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Targeted Pricing, Consumer Myopia and Investment in Customer-Tracking Technology

Irina Baye* Geza Sapi†

February 2014

Abstract

We analyze how consumer myopia influences investment incentives into a technology that enables firms to track consumers' purchases and make targeted offers based on their preferences. In a two-period Hotelling setup firms may invest in customer-tracking technology. If a firm acquires the technology, it can practice first-degree price discrimination among consumers that bought from it in the first period. We distinguish between the cases of all consumers being myopic and when they are sophisticated. In equilibrium firms collect customer data only when consumers are myopic. In that case two asymmetric equilibria emerge, with either one firm investing in customer-tracking technology. We derive several surprising results for consumer policy: *First*, contrary to conventional wisdom, firms are better-off when consumers are sophisticated. *Second*, consumers may be better-off being myopic than sophisticated, provided they are sufficiently patient (the discount factor is high enough). *Third*, in the latter case there is a tension between consumer and social welfare, and correspondingly between consumer and other policies: With myopic consumers, banning customer-tracking would increase social welfare, but may reduce consumer surplus.

JEL-Classification: D43; L13; L15; O30.

Keywords: Price Discrimination, Customer Data, Consumer Myopia.

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† European Commission DG COMP - Chief Economist Team and Düsseldorf Institute for Competition Economics (DICE), Heinrich Heine University of Düsseldorf. E-mail: sapi@dice.uni-duesseldorf.de. The views expressed in this article are solely those of the authors and may not, under any circumstances, be regarded as representing an official position of the European Commission.

1 Introduction

The rapidly improving ability of firms to collect, store and analyze customer data created large opportunities for personalized pricing and other personalized marketing activities. One of the important sources of customer data are loyalty programs, which are particularly widespread in the retail and airline industries (see, for example, Choi, 2013). The CEO of Safeway Inc., the second-largest supermarket chain in the U.S., Steve Burd, said that "*There's going to come a point where our shelf pricing is pretty irrelevant because we can be so personalized in what we offer people*" (Ross, 2013). Airlines have also developed sophisticated techniques to utilize customer insights they obtain from frequent-flyer programs (see, for example, Kolah, 2013). Consumer online purchases and other types of online activities provide further important sources of customer information.[1,2,3]

The increased use of customer data for targeted marketing activities has triggered strong reactions from consumer policy advocates. The debate has been further heated by several incidents where firms collected behavioral data and used it or sold it for marketing purposes without the awareness of consumers.[4] Consumer policy typically regards informing consumers about the consequences of their choices as highest priority and strikes down on fraudulent business practices where firms misguide consumers about these consequences. Limited consumer foresight, either a trait or a result of deliberate marketing strategy, is considered as a main source

[1]One anonymous computer scientist working for online retailers noted that "*...It's common for big retail web sites to direct different users to different deals, offers, or items based on their purchase histories or cookies... And companies frequently offer special deals for customers with a few items in their shopping bags-from discounts on additional items, to free shipping, to coupons for future purchases. Ingenuity, rather than price-tampering, is now the name of the game*" (Klosowski, 2013).

[2]In 2012 Home Depot, an American retailer of home improvement and construction products and services, acquired Blacklocus, a start-up that develops technologies for data-based pricing for retailers using among others customers' online store data (see Taylor, 2012).

[3]Shiller (2013) uses microdata on a large panel of computer users to estimate the profitability of first-degree price discrimination based on different types of user data. He finds that the inclusion of data on the individual web browsing behavior for first-degree price discrimination increases profits much above the level, which is attained when only demographic data is used for tailored pricing.

[4]The Federal Trade Commission, the main consumer policy watchdog, recently investigated fraudulent business practices by a highly popular smartphone application developer. '*Brightest Flashlight*,' an app that allows a phone to be used as a flashlight, deceived consumers about how their geolocation information would be shared with advertisers and other third parties (FTC, 2013). In a similar vein, electronics producer LG was recently accused of its smart TVs secretly recording data on consumer viewing habits that was used to display targeted advertisements, even after consumers opted out from this feature (Adams, 2013).

of consumer harm.[5,6] The argument backing this view is intuitive: If consumers are unable to or wrongly foresee the consequences of their actions, they solve the wrong optimization problem, which per se cannot maximize their true welfare. In this article we argue that this intuition may not always hold: Under very natural circumstances, when firms invest in customer-tracking technology anticipating the reaction (or the absence thereof) of consumers, the latter may be better off being myopic than sophisticated.

In this article we analyze the incentives of competing firms to invest in customer-tracking technology depending on consumer awareness. We consider a two-period model. In the first period each firm decides whether to invest in a technology, which allows a firm to collect information on the preferences of its first-period customers. In the second period firms compete and make use of the collected data for targeted pricing. We consider myopic and sophisticated consumers: The former do not know that the collected data will be used for price discrimination and care only about the current prices. In contrast, sophisticated consumers are informed about the ability of firms to track their behavior and anticipate receiving targeted offers in the future.

