Jorgensen (2005), who show that it emerges by combining a scale-free growth process with a Yule process for the birth of new units. That model is statistical in nature, however, and does not have economic micro-foundations. We provide an economic theory for the DPLN distribution of city sizes by extending the seminal approach by Eeckhout (2004) to incorporate endogenous city creation and age heterogeneity across cities.
place. In fact, the influential contributions by Eeckhout (2004) and by Rozenfeld et al. (2011) use different city size data, and come to divergent conclusions about the appropriate parametrization of the CSD. Using administratively defined US Census places, Eeckhout (2004) shows that the LN closely fits the data, thus providing empirical support for his model. Rozenfeld et al. (2011), in contrast, use a bottom-up approach of constructing area clusters from high resolution data on population density in the US, independently of administrative boundaries. They emphasize that the sizes of area clusters with at least 13,000 inhabitants closely obey to Zipf’s law. Yet, when analyzing the distribution of the entire US population across space, that is, the overall CSD across all clusters, it turns out that Zipf’s law breaks down. Importantly, when fitting the LN to the area clusters data, one also obtains a very poor fit as is shown in Figure 1 below. The LN thus seems to approximate the overall CSD fairly well for one definition of US cities (Census places), but not for the other (area clusters). By contrast, we show that the DPLN distribution closely fits the empirical CSD across all settlements for both definitions of US cities (see Figure 1). Our findings thus suggest that the CSD can be robustly approximated by the same functional form, regardless of which city size data is used. This evidence is also fully in line with, but goes beyond the findings of Rozenfeld et al. (2011): The DPLN is a parametrization for the overall CSD across all clusters that is consistent with their claim that Zipf’s law holds among the large clusters.\footnote{Relatedly, some authors (most notably Levy 2009, Ioannides and Skouras 2013, and Malevergne, Pisarenko and Sornette 2011) have argued that the large Census places also follow a Zipfian power law pattern that is only imperfectly captured by the LN parametrization, even though the LN fits well outside the upper tail. The features of the DPLN are precisely in line with that evidence. The debate between Levy (2009) and Eeckhout (2009) may thus also be settled by our finding that the sizes of Census places are better approximated by a DPLN distribution.}

More generally, the DPLN builds a bridge between the “old” and the “new” literature on city size distributions. It is fully consistent with Zipf’s law for large cities, and incorporates this into a model for the overall size distribution across all cities.

The rest of this paper is organized as follows. In Section 2 we present our evidence on the distribution of city sizes and show that the DPLN fits the empirical data better than other parametrizations. Section 3 turns to our theoretical model of an urban system with endogenous city creation. There we show that age heterogeneity across cities, together with Gibrat’s law, is key to understanding why the DPLN distribution of city sizes emerges. Section 4 presents our novel empirical evidence on the nexus of city age and city size in the US. Finally, Section 5 concludes.

## 2 City size distributions: The evidence

### 2.1 Data

For our empirical analysis of the city size distribution (CSD) we utilize two different definitions of US “cities”: Census places and area clusters. The former dataset refers to the year 2000 and
includes administratively defined settlements according to legal boundaries. It contains 25,359 cities with sizes ranging from one to about 8 million inhabitants (New York City). Comparable data sets on the sizes of administratively defined settlements (not subject to a threshold size) are by now available for many countries. This is a clear advantage. However, a disadvantage is that the boundaries between those units are sometimes quite arbitrary, as two Census places may be considered as separate cities even though they are essentially part of the same city.\footnote{More details about the widely used Census places data can be found in the Geographic Areas Reference Manual available online under http://www.census.gov/geo/www/garm.html. A further problem with this data is that it only represents 74\% of the total US population who reside in incorporated or Census designated places.}

The second dataset has been constructed by (and is explained in detail in) Rozenfeld et al. (2008, 2011). Here, cities are defined by using a clustering algorithm on high resolution data on population densities in the US. We use their benchmark clusters with $\ell=3$ km, which leads to 23,499 cities covering about 96\% of the US population in 2001 and range from one to about 16 million inhabitants (the New York cluster). The advantage of this data is that cities are defined as genuine agglomerations ignoring administrative boundaries, thereby providing a comprehensive portrayal how the US population spreads across space.\footnote{In the top range these area clusters are often coincident with metropolitan statistical areas (MSAs). However, unlike the MSAs, the area clusters data is not subject to a minimum threshold size.} A current disadvantage is that such data is not (yet) available for many countries.

Figure 1 shows kernel density estimates (in logarithmic scale) of the empirical CSDs for both definitions of cities, see the black solid lines. As can be seen, the mean size of area clusters is higher than for the Census places, while the the variance is lower. These distributional features result from the fact that the clustering algorithm tends to connect adjacent places into one agglomeration (the same area cluster), as is explained in detail by Rozenfeld et al. (2008).

2.2 Parameterization and comparison of data fit

We first fit the LN distribution to the data by using maximum likelihood estimation (see Table 1 for the results). Figure 1 depicts the fitted LN distributions as the grey solid lines. For the Census places, the figure corroborates Eeckhout’s (2004) finding: the LN indeed provides a good fit. However, when using the area clusters, the LN plainly fails to match the data. In other words, the LN seems to approximate the overall CSD fairly well only for one definition of US cities (Census places). Once cities are defined as area clusters as in Rozenfeld et al. (2008, 2011), however, the LN is no longer an appropriate parametrization.

Turning to Zipf’s law, it can be easily verified that it closely fits the data when focusing only on large cities (in either definition).\footnote{We have verified the result by Rozenfeld et al. (2011). Using only area clusters that are larger than 13,000 inhabitants, a standard rank-size regression yields a highly significant tail exponent of 0.994 with a $R^2$ level of 0.99.} However, as is clear from Figure 1, outside the upper tail Zipf’s
law eventually breaks down. Hence, Zipf’s law is not a useful description for the distribution of city sizes once smaller settlements are included in the analysis.

Our suggested functional form for the overall CSD is the DPLN distribution, which has been first introduced by Reed (2002) and is further discussed by Reed and Jorgensen (2005). It has the following density function for city sizes \( S \):

\[
f(S) = \frac{\alpha \beta}{\alpha + \beta} \left[ S^{\beta-1} e^{\left(\frac{\beta \mu + \frac{\sigma^2}{2}}{\sigma}\right)} \Phi\left(\frac{\ln(S) - \mu - \beta \sigma^2}{\sigma}\right) + S^{-\alpha-1} e^{\left(\alpha \mu + \frac{\sigma^2}{2}\right)} \Phi\left(\frac{\ln(S) - \mu - \alpha \sigma^2}{\sigma}\right)\right].
\]

(1)

In (1), the \( \Phi \) is the cumulative and \( \Phi^c \) the complementary-cumulative standard normal distribution. The genesis of the DPLN is discussed in detail in the next section. For the moment, it suffices to note some basic properties. It is a four-parameter distribution (\( \alpha, \beta, \mu \) and \( \sigma \)) featuring a lognormal shape in the body and power laws in the tails. More specifically, if \( S \to \infty \) then \( f(S) \sim S^{-\alpha-1} \), and if \( S \to 0 \) then \( f(S) \sim S^{\beta-1} \). The slope parameters of the Pareto tails are thus \( \alpha \) and \( \beta \), while the parameters \( \mu \) and \( \sigma \) pertain to the location and scale of the LN body. In logarithmic scale, the DPLN can be skewed and its kurtosis can have positive or negative excess, that is, it can be more peaked (leptokurtic) or more flat (platykurtic) than the LN.