Our article contributes to the literature on competitive price discrimination with demand-side asymmetries, where consumers can be classified into different groups depending on their preferences for the firms. Thisse and Vives (1988) were the first to show the famous prisoners' dilemma result stating that each firm has a unilateral incentive to price-discriminate, which eventually makes both firms worse-off, because firms end up offering low prices to the loyal consumers of the rival.[7] Most articles in this strand assume that customer data is available exogenously. In our analysis we endogenize firms' ability to collect customer data and show that it is collected in equilibrium only if consumers are myopic. In that case two asymmetric

[5] DG SANCO of the European Commission, Europe's primary consumer policy institution, lists "*limited foresight and consumer myopia*" among the major channels of behavioral biases that give rise to consumer detriment. See http://ec.europa.eu/consumers/strategy/docs/study_consumer_detriment.pdf, p.197. Retrieved January 6, 2014.

[6] In 2012 the European Commission proposed a major reform of the European Union's data protection rules, which will, among others, reinforce consumer privacy in online services. See http://ec.europa.eu/justice/data-protection/. Retrieved February 6, 2014.

[7] A similar contribution is made in Shaffer and Zhang (1995) and Bester and Petrakis (1996). Other papers show that firms' ability to discriminate based on consumer brand preferences does not necessarily lead to a prisoners' dilemma. For example, in Shaffer and Zhang (2000) firms may benefit from the ability to discriminate among the two consumer groups loyal to each of the firms if these groups are sufficiently heterogeneous in the strength of their loyalty. Chen, Narasimhan and Zhang (2001) show that when the targeting ability of one or both firms improves, but remains imperfect, firms' profits may increase. In Shaffer and Zhang (2002) a firm with a stronger brand loyalty may benefit from firms' ability to discriminate among individual consumers based on the strength of brand loyalty.

equilibria emerge, where only one of the firms invests in customer-tracking technology. While this investment is individually profitable, in the spirit of Thisse and Vives firms' joint profits over two periods are lower compared to the no-investment case. However, when consumers are sophisticated, individual incentives to invest vanish, and firms avoid the reduction in joint profits.

Our article is also related to the literature on behavior-based price discrimination, where price discrimination emerges as equilibrium behavior (see, for instance, Fudenberg and Villas-Boas, 2005). We argue that investment incentives into a technology that enables targeted pricing depend crucially on consumer awareness: With sophisticated consumers firms choose not to invest, and price discrimination does not take place in equilibrium. Sophisticated consumers correctly anticipate that a firm holding customer-tracking technology will use the collected data for targeted pricing and reduce their first-period demand respectively. By avoiding investment firms commit not to price discriminate, which restores consumer demand.

Chen and Iyer (2002) and Liu and Serfes (2004) directly address firms' incentives to invest in customer data (technology). Chen and Iyer consider a Hotelling model where firms can invest in a database technology, which allows to reach individual consumers with customized prices. The authors show that full addressability never emerges in equilibrium even when the marginal cost of the database technology is zero, because it leads to a very intense price competition. Similarly, in our model firms never collect data about all consumers in the market. Even when consumers are myopic, only one of the firms invests in equilibrium, because if both firms hold customer-tracking technology, competition would intensify in both periods. Liu and Serfes (2004) also consider a Hotelling model and analyze firms' incentives to acquire data on consumer brand preferences of an exogenously given quality, which can be used for targeted pricing. The authors show that when data quality is low, firms do not acquire customer data in equilibrium. We also find the equilibrium, where firms do not invest in customer-tracking technology and, hence, do not gain customer data, provided consumers are sophisticated.

Finally, our article contributes to the behavioral industrial organization literature, especially to the strand focusing on myopic consumers. Gabaix and Laibson (2006) discuss how consumer myopia can explain the existence of "*shrouded attributes*" for some consumer goods. Myopic consumers buying certain goods (e.g., printers) may not take into account the price of complementary products (e.g., printer cartridges). Gabaix and Laibson show that if the share of myopic consumers is large enough, the shrouded prices equilibrium exists, where firms charge high add-on prices and hide this information from consumers in the primary market. In this equilibrium myopic consumers are worse off compared to sophisticated consumers, because they pay high add-on prices, while the former benefit from the low base-good prices and substitute away from the expensive add-ons. In our analysis myopic consumers can be better off than sophisticated consumers, if the discount factor is large enough. With myopic consumers a firm finds it individually profitable to invest in customer-tracking technology, which, however, decreases firms' joint profits and benefits consumers. When consumers are sophisticated, individual incentives to invest vanish. Hence, we find that firms are always better-off when consumers are sophisticated.[8]

Our article is organized as follows. In the next section we introduce the model. In Section 3 we provide the equilibrium analysis of the second period of the game. In Section 4 we derive the equilibrium of the first period of the game for the case of myopic consumers. In Section 5 we provide the equilibrium analysis of the first period of the game for the case of sophisticated consumers. In Section 6 we compare the equilibrium results for the cases of myopic and sophisticated consumers and analyze firms' incentives to educate consumers. Finally, Section 7 concludes.

2 The Model

We consider a standard Hotelling model where two firms, A and B, sell two versions of the same product. Firms are located at the end points of an interval of unit length with $x_A = 0$ and $x_B = 1$ denoting their locations. There is a mass of consumers normalized to unity. Every consumer is characterized by an address $x \in [0, 1]$ denoting her preference for the ideal product. If a consumer does not buy her ideal product, she has to incur linear transportation costs proportional to the distance to the firm. The utility of a consumer with address x from buying the product of firm $i = A, B$ in period $t = 1, 2$ at the price p_i^t is

$$U_i^t(p_i^t, x) = v - t\,|x - x_i| - p_i^t,$$

[8] There are other studies, which show that firms are not necessarily worse off when consumers become more sophisticated. For example, Eliaz and Spiegler (2011) introduce a model where marketing activities of the firms can influence the set of alternatives, which the boundly rational consumers perceive as relevant for their purchasing decisions. They show that firms' profits may increase when consumers become "more rational."

where $v > 0$ is the basic utility, which is assumed to be large enough such that the market is always covered in equilibrium. A consumer buys from the firm delivering a higher utility.[9]

We consider a two-period game. Initially, firms hold no customer data, but can invest in customer-tracking technology, which allows to collect data on the brand preferences of consumers who buy from them in the first period.[10] In the second period the firm(s) with customer data can engage in first-degree price discrimination among consumers whose data it (they) have. Precisely, the timing is as follows.

Period 1:

Stage 1 (Investment). Firms decide independently and simultaneously whether to invest in customer-tracking technology.

Stage 2 (Competition with uniform prices). First, firms publish independently and simultaneously their uniform prices. Consumers then observe these prices and make their purchasing decisions.

Period 2:

Stage 1 (Competition with uniform prices and discounts). Firms independently and simultaneously choose their uniform prices. Subsequently, the firm(s) with customer data issues (issue) discounts to consumers. Finally, consumers make their purchasing decisions.

The timing of the competition stage in Period 2 is consistent with a large body of literature on competitive price discrimination where firms make their targeted offers after setting regular prices (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 1995, 2002; Liu and Serfes, 2004, 2005; Choudhary et al., 2005).[11] It reflects the observation that discounts issued to finer consumer

[9] We follow Liu and Serfes (2006) and use two tie-breaking rules. Assume that both firms offer equal utilities. In this case a consumer chooses the closer firm if both firms hold (or both firms do not hold) customer-tracking technology (if $x = 1/2$, then the consumer visits firm A). Second, a consumer chooses the firm holding customer-tracking technology, if the other firm does not have it.

[10] In the literature on behavior-based price discrimination one usually assumes that in the second period a firm can only distinguish among consumers who bough from it and the rival in the first period (see, for instance, Fudenberg and Villas-Boas, 2005). We follow Liu and Serfes (2006) and assume that in the first period firms collect data on the preferences of their customers. This assumption relies on the observation that modern information technologies allow firms to learn more about the own customers than just distinguishing them from those of the rival. For example, cookies that collect data on consumers' web browsing behavior or consumer profiles in social networking websites can serve as sources of additional data on consumers' preferences.

[11] Note that this timing is equivalent to the following: *i)* in the subgame where both firms hold customer-tracking technology, firms choose all the prices simultaneously, and *ii)* in the subgames where only one firm holds customer-tracking technology, the firm without data chooses its prices first, and the other firm follows.

groups can be changed easier than prices targeted at broader consumer groups. Moreover, if firms decide simultaneously on regular prices and discounts, no Nash equilibrium in pure strategies may exist.

We assume that firms maximize the discounted sum of profits over two periods using common discount factor $0 \leq \delta \leq 1$. We distinguish between two cases, with myopic and sophisticated consumers. The former take into account only prices in the first period while making purchases in that period, because they do not realize that the firm(s) holding customer-tracking technology will use customer data collected in the first period for price discrimination in the second period. In contrast, sophisticated consumers maximize the discounted sum of utility over both periods. As is common in the literature, we assume that sophisticated consumers use the same discount factor as the firms (see, for instance, Fudenberg and Tirole, 2000). We will also use this discount factor to compute the discounted consumer surplus over two periods when consumers are myopic.

We seek for a subgame-perfect Nash equilibrium and start the analysis from the second period.[12]

3 Equilibrium Analysis of the Second Period

Depending on firms' choices whether to invest in customer-tracking technology in the investment stage of the first period, three types of subgames can emerge: *i)* subgame, where only one of the firms invested, *ii)* subgame, where both firms invested, *iii)* subgame, where none of the firms invested. In the latter case our game reduces to two independent static Hotelling models, where in equilibrium firms charge prices $p_A^t = p_B^t = 1/2$ ($t = \{1, 2\}$), and each firm serves half of the market. To the subgames *i)* and *ii)* we will refer as *asymmetric* and *symmetric* subgames and will denote them with the subscripts "As" and "S," respectively. We will assume that it is firm A, which holds customer-tracking technology in the asymmetric subgame.