**Figure 1: Kernel density estimates and fitted LN + DPLN distributions**

It is straightforward to estimate the parameters of the DPLN as given in (1) by maximum likelihood (see Table 1 for the estimation results). We depict the fitted DPLN distributions in Figure 1 as the dashed black lines. As can be seen, the DPLN provides a very close fit to

---

8 We utilize the log-likelihood function and the corresponding starting values as proposed by Reed (2002).
the empirical CSD for both definitions of US cities. Certainly the DPLN does a better job than the LN. For the area clusters this is self-evident by visual inspection. For the Census places, the performance difference is less pronounced. Still, the DPLN clearly fits better than the LN. Importantly, this is also true when taking into account that the DPLN is a more flexible function form with two more parameters. This improvement can be seen from the Akaike (AIC) and the Bayesian information criterion (BIC), which are also reported in Table 1. Both clearly favor the DPLN over the LN parametrization. Standard statistical specification tests convey the same message: for both data sets, the LN is rejected much earlier than the DPLN.9

Table 1: Data and estimated parameters

<table>
<thead>
<tr>
<th></th>
<th>Area clusters</th>
<th>Places</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>23,499</td>
<td>25,359</td>
</tr>
<tr>
<td>coverage</td>
<td>0.96</td>
<td>0.74</td>
</tr>
<tr>
<td>Min</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max</td>
<td>15,594,627</td>
<td>8,008,278</td>
</tr>
<tr>
<td>DPLN</td>
<td>LN</td>
<td>DPLN</td>
</tr>
<tr>
<td>α</td>
<td>1.659</td>
<td>-</td>
</tr>
<tr>
<td>β</td>
<td>1.830</td>
<td>-</td>
</tr>
<tr>
<td>μ</td>
<td>8.370</td>
<td>8.427</td>
</tr>
<tr>
<td>σ</td>
<td>0.155</td>
<td>0.911</td>
</tr>
<tr>
<td>AIC</td>
<td>450,996</td>
<td>458,347</td>
</tr>
<tr>
<td>BIC</td>
<td>451,028</td>
<td>458,363</td>
</tr>
<tr>
<td>ln(Lj^i)</td>
<td>-225,493.9</td>
<td>-229,171.3</td>
</tr>
</tbody>
</table>

Legend: N is the number of data points (cities), coverage is the percentage of the total US population represented by the data set. Min and Max are the population size of the smallest and the largest settlement. Parameters are estimated with maximum likelihood. ln(Lj^i) is the log-likelihood of distribution j = LN; DPLN for the respective dataset. The Akaike information criterion for dataset i and distribution j is computed as AIC_i^j = 2k_j - 2ln(Lj^i), and the Bayesian information criterion as BIC_j^i = k_j·ln(N^i) - 2ln(Lj^i), with k_j denoting the number of parameters of distribution j. Both criteria favor the distribution j that yields the lower value.

9We have performed Kolmogorov-Smirnov tests by drawing 1000 random samples of size 1000 from both datasets, and for the two hypothesized parametrizations. Using the area cluster (Census places) data we obtain an average p-value of 0.34 (0.41) for the null that the data follows the DPLN. For the null that the data follows the LN we get a p-value much below 0.001 for both datasets. We hence cannot reject the DPLN, while the LN is strongly rejected.
2.3 Discussion

Summing up, the DPLN is a better approximation of the empirical US city size distribution than the LN parametrization, also in terms of "adjusted fit". The better performance of the DPLN also holds for other countries. Giesen, Suedekum and Zimmermann (2010) analyze the CSDs of seven other economies, using data on administratively defined cities comparable to the Census places. Using various model selection tests, they show that the DPLN outperforms the LN in terms of adjusted fit for almost all countries (the only exception is Switzerland). However, they do not explain why DPLN distributed city size should emerge in an urban system. The theory developed in this paper provides such an explanation.

An additional empirical contribution of this paper is to show that the superior fit of the DPLN also holds for the recently developed US area clusters data which were not included in Giesen et al. (2010). For this data, the LN provides a poor fit even in absolute terms while the DPLN fits nicely. This evidence, moreover, complements Rozenfeld et al.’s (2011) findings. They only focus on the upper tail of the size distribution, and find that Zipf’s law performs well there. They do not suggest an appropriate functional form for the overall size distribution across all clusters, however. Our findings show that the DPLN performs well in that respect, and this is fully consistent with Rozenfeld et al.’s (2011) evidence for Zipf’s law among large clusters.\textsuperscript{10}

Finally, the DPLN also outperforms other parametrizations that have been suggested. In particular, Ioannides and Skouras (2013) suggest a mixture of LN and Pareto as the appropriate functional form for the overall CSD, and estimate several versions of it using the US Census places and area clusters data. However, while their ad hoc parametrizations fit better than the LN, they deliver a worse fit for both data sets than the DPLN.\textsuperscript{11} In addition, González-Val, Sanz and Ramos (2013) compare the DPLN and three other parametrizations for the overall CSD, using data from Italy, Spain and the US. They find that the DPLN consistently delivers a better fit than the competing distributions in all three countries.

3 The model

We now turn to the theory and explain why the DPLN distribution for city sizes may emerge. This explanation rests on the nexus of city age and city size. Before we describe our dynamic economic

\textsuperscript{10}Rozenfeld et al. (2011) also provide data for area clusters in Great Britain. We have used that data as well, and obtained the consistent result that the DPLN provides a very good fit while the LN fits poorly. Detailed results for the case of Great Britain are available upon request.

\textsuperscript{11}This can be immediately seen by comparing their Table 1 with our Table 1 above. For the LN distribution we obtain exactly the same results as they do, since we have used the same data. Comparing the log-likelihood, the AIC and the BIC for their parametrizations with our values presented above, it follows that the DPLN is more successful in matching the empirical CSDs than their ad hoc mixture model.
model in Section 3.2., it is useful to first discuss some background about the stochastic foundations of CSDs in an urban system.

### 3.1 Background

Gibrat’s law states that the growth rate of a city is independent of its current size. In this subsection, we first describe what this implies for the stochastic evolution of the size of a single city, and then turn to the overall city size distribution in the economy.

Let $S(i, t)$ be the size of city $i$ at time $t$, and let $\epsilon(i, t) = \dot{S}(i, t)/S(i, t)$ denote the population growth rate between $t$ and $t + dt$. The “pure” Gibrat’s law is satisfied in continuous time when $\epsilon(i, t)$ follows a geometric Brownian motion of the following form:

$$\epsilon(i, t) = \gamma \cdot dt + \varsigma \cdot dB(i, t), \tag{2}$$

where $B(i, t)$ is a Wiener process, $\gamma \geq 0$ is the positive drift, and $\varsigma > 0$ is the variability of this stochastic urban growth process.