Let $\alpha^1(p_A^1, p_B^1)$ denote the market share of firm A in the first period, to which we will sometimes refer with α^1 to simplify the notation. We will assume that consumers with brand

[12] Unlike in Fudenberg and Tirole (2000) who have a game with incomplete information and, hence, solve for a perfect Bayesian equilibrium, our game is a game with complete information. In Fudenberg and Tirole (2000) a firm knows only whether a given consumer bough from it or from the rival in the first period. Hence, firms should form beliefs about the preferences of consumers in those two groups. We assume, in contrast, that customer-tracking technology allows firms to observe the preferences of consumers it served in the first period. Since the market is always covered in equilibrium, all other consumers bough from the rival, such that a firm holding customer-tracking technology also knows which consumers were served by the rival.

preferences $x \leq \alpha^1$ ($x > \alpha^1$) bought from firm A (B) in the first period.[13] To the former (the latter) we will refer as the turf of firm A (B). Similarly, we will denote the market share of firm A in the second period as $\alpha^2(p_A^2, p_B^2)$. Furthermore, consumers with $x \leq 1/2$ ($x > 1/2$) we will call the loyal consumers of firm A (B).

Asymmetric subgame. In the second period firm A can discriminate among consumers on its turf and has to charge a uniform price to consumers on the turf of firm B. Firm B, in contrast, has to offer a uniform price to all consumers. The following proposition characterizes the equilibrium of the second period for any α^1.

Lemma 1. *(Second period. Asymmetric subgame.) Assume that only firm A invested in customer-tracking technology in the first period. The equilibrium of the second period depends on the size of firm A' turf as follows.*
i) If it is relatively small, $\alpha^1 \leq (3-\sqrt{2})/2$, firm B loses consumers on its turf and serves those with $x > (5+2\alpha^1)/8$. Firm B charges the price $p_B^{2,As}(\alpha^1) = t(3-2\alpha^1)/2$. The discriminatory price of firm A is $p_A^{2,As}\left(x;\alpha^1\right) = t\left(3-2\alpha^1\right)/2 + t(1-2x)$, on the turf of firm B it charges the price $p_A^{2,As}\left(x;\alpha^1\right) = t\left(5-6\alpha^1\right)/4$. Firms realize profits $\Pi_A^{2,As}(\alpha^1) = t\left[-28\left(\alpha^1\right)^2 + 20\alpha^1 + 25\right]/32$ and $\Pi_B^{2,As}(\alpha^1) = t(3-2\alpha^1)^2/16$.
ii) If it is relatively large, $\alpha^1 > (3-\sqrt{2})/2$, firm A loses consumers on its turf and serves those with $x \leq 3/4$. Firm B charges the price $p_B^{2,As}(\alpha^1) = t/2$. The discriminatory price of firm A to consumers with $x \leq 3/4$ is $p_A^{2,As}(x;\alpha^1) = t/2 + t(1-2x)$, to all other consumers firm A charges the price $p_A^{2,As}(x;\alpha^1) = 0$. Firms realize profits $\Pi_A^{2,As}(\alpha^1) = 9t/16$ and $\Pi_B^{2,As}(\alpha^1) = t/8$.

Proof. See Appendix.

In the asymmetric subgame firm A has a competitive advantage as it holds customer data and can better target consumers. When α^1 increases, firm A gets more data and can also on average better estimate the preferences of consumers on the turf of firm B. This allows firm A to gain consumers on the turf of firm B, however, only if α^1 is not large. In equilibrium, the uniform price of firm B has to strike an optimal balance between gaining new market shares and extracting rents from its most loyal consumers. When α^1 is low ($\alpha^1 \leq (3-\sqrt{2})/2$), firm A holds data on consumers which are relatively loyal to it and competing for whom is costly for firm B, such that the latter follows the rent-extraction strategy, charges a relatively high uniform price

[13] We will prove below that this holds in any subgame, symmetric and asymmetric.

and loses consumers on its turf. When α^1 is large ($\alpha^1 > (3-\sqrt{2})/2$), the ability of firm A to compete for the loyal consumers of the rival increases, such that firm B is forced to protect its market shares by charging a relatively low uniform price, and its market shares increase. In that case firms' second-period market shares do not depend on α^1, because the non-discriminatory price of firm A is zero, and the price of firm B does not depend on α^1 directly, only through the equilibrium price of firm A on B's turf, because firm B targets the most loyal consumers on its turf.

We now consider how firms' profits, $\Pi_A^{2,As}(\cdot)$ and $\Pi_B^{2,As}(\cdot)$, change with α^1. On the interval $\alpha^1 \leq (3-\sqrt{2})/2$, where the second-period market share of firm A increases in α^1, the profit of firm A first increases and then starts to decrease. The latter happens because the uniform price of firm B, which decreases in α^1, puts a downward pressure on firm A's discriminatory prices. When firm B switches to a market-protection strategy (at $\alpha^1 = (3-\sqrt{2})/2$), the profit of firm A decreases abruptly and does not change with a further increase in α^1, because both its prices and market shares do not change in α^1. On the interval $\alpha^1 \leq (3-\sqrt{2})/2$ both the uniform price and the market share of firm B decrease in α^1, so does its profit. On the interval $\alpha^1 > (3-\sqrt{2})/2$, where firm B adopts a market-protection strategy both its uniform price and the market share remain constant, so that the profit of firm B does not change in α^1 either.

Symmetric subgame. In the second period each firm can discriminate among consumers on its turf. The following lemma states the equilibrium of the second period depending on α^1.