Assume that the initial size of city $i$ in logarithmic scale at the time of birth, $\ln S(i, 0)$, is drawn from some distribution with finite mean $s_0 > 0$ and variance $\sigma_0^2 \geq 0$. Now move ahead in time and consider the probability distribution for the size of that city at time $T$. It follows from the central limit theorem and standard Itô calculus that the (log) size of that city in $T$ can be described by the following size probability distribution:

$$\ln S(i, T) \sim N\left(s_0 + \mu_t(T), \sigma_0^2 + \sigma_t^2(T)\right), \tag{3}$$

with:

$$\mu_t(T) = \left((\gamma - \varsigma^2/2) \cdot T\right)$$

and

$$\sigma_t^2(T) = \varsigma^2 \cdot T. \tag{4}$$

The expected size of a city, conditional on its age $T$, is thus $E[S(i, T)] = \exp(s_0 + \sigma_0^2/2 + \gamma \cdot T)$. Provided $\gamma > 0$, this shows that older cities are larger on average since they had longer time to grow under the process specified in (2). The conditional variance of city sizes is also larger for older cities, since they were exposed to random shocks for a longer time.

Turning to the country’s overall CSD in a given point in time, this is the mixture of the size probability distributions of all cities that exist at that time. Suppose for the moment that all cities have the same age $T = \bar{T}$. In that case, it is easy to see from (3) and (4) that all city-specific size probability distributions are LN with the same parameters $s_0 + \mu_t(\bar{T})$ and $\sigma_0^2 + \sigma_t^2(\bar{T})$. The overall CSD that results from a mixture of these identical distributions is then itself also LN with parameters $s_0 + \mu_t(\bar{T})$ and $\sigma_0^2 + \sigma_t^2(\bar{T})$. For the more general case with age heterogeneity across cities and strictly positive drift in the stochastic growth process, however, the overall CSD is not a LN but a mixture of different LN distributions with parameters dependent on the city’s age.\(^\text{12}\)

\(^{12}\)Stated differently, the conditional CSD across all cities with the same age $T$ is a LN when urban growth follows Gibrat’s law as in (2). However, the unconditional CSD across all cities is in general not a LN.
In fact, the overall CSD in a given point in time, \( f(S) \), can be written as the Riemann-Stieltjes integral of the LN with respect to the distribution of the mixing parameter \( T \). Let this distribution, which in our context is the city age distribution, be denoted by \( g(T) \). We then have

\[
f(S) = \int LN(S; s_0 + \mu_t(T), \sigma_0^2 + \sigma_t^2(T)) \, dg(T).
\]

(5)

For particular cases of the distribution \( g(T) \) this integral in (5) can be solved analytically. In particular, assume that the mixing parameter \( T \) is exponentially distributed with shape parameter \( \lambda \), that is, \( g(T) = \exp(T; \lambda) \). As is shown by Reed (2002, 2003), the DPLN as given in (1) is then the solution for this density function \( f(S) \) (see the appendix for details).

Under which conditions would the city age distribution \( g(T) \) follow an exponential form? In a dynamic context, this arises if the mass (the “number”) of cities is increasing at a constant rate, where this rate \( \lambda \) is, in turn, the shape parameter of the exponential. To see this, let the number of cities existing at time \( t \) be denoted as \( N(t) = N_0 e^{\lambda t} \), so that the cumulative distribution of the birth year \( \tau \) can be calculated as \( P(Y \leq \tau) = \frac{N_0}{N} = e^{\lambda \tau - \lambda t} \). The age \( T \) of a city is then defined as \( T = t - \tau \), and using \( Z = t - Y \), the cumulative age distribution is thus characterized by \( P(Z \leq T) = 1 - e^{-\lambda(T)} \). The shape of the city age distribution \( g(T) \), and hence the shape of the resulting asymptotic size distribution \( f(S) \), are therefore determined by the (constant) city creation rate \( \lambda \). In particular, it can be shown (see the appendix) that the slope parameters of the DPLN (\( \alpha \) and \( \beta \)) are increasing in \( \lambda \), so that the CSD has fatter tails the lower \( \lambda \) is. Intuitively, if \( \lambda \) is very low, the upper tail of the CSD is dominated by a small number of very old cities which tend to be very large. Vice versa, the higher \( \lambda \) is, the thinner is the upper tail of the DPLN since the age heterogeneity across cities is lower.\(^{13}\)

Notice further that an exponential city age distribution does not require sustained growth in the mass of cities. Consider, for example, a scenario where the number of cities first grows exponentially in an early phase of history (say, for \( t < \hat{t} \)), but city creation then stops at \( t = \hat{t} \) and the number of cities stays fixed afterwards. Such a scenario seems to roughly match the experience of many European countries with mature urban systems. In such a case, the city age distribution \( g(T) \) is still a shifted exponential distribution,\(^{14}\) and the mixing of the city-specific size probability distributions works analogously in that case. City sizes \( f(S) \) thus still converge to a DPLN distribution, although absolute size differences between cities fan out by the variance of the growth process in (2).

---

\(^{13}\)In the limit with \( \lambda \to \infty \), all cities have the same age and the DPLN turns to a LN. The scenario studied by Eeckhout (2004) corresponds to this case with a degenerate age distribution \( g(T) = \hat{T} \). In addition, \( \gamma = 0 \) is assumed in his framework. In that case, even if there were age heterogeneity, there would be no positive correlation between city age and expected city size although older cities would have a higher variance in their size probability distributions. In Section 4 we provide empirical evidence that age and size are positively correlated across US cities.

\(^{14}\)There are no cities younger than \( \hat{T} = (t - \hat{t}) \) at \( t \), while age is exponentially distributed for cities older than \( \hat{T} \).
Summing up, there are two important insights to bear in mind for an urban system where the pure Gibrat’s law with positive drift (the growth process in (2) with \( \gamma > 0 \)) holds:

1. The urban system only converges to an overall CSD with \( LN \) distributed city sizes if there is no city creation and all cities have the same age.

2. The urban system converges to an overall CSD with \( DPLN \) distributed city sizes if cities are created at a constant rate (at least up to some point in time), so that the city age distribution is described by a (shifted) exponential.

For other city creation dynamics, a different age distribution \( g(T) \) would result. In such a case, one can typically not obtain an analytical expression for the respective asymptotic city size distribution \( f(S) \) from (5), see Section 4.3 for further discussion of this issue.

### 3.2 An urban system with endogenous city creation

We now develop an economic model of an urban system. Our starting point is a continuous time version of the urban growth framework by Eeckhout (2004). We extend that model to incorporate exogenous population growth and technological progress, as well as endogenous city creation.

**Basic setup** Consider an economy with a total population \( S(t) \) that is growing at the exogenous rate \( g_S > 0 \). The economy consists of a continuum of \( N(t) \) locations/cities at time \( t \). Firms produce a perfectly tradeable commodity using labor only, and operate under perfect competition.