Lemma 2. *(Second period. Symmetric subgame.) Assume that both firms invested in customer-tracking technology in the first period. The equilibrium of the second period depends on the size of firm A's turf as follows.*

i) If $\alpha^1 \leq 1/2$, then on the turf of firm A firms charge prices $p_A^{2,S}(x;\alpha^1) = t(1-2x)$ and $p_B^{2,S}(x;\alpha^1) = 0$, where firm A serves all consumers. On the turf of firm B firms charge prices $p_A^{2,S}(x;\alpha^1) = t(1-2\alpha^1)/2$ and $p_B^{2,S}(x;\alpha^1) = t(1-2\alpha^1)/2 + t(2x-1)$, where firm A serves consumers with $x < (2\alpha^1+1)/4$. Firms realize profits $\Pi_A^{2,S}(\alpha^1) = t\left[-4(\alpha^1)^2 + 4\alpha + 1\right]/8$ and $\Pi_B^{2,S}(\alpha^1) = t\left[4(\alpha^1)^2 - 12\alpha + 9\right]/16$.

ii) If $\alpha^1 > 1/2$, then on the turf of firm A firms charge prices $p_A^{2,S}(x;\alpha^1) = t(1-2x) + t(2\alpha^1-1)/2$ and $p_B^{2,S}(x;\alpha^1) = t(2\alpha^1-1)/2$, where firm A serves consumers with $x \leq (2\alpha^1+1)/4$. On the turf of firm B firms charge prices $p_A^{2,S}(x;\alpha^1) = 0$ and $p_B^{2,S}(x;\alpha^1) = t(1-2\alpha^1)/2$, where firm B serves all consumers. $\Pi_A^{2,S}(\alpha^1) = t\left[4(\alpha^1)^2 + 4\alpha + 1\right]/16$ and

$\Pi_B^{2,S}\left(\alpha^1\right)=t\left[-4\left(\alpha^1\right)^2+4\alpha+1\right]/8$ *are firms' profits over two periods.*

Proof. See Appendix.

In equilibrium in the symmetric subgame a firm never loses consumers on its turf if it only served the own loyal consumers, because it has data on their precise brand preferences and can undercut any uniform price of the rival. The latter cannot then do better than charging the price of zero on a firm's turf. In contrast, a firm always loses consumers on its turf if it served some of the rival's loyal consumers in the first period. In equilibrium the rival targets its loyal consumers on a firm's turf and always makes some of them switch.

We now turn to the change in firms' profits depending on α^1. As firms are symmetric, we only consider firm A. If $\alpha^1 \leq 1/2$, the profits of firm A increase in α^1 for two reasons. First, firm A is able to extract more rents on its turf, because it gains more data on its loyal consumers, and the negative competition effect is absent as firm B always charges the price of zero there. Also, firm A increases its market shares on firm B's turf. If $\alpha^1 > 1/2$, the profits of firm A increase in α^1, although it loses market shares. This is due to higher rents firm A gets on its turf, because it faces a positive competition effect as firm B targets with the non-discriminatory price its most loyal consumers there with an address close to α^1. As a result, the profits of firm A increase for any α^1. Nevertheless, firm A's profits in a symmetric subgame reach the profit level in the subgame where none of the firms invested in customer-tracking technology only if it holds data on more than 90 percent of consumers in the market (precisely, if $\alpha^1 > \sqrt{2}-1/2$). The reason is that in the symmetric subgame every firm can distinguish between its own loyal consumers and those of the rival, and prices aggressively the latter group. Precisely, for any $\alpha^1 \leq 1/2$ firm B charges the price of zero to the loyal consumers of firm A on the latter's turf. Thisse and Vives (1988) first identified this negative competition effect driven by the availability of data on consumers' brand preferences.

Our results are different from Fudenberg and Tirole (2000), where in the second period firms can only distinguish between consumers on the two turfs. In their model a firm may lose consumers on its turf even if it contains only its loyal consumers, because a firm does not have data on their precise brand preferences and has to charge a uniform price. Then if its turf is relatively large, a firm prefers to extract rents from its more loyal consumers, while the less loyal consumers switch to the rival. Also, different from our result on a positive monotone relationship between the size of a firm's turf and its second-period profits, in the case of a uniform consumer distribution in Fudenberg and Tirole this relationship is U-shaped. Profits are lowest when firms' turfs are of equal sizes, in which case every firm can perfectly discriminate among its own loyal consumers and those of the rival, and competition is most intense. Profits are highest if one of the firms served all consumers in the first period, because this outcome is least informative leading to the weakest competition. In contrast, if a firm did not serve any consumers in the first period, in our model the rival holds the largest data leading to the most intense competition.

4 Equilibrium Analysis of the First Period with Myopic Consumers

Myopic consumers do not foresee that the firm, which invested in customer-tracking technology, will use the data collected in the first period for price discrimination in the second period. Hence, the address of the indifferent consumer in first period, $\alpha^1(p_A^1, p_B^1)$, is given by a standard expression: $\alpha^1(p_A^1, p_B^1) = 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$. The following lemma summarizes our results on the equilibrium in the asymmetric subgame with myopic consumers.