The wage \( w(i, t) \) is equal to the marginal product of labor in location \( i \) and time \( t \) and depends positively on the city’s overall productivity \( A(i, t) \) and on the current city size \( S(i, t) \). The positive effect of \( S(i, t) \) on \( w(i, t) \) represents a localized agglomeration externality. At the same time, within each city, agents consume land and have to commute to work, thereby losing effective working time. This represents a negative size externality from congestion: land prices are higher, and more time is lost for commuting in larger cities. For simplicity, we consider the same functional forms for the localized externalities as used in the baseline model by Eeckhout (2004). Indirect utility in city \( i \) at time \( t \) can then be written as

\[
V(i, t) = \Phi \left( A(i, t) \cdot S(i, t)^{-\Theta} \right)^\alpha,
\]

where \( \alpha, \Theta, \) and \( \Phi \) are positive parameters that are the same across cities and time. Note that utility \( V(i, t) \) is decreasing in the local population size \( S(i, t) \).

With respect to productivity \( A(i, t) \), we assume that locations are hit by idiosyncratic and permanent i.i.d. shocks. More specifically, we assume a Brownian motion

\[
\frac{\dot{A}(i, t)}{A(i, t)} = \epsilon A(i, t) = g_A \cdot dt + \varsigma_A \cdot dB(i, t).
\]
The positive drift $g_A > 0$ captures the expected productivity growth in the economy, while $\varsigma_A > 0$ is the variability of this stochastic growth process. The term $A(i,t)$ in (6) then reflects the history of productivity shocks in city $i$ up to time $t$, and $V(i,t)$ is increasing in $A(i,t)$. That is, utility is higher in cities with higher accumulated productivity.

A few comments are in order about this basic setup. First, the negative size externality dominates at the city level. Hence, as in Eeckhout (2004), our model does not feature a U-shaped net agglomeration curve á la Henderson (1974). Notice, however, that this is not overly restrictive since Henderson (1974) has shown in his seminal paper that all cities must be located on the downward-sloping range of that curve in order for a spatial equilibrium (with two or more cities) to be stable. Our model therefore can be thought of as a parsimonious specification for such a constellation. Notice, further, that wages and productivity are still higher in larger cities, ceteris paribus, which is due to the positive agglomeration externality and consistent with abundant evidence on the urban wage premium (Glaeser and Marè, 2001). Second, as Gabaix (1999), Eeckhout (2004), Rossi-Hansberg and Wright (2007), and others, our model does not take a stance on the nature of the random productivity shocks. Our specification may provide a short-cut for a variety of micro-foundations, however, such as changes in localized production amenities, technological innovations causing relocation of firms, city-specific productivity realizations for particular matches of firms and workers, and so on. Finally, as in those and most other models from the urban growth literature, we do not explicitly analyze where in space the cities are located. Yet, when considering the birth of new cities below, we assume that cities starts off from a randomly drawn initial productivity. This initial productivity may reflect (at least implicitly) some deep characteristics of the city’s location (Bleakley and Lin, 2012). Thereby we allow geographical factors to play some role, although we do not consider a screening of potential locations for suitability of city creation.

Spatial equilibrium Workers are freely mobile so that indirect utility is equalized across all cities at each point in time. Using the property that $V(i,t) = V(j,t)$ for all $i$ and $j$, it can be shown (see Giesen 2012) that the economy-wide indirect utility level in the spatial equilibrium is:

$$V^*(t) = \Phi \left( A(t) \cdot S(t)^{-\Theta} \right)^{\alpha}, \quad \text{where} \quad A(t) = \left( \int_{i=0}^{N(t)} A(i,t)^{1/\Theta} \, di \right)^{\Theta} \quad (8)$$

The equilibrium size of a single city then reflects its relative productivity level, $S(i,t)^*/S(t) = (A(i,t)/A(t))^{1/\Theta}$, and it immediately follows from this relationship that Gibrat’s law holds since $A(i,t)$ evolves randomly around the common trend $g_A$. Furthermore, it follows from (8) that $V(t)^*$ is decreasing in $S(t)$. If more workers have to be fitted into a fixed set of cities, city sizes would rise proportionally and all individuals end up worse off because of the pervasive negative size externality. Since the total population grows at the rate $g_S > 0$, welfare would thus decrease.

---

15See Hsu (2012) for a recent model that addresses the spatial dimension of the CSD.
over time, ceteris paribus. Vice versa, $V(t)^*$ is increasing in $A(t)$. Expected productivity growth $g_A > 0$ thus raises welfare over time, ceteris paribus, since it increases wages everywhere.

**Endogenous city creation and growth in new cities**  The formation of new cities in an urban system has been analyzed ever since the classical contributions by Henderson (1974) and Fujita (1978). Those contributions have shown that decentralized market allocations are typically characterized by an inefficient number of cities with inefficient sizes, and triggered a series of papers which analyze different arrangements how the involved externalities can be internalized. Those aspects are not the focus of this paper, but our main interest is the city age distribution that arises endogenously from the dynamics of city formation. As our benchmark, we therefore take the simplest possible approach and consider a forward-looking social planner who creates the efficient number of cities over time. Below we then briefly discuss also other mechanisms for city creation.

Assume there is a large amount of featureless land where the planner can form cities. The creation of every new city imposes sunk resource costs $F$ for developing infrastructure, the housing stock, and so on, that are borne by the currently alive population.\(^\text{16}\) Whenever the planner creates a new city, its initial productivity $A_{i,0}$ is drawn from some distribution with finite mean $A_0 > 0$ and variance $\sigma_{A0}^2 > 0$. As said before, this $A_{i,0}$ may reflect, in a stylized way, some deep location characteristics. Afterwards, productivity in those new cities evolves just as in any other city, namely, according to the Brownian motion (7).

At the time of creation, a new city is initially empty and, hence, offers very high utility. There is inflow of population from the established cities until a new spatial equilibrium is reached. This induced inflow is stronger, the higher is the realization of $A_{i,0}$. That is, the city’s starting size $S_{i,0}$ reflects its initial productivity draw, and the new city exhibits strong growth during the transition towards this starting size. In the model, this transition works instantaneously because of free mobility. If the transition would require some time, which is likely to be the case in reality, young cities would then initially exhibit very high growth rates in their early times. Eventually though, they revert to the growth rate of the established cities. Such a pattern, where Gibrat’s law holds in the long run but where young cities (which tend to be relatively small) initially grow faster, is consistent with recent empirical evidence on US urban growth over the last two centuries.\(^\text{17}\)

**Social planner’s problem**  Let $x(t)$ denote the mass of cities that the planner creates between $t$ and $t+dt$, which adds to the stock of existing cities $N(t)$. The formation of every new city raises the country’s normed productivity $A(t)$ and equilibrium utility $V(t)$, since the population can spread across more cities. Specifically, using (8), equilibrium utility can be rewritten as $V(t)^* = \Phi \cdot \Omega(t)^{\alpha \Theta}$ where

\(^{16}\)If city formation were costless, the planner would create an infinite number of infinitely small cities.