Lemma 3. *(First period. Asymmetric subgame. Myopic consumers.) Assume that only firm A invested in customer-tracking technology and consumers are myopic. In equilibrium in the first period prices are $p_A^{1,As}(\delta) = t\left(24 + 5\delta - 4\delta^2\right)/(5\delta + 24)$ and $p_B^{1,As}(\delta) = t\left(24 - \delta - 4\delta^2\right)/(5\delta + 24)$, where firm A serves consumers with $x \leq \alpha^{1,As}(\delta)$ and $\alpha^{1,As}(\delta) = (24 - \delta)/(10\delta + 48)$. Profits in the first period are $\Pi_A^{1,As}(\delta) = t(24 - \delta)\left(-4\delta^2 + 5\delta + 24\right)/[(5\delta + 24)(10\delta + 48)]$ and $\Pi_B^{1,As}(\delta) = t(11\delta + 24)\left(24 - 4\delta^2 - \delta\right)/[(5\delta + 24)(10\delta + 48)]$. The discounted sum of firms' profits in both periods is $\Pi_A^{1+2,As}(\delta) = t\left(79\delta^3 + 710\delta^2 + 2208\delta + 1152\right)/\left[4(5\delta + 24)^2\right]$ and $\Pi_B^{1+2,As}(\delta) = t\left(-12\delta^3 + 85\delta^2 + 528\delta + 576\right)/\left[2(5\delta + 24)^2\right]$.*

Proof. See Appendix.

Since firms maximize the discounted sum of their profits, they distort first-period prices for higher second-period profits. The profits of firm B in the second period decrease in the size of firm A's turf (provided α^1 is not very large, which is the case in equilibrium). Then in the first period firm B charges a relatively low price to prevent the rival from gaining much customer data. In contrast, firm A charges a relatively high price in the first period, although it means obtaining less customer data, because this secures higher second-period profits by making firm B price less aggressively then. As a result, in the first period firm A serves less consumers than the rival and gains data only on its most loyal consumers.

As firms distort their first-period prices to increase profits in the second period, each reaps lower profits in the first period than in the subgame where neither firm invests in customer-tracking technology. However, in the second period firm A gains higher profits due to its informational advantage, and its discounted profits over two periods are higher, while the profits of firm B over two periods are lower compared to the subgame where firms do not invest. We next consider the symmetric subgame with myopic consumers. The following lemma summarizes our results.

Lemma 4. *(First period. Symmetric subgame. Myopic consumers.) Assume that both firms invested in customer-tracking technology and consumers are myopic. Two asymmetric equilibria exist where in the first period firms charge prices* $p_i^{1,S}(\delta) = t\left(12-\delta-\delta^2\right)/(12+\delta)$ *and* $p_j^{1,S}(\delta) = t\left(12-3\delta-\delta^2\right)/(12+\delta)$*, and* $p_i^{1,S}(\delta) > p_j^{1,S}(\delta)$*,* $i,j = \{A,B\}$ *and* $i \neq j$*. Firm* i *serves consumers with* $x \leq (12-\delta)/[2(12+\delta)]$*. In the first period firms realize profits* $\Pi_i^{1,S}(\delta) = t(12-\delta)\left(12-\delta^2-\delta\right)/\left[2(\delta+12)^2\right]$ *and* $\Pi_j^{1,S}(\delta) = 3t(\delta+4)\left(12-\delta^2-3\delta\right)/\left[2(\delta+12)^2\right]$*. Firms' profits over two periods together are* $\Pi_i^{1+2,S}(\delta) = t\left(\delta^3+2\delta^2+96\delta+288\right)/\left[4(\delta+12)^2\right]$ *and* $\Pi_j^{1+2,S}(\delta) = t\left(-\delta^3+3\delta^2+72\delta+144\right)/\left[2(\delta+12)^2\right]$*.*

Proof. See Appendix.

As shown in Lemma 2, each firm's profits in the second period increase monotonically in the size of its turf, such that each firm has an incentive to charge a relatively low price in the first period to gain more customer data. Interestingly, we get two asymmetric equilibria in the first period, where firms charge different prices. This result is driven by the fact that the second-period profit of a firm is given by two different functions depending on whether a firm had a larger or a smaller market share in the first period. The firm with a smaller turf has a low incentive to decrease its first-period price, because its second-period profits would increase slowly. On the other hand, the firm with a larger turf has a low incentive to increase its first-period price, because its second-period profits would decrease substantially then.

Firms are worse-off in both periods compared to the subgame where they do not hold customer-tracking technology and realize the profit of $t/2$. Second-period profits are low due to the negative competition effect driven by price discrimination described above. First-period profits are low, because firms compete intensively for market shares to collect more customer data. There is a similar result in two-period models where consumers have switching costs.

There firms compete in the first period to lock-in more consumers and gain lower profits than in the static game (see, for instance, Klemperer, 1995). However, different from those models in our case profits are also lower in the second period compared to the static profits. In the next proposition we summarize firms' incentives to invest in customer-tracking technology in the first period. With the subscript "m" we will refer to the equilibrium values when consumers are myopic.