\(^{17}\)See, in particular, Desmet and Rappaport (2012) and González-Val et al. (2012).
\[ \Omega(t) = \int_{i=0}^{N(t)} \frac{A(i,t)^{1/\Theta} \, di}{S(t)}. \]  

This state variable evolves according to

\[ \dot{\Omega}(t) = \left( \frac{(1 + g_A)^{1/\Theta}}{1 + g_S} - 1 \right) \Omega(t) + \frac{x(t) \cdot A_0^{1/\Theta}}{S(t)}. \]  

(10)

The first term in (10) entails the exogenous growth rate of the (transformed) equilibrium utility for a fixed set of cities, which is increasing in \( g_A \) and decreasing in \( g_S \). The (positive) second term is the expected benefit from developing new cities.

The forward-looking planner chooses the time-path of city creation \( x(t) \) in order to maximize overall welfare, taking into account the real resource costs of city creation. The present-value Hamiltonian of this dynamic problem can be written as follows,

\[ H(t) = e^{-(\rho - g_S)t} \left( V(t)^* - \frac{x(t) \cdot \chi F}{S(t)} \right) + \lambda(t) \cdot \dot{\Omega}(t), \]  

(11)

where \( \rho > g_S > 0 \) is the time discount rate, \( \chi \) is the marginal utility of income that is assumed fixed, and \( \lambda(t) \) is the costate variable. The planner maximizes (11) subject to the transition equation (10) and \( x(t) \geq 0 \). This is a standard optimal control problem, and it can be shown that the planner creates cities so as to smooth utility over time. It becomes \( V^* = \Phi \cdot \Omega^* \cdot \alpha \Theta \), where

\[ \Omega^* = \left( \frac{\alpha \Theta \Phi \cdot A_0^{1/\Theta}}{\chi F} \cdot \frac{1 + g_S}{(1 + \rho - g_S)(1 + g_S) - (1 + g_A)^{1/\Theta}} \right)^{\frac{1}{1/\Theta}} \]  

(12)

The time path of city creation is then given by

\[ x^*(t) = e^{g_S \cdot t} \cdot \left[ \left( 1 - \frac{(1 + g_A)^{1/\Theta}}{1 + g_S} \right) \cdot \frac{S_0}{A_0^{1/\Theta}} \right] \cdot \Omega^*. \]  

(13)

The condition \( x^*(t) \geq 0 \) requires that \( (1 + g_A)^{1/\Theta} < (1 + g_S) \), i.e., population growth must be sufficiently strong relative to exogenous productivity growth. We assume that this is the case. It then follows from (12) and (13) that the mass of created cities is higher at every point in time the higher is \( g_A \) and the lower is \( F \). Most importantly, it follows from (13) that \( \dot{x}(t)/x(t) = g_S \).

In other words, the planner creates cities at a constant rate, namely the country’s population growth rate. Productivity growth \( g_A \) positively affects the level of city creation, but not its growth rate. Finally, when the mass of new born cities increases at a constant rate, so does the total number of cities. Specifically, we have \( \dot{N}(t)/N(t) = x(t)/N(t) \) which becomes \( \frac{e^{(t \cdot g_S)}}{e^{(t \cdot g_S) - 1}} \cdot g_S \) and thus (quickly) converges to \( g_S \).
City age and city size distribution  The efficient number of cities thus increases at a constant rate in our urban system. The planner chooses this time path of city creation in view of the constant growth of the economy’s total population, since the creation of new cities avoids crowding in the established cities and thereby leads to utility smoothing for all generations over time.

As shown before, this constant growth in the mass of cities endogenously leads to an exponential city age distribution, \( g(T) = \exp(T; \lambda) \), where \( \lambda = g_S \) in our model. Moreover, since population growth among established cities obeys to the pure Gibrat’s law, as they are hit by idiosyncratic productivity shocks, city sizes will thus converge to a DPLN distribution: The city-specific size probability distributions follow a LN because of Gibrat’s law, with mean and variance increasing by the city’s age \( T \). These city-specific distributions are then mixed according to the exponential age distribution, which in turn leads to the DPLN distribution for city sizes (see Section 3.1). The lower is \( g_S \), the more characteristic is the DPLN shape and the fatter are the tails of the CSD.

Recall that the DPLN would also emerge if the city age distribution were a shifted exponential. That age distribution would result in our model if the population grows at the rate \( g_S > 0 \) for \( t < \hat{t} \), but when growth unexpectedly stops at \( \hat{t} \) and the overall population remains constant afterwards. Then, at \( \hat{t} \), the planner stops creating cities so that their total mass remains fixed from there on.

Decentralized city creation  Notice that a city is a pure public good in our model. Hence, a single individual has no incentive to invest in the formation of new cities. To see this, suppose that city creation is decided upon in a fully decentralized fashion. Let \( x_j(t) \) denote the mass of cities created by some individual \( j \), and \( x_{-j}(t) \) the mass of cities created by others. The optimal control problem for individual \( j \) can then be described by the following Hamiltonian:

\[
H_j(t) = e^{-(\rho - t)} (V(t)^* - x_j(t) \cdot \chi F) + \lambda(t) \left[ g_O \cdot \Omega(t) + \frac{A_0^{1/\Theta} (x_j(t) + S_{-j}(t)x_{-j}(t))}{S(t)} \right]
\]

where \( g_O = \left( \frac{(1+g_A)^{1/\Theta}}{1+g_S} - 1 \right) \) and \( S_{-j}(t) \approx S(t) \) is the population size except for \( j \). A single individual thus bears the full costs of city creation, but shares the benefits with all others due to free mobility. It is straightforward to show from (14) that \( x_j(t) = x_{-j}(t) = 0 \), so that the total mass of cities stays fixed at some initial level \( N_0 \). This extreme underinvestment results from a standard free rider problem, which even becomes more severe over time as the population is growing.

This inefficiency of the decentralized market allocation can be addressed in various ways. For example, there may be a government which coordinates the city formation process by collecting taxes that are spent on the development costs. It is beyond the scope of this paper, however, to analyze the details of such institutional arrangements to implement the efficient allocation.\(^{18}\) We just suffice with pointing out that, with an appropriately designed policy scheme, constant growth in the mass of cities would also result as an equilibrium outcome in the urban system.

\(^{18}\)See Henderson and Venables (2009) for a recent analysis of related issues in a dynamic context.
Summing up, in our urban system (constant) population growth induces (constant) growth in
the number of cities, and hence age differences between cities. Such a result can, supposedly, also
be derived from many alternative urban models that do not share the specificities of our simple
framework. Importantly, these dynamics of city creation, in turn, generate an exponential city age
and a DPLN city size distribution which has been the main goal for our theoretical analysis.

4 City age and city size in the US urban system

In this final section we empirically address age heterogeneity across American cities. Thereby we
provide evidence for the key new building block of our model, and we discuss this evidence in the
light of our theoretical approach.

Although little is known so far about the number or the age structure of cities in an economy,
we are not the first to address those issues. Among the few existing papers are Dobkins and
Ioannides (2001), Henderson and (2007), González-Val et al. (2012) and Desmet and Rappaport
(2012). These studies clearly show that the number of cities has grown over time, which implies
that cities differ by age. However, Dobkins and Ioannides (2001) and Henderson and Wang (2007)
only include cities in their analysis that are larger than a certain threshold size. Their information
thus refers to the date when the city’s size has crossed the threshold, but not to the city’s actual
creation. Desmet and Rappaport (2012) and González-Val et al. (2012) comprehensively count the
number of all US counties or, respectively, Census places that exist in a given time, thereby giving
a more comprehensive picture of city ages in the US.19 However, they focus on age-dependent
patterns of urban growth as discussed before, but do not address the correlation of city age and
the current city size which is one of our main aims.

4.1 Data

Our data traces the actual foundation dates of American cities, which correspond in their definition
to the US Census places, so that the city age data is compatible with the previously used city size
data for the year 2000.20 Among historians, there is no agreement on the precise meaning of the
term “foundation date” for a city. Some claim it to be the date when the first settlers arrived
at the site, when the deed for the land was granted, or when the first building was completed.
However, such dates are typically unknown. As the birth date of the respective US Census place,
we therefore consider the foundation date of the administrative municipality, that is, the earliest
date of self-government or incorporation.

19Desmet and Rappaport (2012) use data for US counties and find that their number has increased from about
300 in 1800 to more than 3,000 in the year 2000. González-Val et al. (2012) report that the number of incorporated
Census places has increased from 10,496 to 19,211 in the period 1900–2000.

20Explicit information about the history of the area clusters is, unfortunately, not available.
We use data from the commercially sold *Cities Databank™* that has extensively collected this information, drawing on official sources including legal citation of a law, court order, county commission order, or city charter, as well as by examining standard library citations for an original published source, or a legally designated source or repository, and the laws of each legislative session of each territory, colony and State.\textsuperscript{21} The data base, in total, includes 10,417 US Census places, together accounting for around 148 million citizens, roughly half of the total US population in 2000.\textsuperscript{22} For the vast majority of cases (> 99%), the foundation date refers to the earliest incorporation or self-governing date of the respective place.\textsuperscript{23} In a few instances, a merger or a consolidation of several Census places to a single one occurred. In those cases, we use wherever possible the incorporation date of the oldest involved place. For example, the foundation of New York (2/12/1654) refers to the first date of self-government of a legislative body in the city. The consolidation of the boroughs Manhattan, Brooklyn, Richmond (Staten Island), Queens and Bronx to the Greater New York Area, the currently defined Census place, only occurred in 1898 which would not be an appropriate description of the city’s foundation.\textsuperscript{24} We exclude 15 cities where the foundation date is given by a later than the first incorporation, or where an unclear merger happened. Thereby we end up with 10,402 Census places for which we have reliable information on their foundations.

4.2 Empirical analysis

Table 2 gives a first overview and reports the birth dates of some selected US cities. The oldest ones – including Boston – were 370 years old in the year 2000. The youngest cities have just been founded at that time. Among the largest US cities, New York is the oldest with age 346, while Chicago, Houston and Los Angeles are in the middle of the spectrum with ages between 150 and 166 years in the year 2000. Las Vegas has been founded more than 250 years later than New York.

\textsuperscript{21}To the best of our knowledge, this data set has not been analyzed in the economics literature so far.

\textsuperscript{22}This sample is representative for the universe of all US Census places considered above. The sample’s smallest city has 6 inhabitants in 2000, so even the very small places are included. Furthermore, it can be shown that the CSD among those cities looks very similar to the overall CSD across all Census places, and that the DPLN distribution delivers a better data fit than the LN also for this sub-sample of cities.

\textsuperscript{23}Incorporations under colonial law were often limited by the terms of the royal charter of the colony. Some cities were also initially chartered by the British or Dutch crowns, by the royal governor of the colony, or by the colonial legislature. In such cases, the earlier charter date is used as the foundation date if self-government was provided to the city. An example is Boston, which has been self-governing since 1630 but was incorporated only in 1822.

\textsuperscript{24}There are two further data issues concerning border modifications over time. First, in the case of small annexations that did not significantly change the appearance of the annexing place (e.g., a large Census place A swallows a small one B), we use the initial foundation of the surviving place A. Second, in the very rare event where one place A is divided into several ones, we use the date of division as the birth date of the resulting places B and C.
Table 2: Age and size of some selected US cities

<table>
<thead>
<tr>
<th></th>
<th>City (Census Place)</th>
<th>Foundation date</th>
<th>Population size in 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Watertown, MA</td>
<td>9/17/1630</td>
<td>32,986</td>
</tr>
<tr>
<td>2</td>
<td>Boston, MA</td>
<td>10/29/1630</td>
<td>589,141</td>
</tr>
<tr>
<td>3</td>
<td>Hampton, NH</td>
<td>4/19/1639</td>
<td>14,937</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>New York City, NY</td>
<td>2/12/1654</td>
<td>8,008,278</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Philadelphia, PA</td>
<td>11/8/1701</td>
<td>1,517,550</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,830</td>
<td>Chicago, IL</td>
<td>8/12/1833</td>
<td>2,896,016</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,181</td>
<td>Houston, TX</td>
<td>6/5/1837</td>
<td>1,953,631</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,999</td>
<td>Los Angeles, CA</td>
<td>4/4/1850</td>
<td>3,694,820</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8,468</td>
<td>Las Vegas, NV</td>
<td>3/16/1911</td>
<td>478,434</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10,400</td>
<td>Fountain Lake, AR</td>
<td>07/26/1999</td>
<td>409</td>
</tr>
<tr>
<td>10,401</td>
<td>Sammamish, WA</td>
<td>8/31/1999</td>
<td>34,104</td>
</tr>
<tr>
<td>10,402</td>
<td>Palm Coast, FL</td>
<td>12/31/1999</td>
<td>32,732</td>
</tr>
</tbody>
</table>

Legend: Table reports the foundation date and the population size in 2000 of selected US Census Places. Foundation dates are taken from the Cities Databank™ and refer to the earliest date of incorporation or self-government of the municipality. Data collection ends as of 12/31/1999.