Proposition 1. *(Myopic consumers. Investment incentives and welfare.) If consumers are myopic, two asymmetric equilibria exist, where one of the firms invests in customer-tracking technology.* $\Pi_{A,m}^{1+2}(\delta) = t\left(79\delta^3 + 710\delta^2 + 2208\delta + 1152\right) / \left[4\left(5\delta + 24\right)^2\right]$ *is the profit over two periods of the investing firm and* $\Pi_{B,m}^{1+2}(\delta) = t\left(-12\delta^3 + 85\delta^2 + 528\delta + 576\right) / \left[2\left(5\delta + 24\right)^2\right]$ *is the profit over two periods of the firm which does not invest. The discounted social welfare and consumer surplus over two periods are given by* $SW_m^{1+2}(\delta) = v(1+\delta) - t\left(26\delta^3 + 325\delta^2 + 960\delta + 576\right) / \left[4\left(5\delta + 24\right)^2\right]$ *and* $CS_m^{1+2}(\delta) = v(1+\delta) - t\left(81\delta^3 + 1205\delta^2 + 4224\delta + 2880\right) / \left[4\left(5\delta + 24\right)^2\right]$*, respectively.*

Proof. See Appendix.

If the rival does not invest in customer-tracking technology, a firm has a unilateral incentive to do that. As we showed in Lemma 3, in that case a firm realizes lower profits in the first period, which are outweighed by higher second-period profits driven by its informational advantage. However, a firm does not have an incentive to invest in customer-tracking technology if the rival does the same, because competition would then intensify in both periods. As a result, in equilibrium only one of the firms invests.[14] This result is similar to Chen and Iyer (2002) where ex-ante symmetric firms make asymmetric investments in customer data to mitigate competition.

While over two periods the firm holding customer-tracking technology realizes higher profits, firms' joint profits are lower compared to the case where firms do not invest in customer-tracking technology. In equilibrium social welfare is also smaller compared to the case without investment. This is because the asymmetric investment decisions boil down into asymmetric market shares in both periods such that some consumers do not buy from their most preferred firms implying allocative inefficiency. Consumer surplus can be higher than in the case where firms do not

[14] In reality we often observe that firms differ in their abilities to collect and analyze customer data for targeted pricing. The most prominent example is the UK's retail industry, where Tesco, the world's third largest supermarket group, became the leading supermarket chain in the UK after the successful introduction of a loyalty card (see Winterman, 2013). Using is loyalty card Tesco collects data on consumers' preferences and based on that data designs individual discounts and rewards to consumers.

invest in customer-tracking technology, if $\delta > \left(3\sqrt{69} - 15\right)/11 \approx 0.9$. In that case consumers benefit more from lower payments to the firms than they lose from higher transportation costs.

5 Equilibrium Analysis of the First Period with Sophisticated Consumers

Sophisticated consumers correctly anticipate that a firm holding customer-tracking technology will use the data collected in the first period for targeted pricing in the second period and adapt respectively the demand in the first period. We will consider again in turn each subgame (asymmetric and symmetric) and will start with the derivation of consumer demand in the first period.

Asymmetric subgame. The following lemma states consumer demand in the first period in the asymmetric subgame.

Lemma 5. *(First period. Asymmetric subgame. Sophisticated consumers. Demand.) Assume that only firm A invested in customer-tracking technology and consumers are sophisticated. Then the demand of firm A in the first period is given by:*

$$\alpha^{1,As}(p_A^1, p_B^1) = \begin{cases} 1 & \text{if} \quad p_A^1 < p_B^1 - t \\ \frac{1}{2} + \frac{p_B^1 - p_A^1}{2t} & \text{if} \quad p_B^1 - t \leq p_A^1 < p_B^1 - t(2-\sqrt{2}) \\ \frac{t(4-5\delta)+4(p_B^1 - p_A^1)}{2t(4-3\delta)} & \text{if} \quad p_B^1 - t(2-\sqrt{2}) - \frac{\delta t(3\sqrt{2}-4)}{4} \leq p_A^1 \leq p_B^1 + \frac{t(4-5\delta)}{4} \\ 0 & \text{if} \quad p_A^1 > p_B^1 + \frac{t(4-5\delta)}{4}. \end{cases} \tag{1}$$

Proof. See Appendix.