In Table 3 we summarize some features of the US city age distribution. The data clearly shows the development of the country from the East to the West. The average age of US cities in 2000 was 125.7 years. Cities in the “frontier” States in the South-West and West are on average much younger than that, however, while cities in the more traditional States in the Mid-West and along the East Coast (particularly in New England) are older. This is shown in the second and third row, where we divide the US into those two parts. Table 3 also shows differences in the shape of the city age distribution across the two groups of US Federal States. The distribution is positively skewed among the cities in the traditional States, while it has negative skewness in the frontier States. This is also shown in Figure 2, where we graphically illustrate the city age distributions. In the traditional US States, only few cities are younger than 100 years old in the year 2000. The distribution exhibits a peak in the range between 110-120 years, and then has some very old cities to the far right in the upper tail. In the frontier States, on the other hand, the bulk of cities is younger than 100 years, and only few are older than 150 years. The shape of the age distribution for the US as a whole resembles the one in the traditional States, with the young cities from the frontier States showing up in the lower tail.
Table 3: City age distribution and correlation between age and size

<table>
<thead>
<tr>
<th></th>
<th>number of cities</th>
<th>mean age</th>
<th>std. deviation</th>
<th>skewness</th>
<th>age-size correl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL</td>
<td>10,402</td>
<td>125.7</td>
<td>49.49</td>
<td>.385</td>
<td><strong>0.571</strong></td>
</tr>
<tr>
<td>traditional States (East Coast &amp; Mid-West)</td>
<td>8,008</td>
<td>136.3</td>
<td>47.67</td>
<td>.463</td>
<td><strong>0.593</strong></td>
</tr>
<tr>
<td>frontier States (West &amp; South-West)</td>
<td>2,394</td>
<td>90.2</td>
<td>37.50</td>
<td>-.319</td>
<td><strong>0.546</strong></td>
</tr>
</tbody>
</table>

Legend: Table reports the number cities, mean age, standard deviation and skewness of the age distribution of US Census places. Foundation dates are taken from the Cities Databank°°M and refer to the earliest date of incorporation or self-government of the municipality. The last column reports the correlation between log age (in years as of 2000) and log population size in 2000, controlling for Federal State fixed effects to account for State-specific differences in incorporation laws. The second row refers to cities from the following States: AL, CT, DE, FL, GA, IA, IL, IN, KY, MA, MD, ME, MI, MN, MO, MS, NC, NH, NJ, NY, OH, PA, RI, SC, TN, VA, VT, WI, WV. The third row refers to cities from the following States: AK, AR, AZ, CA, CO, HI, ID, KS, LA, MT, ND, NE, NM, NV, OK, OR, SD, TX, UT, WA, WY.

Figure 2: US city age distribution (kernel density)

Finally, recall from the previous section that a positive correlation between city age and city size plays an important role in the genesis of the DPLN city size distribution. As can be seen in Table 3, this positive correlation is strongly empirically supported both for the traditional and the frontier States. The elasticity of current city size with respect to city age is estimated to be 0.571 for the US urban system as a whole, and is highly statistically significant. That is, doubling the age of a city is – on average – associated with an increase of the city’s current population size by about 57%. In the traditional States, the elasticity is a bit higher (0.593) and in the frontier States it is a bit lower (0.546), but in both cases there is a notably positive and highly significant relationship between city age and city size in the data.

---

25In the log size-log age regression we have controlled for Federal State fixed effects, in order to take into account State-specific differences in historical incorporation legislations which affect the measured city foundation dates.
4.3 Discussion

Using the novel US city age data, we thus find empirical support for several features and predictions of our theoretical model. There is vast age heterogeneity across American cities, and older cities tend to be larger than younger ones. The country’s overall distribution of city sizes is, therefore, also affected by the city age profile in the economy. Evidence is more mixed when it comes to the precise functional form of the city age distribution. Our model shows that the DPLN distribution for city sizes emerges when city ages follow a (possibly shifted) exponential distribution. As can be seen in the left panel of Figure 2, such an exponential shape may not perfectly fit the city age data, mainly because of the mass of young cities in the lower tail. However, it can be argued that a shifted exponential still yields a reasonable approximation. The empirical age distribution roughly starts at a minimum city age of around 100 years, and is then clearly right-tailed as indicated in Table 3. Those features are in line with the parametrization of a shifted exponential, which delivers a decent approximation of the city age data particularly for the cities in the mature part of the US urban system (see middle panel of Figure 2).

Notice that those deviations are not necessarily a refutation of our theoretical model. We have shown that constant growth in the mass of cities (and hence, a pure exponential shape of \( g(T) \)) would result as the efficient allocation in an urban system with constant population growth up to some point in time. Irregularities in the dynamics of population growth, or failures in the policy scheme applied to implement the social optimum as the decentralized market allocation, would also distort the shape of the city age distribution. The resulting CSD would then also not be the DPLN, but something more complex.

One possible strategy could be to estimate a distribution \( \tilde{g}(T) \) that closely matches the city age data, and then to mix age-specific LN size probability distributions as in (5), under the assumption that the mixing parameter \( T \) is distributed according to this function \( \tilde{g}(T) \). The disadvantage of such an approach, however, is that an analytical expression for the resulting size distribution \( \tilde{f}(S) \) is then, in general, no longer available and \( \tilde{f}(S) \) can only be obtained via simulation.

The DPLN, by contrast, can be solved in closed form, and it can be readily taken to the data by using standard methods. As shown in Section 2, it achieves a considerable edge over the LN and other parametrizations in terms of data fit to the empirical CSD, yet without being computationally much more difficult to handle. This advantage would disappear when both the age distribution \( \tilde{g}(T) \) and the resulting asymptotic city size distribution \( \tilde{f}(S) \) have to be simulated. We therefore believe that our theory-based approach to derive the DPLN distribution for city sizes is more attractive than a pure simulation approach, even if the exponential city age distribution does not fit the empirical age data perfectly.
5 Conclusions

Recently, there has been a lively discussion about city size distributions. Our research can potentially resolve several controversial issues from this literature. First, our results show that the same functional form – the DPLN distribution – closely approximates the empirical city size data, regardless of whether cities are economically or administratively defined. Second, the DPLN unifies the lognormal distribution suggested by Eeckhout (2004) and the Pareto distribution (Zipf’s law) advocated by Gabaix (1999), Rozenfeld et al. (2011), and many others, in a single framework of an urban system, thereby building a bridge between those two views.

The main aim of this paper was to provide economic foundations where this DPLN distribution of city sizes comes from. One crucial building block is age heterogeneity across cities, which emerges in our model as a growing population allocates over an endogenously determined set of locations. A second important feature is the positive correlation of city age and city size. Finally, the model predicts that cities grow with the same expected rate in the long run (Gibrat’s law), but that young cities may grow faster in the beginning. As we show in this paper, these building blocks of the DPLN size distribution are all empirically relevant. In particular, using novel data on the foundation dates of American cities, we indeed find strong age differences, and that older cities in the US tend to be larger than younger ones.

References


Appendix A: Genesis of the DPLN

Instead of directly deriving the density function of the DPLN by solving the Riemann-Stieltjes integral given in (5), one can make use of the respective moment generating function (mgf). Reed (2002) shows the mgf of a city with distribution according to equation (3) and age $T$ is given by

$$M_{\log(S_T)}(\theta) = \exp \left( s_0 \theta + \frac{\sigma^2_0 \theta^2}{2} + \left( \gamma - \frac{\varsigma^2}{2} \right) \theta + \frac{\theta^2 \varsigma^2}{2} \cdot T \right) \quad (15)$$

and the corresponding mgf of the overall distribution, under which $T$ is also a random variable, is

$$M_{\log(S)}(\theta) = \exp \left( s_0 \theta + \frac{\sigma^2_0 \theta^2}{2} \right) \cdot M_T \left( \left( \gamma - \frac{\varsigma^2}{2} \right) \theta + \frac{\theta^2 \varsigma^2}{2} \right). \quad (16)$$

Under the assumption that $T$ follows an exponential distribution, the mgf of time becomes

$$M_T(\theta) = \frac{\lambda}{\lambda - \theta} \quad \text{and therefore}$$

$$M_{\log(S)}(\theta) = \frac{\exp \left( s_0 \theta + \frac{\sigma^2_0 \theta^2}{2} \right)}{\lambda - \left( \gamma - \frac{\varsigma^2}{2} \right) \theta - \frac{\theta^2 \varsigma^2}{2}}, \quad (17)$$

which can be simplified by using a partial decomposition (see Appendix B) to

$$M_{\log(S)}(\theta) = \exp \left( s_0 \theta + \frac{\sigma^2_0 \theta^2}{2} \right) \cdot \frac{\alpha \beta}{(\alpha - \theta)(\beta + \theta)}. \quad (18)$$

This shows that the distribution of $\log(S)$ is the convolution of a normal distribution with an asymmetric Laplace distribution, since $\exp \left( s_0 \theta + \frac{\sigma^2_0 \theta^2}{2} \right)$ is the mgf of a normal distribution and $\frac{\alpha \beta}{(\alpha - \theta)(\beta + \theta)}$ is the mgf of an asymmetric Laplace distribution. The respective distribution of $S$, as represented in equation (1), is then obtained by transforming log city sizes to levels.

Appendix B: Specifics of $\alpha$ and $\beta$

The parameters $\alpha$ and $\beta$ are time constant collections of the parameters $\gamma$, $\varsigma$ and $\lambda$, which govern the growth process. They are determined in the above partial decomposition of the mgf of the DPLN, which reduces equation (17) to (18). Therein, the parameters $\alpha$ and $-\beta$ are the roots of the characteristic equation

$$\left( \gamma - \frac{\varsigma^2}{2} \right) \theta + \frac{\sigma^2 \theta^2}{2} - \lambda = 0$$

given by

$$\alpha = \frac{-2\gamma + \varsigma^2 + \sqrt{(-2\gamma + \varsigma^2)^2 + 8\varsigma^2 \lambda}}{2\varsigma^2} \quad \text{and} \quad \beta = \frac{2\gamma - \varsigma^2 + \sqrt{(-2\gamma + \varsigma^2)^2 + 8\varsigma^2 \lambda}}{2\varsigma^2}.$$

As can be seen, $\alpha$ and $\beta$ are increasing in $\lambda$. Therefore, in the limit where $\lambda \to \infty$ this translates into $\alpha \to \infty$ and $\beta \to \infty$ and the DPLN turns to a LN, as the mgf of the DPLN in equation (18) converges to the mgf of a normal distribution.
<table>
<thead>
<tr>
<th>Number</th>
<th>Author(s)</th>
<th>Title</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>Giesen, Kristian and Suedekum, Jens</td>
<td>City Age and City Size</td>
<td>November 2013.</td>
</tr>
<tr>
<td>117</td>
<td>Sapi, Geza and Suleymanova, Irina</td>
<td>Consumer Flexibility, Data Quality and Targeted Pricing</td>
<td>November 2013.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Forthcoming in: European Economic Review.</td>
<td></td>
</tr>
<tr>
<td>115</td>
<td>Baumann, Florian, Denter, Philipp and Friehe Tim</td>
<td>Hide or Show? Endogenous Observability of Private Precautions Against Crime When Property Value is Private Information</td>
<td>November 2013.</td>
</tr>
<tr>
<td>113</td>
<td>Aguzzoni, Luca, Argentesi, Elena, Buccirossi, Paolo, Ciari, Lorenzo, Duso, Tomaso, Tognoni, Massimo and Vitale, Cristiana</td>
<td>They Played the Merger Game: A Retrospective Analysis in the UK Videogames Market</td>
<td>October 2013.</td>
</tr>
<tr>
<td>110</td>
<td>Baumann, Florian and Friehe, Tim</td>
<td>Competitive Pressure and Corporate Crime</td>
<td>September 2013.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Forthcoming in: Industrial and Corporate Change.</td>
<td></td>
</tr>
<tr>
<td>106</td>
<td>Baumann, Florian and Friehe, Tim</td>
<td>Design Standards and Technology Adoption: Welfare Effects of Increasing Environmental Fines when the Number of Firms is Endogenous</td>
<td>September 2013.</td>
</tr>
</tbody>
</table>


Baumann, Florian and Friehe, Tim, Status Concerns as a Motive for Crime?, April 2013.


Baumann, Florian and Friehe, Tim, Private Protection Against Crime when Property Value is Private Information, April 2013. Published in: International Review of Law and Economics, 35 (2013), pp. 73-79.


Jovanovic, Dragan, Mergers, Managerial Incentives, and Efficiencies, April 2013.


84 Bataille, Marc and Steinmetz, Alexander, Intermodal Competition on Some Routes in Transportation Networks: The Case of Inter Urban Buses and Railways, January 2013.


82 Regner, Tobias and Rie ner, Gerhard, Voluntary Payments, Privacy and Social Pressure on the Internet: A Natural Field Experiment, December 2012.


80 Baumann, Florian and Fri ehe, Tim, Optimal Damages Multipliers in Oligopolistic Markets, December 2012.


78 Baumann, Florian and Heine, Klaus, Innovation, Tort Law, and Competition, December 2012. Forthcoming in: Journal of Institutional and Theoretical Economics.

77 Coenen, Michael and Jovanovic, Dragan, Investment Behavior in a Constrained Dictator Game, November 2012.


73 Riener, Gerhard and Wiederhold, Simon, Heterogeneous Treatment Effects in Groups, November 2012.


71 Muck, Johannes and Heimeshoff, Ulrich, First Mover Advantages in Mobile Telecommunications: Evidence from OECD Countries, October 2012.


Regner, Tobias and Riener, Gerhard, Motivational Cherry Picking, September 2012.


Riener, Gerhard and Wiederhold, Simon, Team Building and Hidden Costs of Control, September 2012.


Dewenter, Ralf, Jaschinski, Thomas and Kuchinke, Björn A., Hospital Market Concentration and Discrimination of Patients, July 2012.


Dewenter, Ralf and Heimeshoff, Ulrich, More Ads, More Revs? Is there a Media Bias in the Likelihood to be Reviewed?, June 2012.


Benndorf, Volker and Rau, Holger A., Competition in the Workplace: An Experimental Investigation, May 2012.


48 Herr, Annika and Suppliet, Moritz, Pharmaceutical Prices under Regulation: Tiered Co-payments and Reference Pricing in Germany, April 2012.


34 Christin, Cémence, Nicolai, Jean-Philippe and Pouyet, Jerome, The Role of Abatement Technologies for Allocating Free Allowances, October 2011.


31 Hauck, Achim, Neyer, Ulrike and Vieten, Thomas, Reestablishing Stability and Avoiding a Credit Crunch: Comparing Different Bad Bank Schemes, August 2011.


26 Balsmeier, Benjamin, Buchwald, Achim and Peters, Heiko, Outside Board Memberships of CEOs: Expertise or Entrenchment?, June 2011.