If $\delta = 0$, demand (1) yields a standard expression for the market share of firm A in the first period: $\alpha^1(p_A^1, p_B^1) = 1/2 + \left(p_B^1 - p_A^1\right)/(2t)$. Otherwise, it is different from the latter in two ways: First, it is discontinuous and second, it is given by a correspondence such that if firm A charges a moderate price, $p_B^1 - t(2-\sqrt{2}) - \delta t(3\sqrt{2}-4)/4 \leq p_A^1 < p_B^1 - t(2-\sqrt{2})$, it can gain either relatively few or many consumers. Both properties are related to the discontinuity of the optimal strategy of firm B in the second period at the point $\alpha^1 = \left(3-\sqrt{2}\right)/2$, where firm B switches from a rent-extraction ($p_B^{2,As}(\alpha^1) = t(3-2\alpha^1)/2$) to a market-protection strategy ($p_B^{2,As}(\alpha^1) = t/2$). If $\alpha^1 \leq \left(3-\sqrt{2}\right)/2$, in the second period firm A expands its market shares and charges positive prices to all consumers whose preferences it learns. In that case there is a disadvantage of buying at firm A in the first period related to preference revealing, such that the indifferent consumer should have a relatively strong preference for firm A implying a relatively large market share of firm A. If $\alpha^1 > \left(3-\sqrt{2}\right)/2$, in the second period firm B expands its market shares, and firm A charges the price of zero to the indifferent consumer. In that case there is no disadvantage of buying at firm A in the first period related to preference revealing, and the indifferent consumer can have a relatively weak preference for firm A implying a relatively small market share of firm A. Hence, under a moderate first-period price the market share of firm A can be either relatively large or small. In the latter case consumers correctly anticipate that they will receive targeted offers in the second period based on the revealed preferences in case of buying at firm A and reduce the first-period demand respectively.

If $\alpha^1 \leq \left(3-\sqrt{2}\right)/2$, firm A faces in the first period a *more* elastic demand than if $\alpha^1 > \left(3-\sqrt{2}\right)/2$. In the former case upon buying at firm A (B) in the first period, in the second period the indifferent consumer buys at firm A at a discriminatory (non-discriminatory) price. Both prices decrease in the address of the indifferent consumer (the market share of firm A in the first period). However, the discriminatory price decreases more because it is targeted directly at that consumer. Hence, when the first-period market share of firm A gets larger, the difference between the discriminatory price of firm A and non-discriminatory price of firm B in the second period decreases as well as the disadvantage related to preference revealing to firm A. As a result, for a given price reduction by firm A more consumers want to buy from it in the first period than in the case $\alpha^1 > \left(3-\sqrt{2}\right)/2$, where there is no disadvantage related to preference revealing to firm A. In the next lemma we characterize the equilibrium of the first period.

Lemma 6. *(First period. Asymmetric subgame. Sophisticated consumers.) Assume that only firm A invested in customer-tracking technology and consumers are sophisticated. The equilibrium of the first period depends on the discount factor as follows.*

i) If $\delta \leq (70-2\sqrt{721})/21$, then in the first period firms charge prices $p_A^{1,As}(\delta) = t(96 - 140\delta + 21\delta^2)/(96-52\delta)$ and $p_B^{1,As}(\delta) = t(96 - 132\delta + 25\delta^2)/(96-52\delta)$, the market share of firm A is $\alpha^{1,As}(\delta) = (24-23\delta)/(48-26\delta)$. $\Pi_A^{1+2,As}(\delta) = t\left(395\delta^3 - 404\delta^2 - 1536\delta + 2304\right)/\left[8\left(13\delta-24\right)^2\right]$ and $\Pi_B^{1+2,As}(\delta) = t\left(53\delta^3 + 228\delta^2 - 2304\delta + 2304\right)/\left[8\left(13\delta-24\right)^2\right]$ are firms' profits over two periods.

ii) If $(70-2\sqrt{721})/21 < \delta < 6/7$, then in the first period firms charge prices $p_A^{1,As}(\delta) = 0$ and

$p_B^{1,As}(\delta) = t(16-24\delta+7\delta^2)/(32-28\delta)$, *the market share of firm A is* $\alpha^{1,As}(\delta) = (7\delta-6)/(7\delta-8)$. *Over two periods firms realize profits* $\Pi_A^{1+2,As} = t\delta\left(833\delta^2 - 2408\delta + 1552\right) / \left[32\left(7\delta-8\right)^2\right]$ *and* $\Pi_B^{1+2,As} = t\left(-7\delta^2 + 8\delta + 16\right) / \left[16\left(8-7\delta\right)\right]$.

iii) If $\delta \geq 6/7$, *then in the first period firms charge prices* $p_A^{1,As}(\delta) = 0$ *and* $p_B^{1,As}(\delta) = t(5\delta-4)/4$, *the market share of firm A is* $\alpha^{1,As}(\delta) = 0$. *Firms' profits over two periods are* $\Pi_A^{1+2,As}(\delta) = 25\delta t/32$ *and* $\Pi_B^{1+2,As}(\delta) = (29\delta - 16)\,t/16$.

Proof. See Appendix.

Firm A charges a positive price and serves some consumers in the first period only if the discount factor is sufficiently small ($\delta \leq (70 - 2\sqrt{721})/21$). Under a higher discount factor ($(70 - 2\sqrt{721})/21 < \delta < 6/7$) firm A has to reduce its first-period price to zero to attract some consumers. Finally, if the discount factor is large ($\delta \geq 6/7$), firm A does not serve any consumers in the first period although it charges the price of zero while the rival's price is positive. Sophisticated consumers correctly anticipate that if they buy at firm A in the first period, it will discriminate in the second period based on their preferences, and reduce the demand for firm A. As a result, under any discount factor over two periods firm A realizes lower profits than in the subgame where neither firm holds customer-tracking technology. This is different in the asymmetric subgame with myopic consumers, where lower first-period profits of firm A are compensated by higher profits in the second period. With myopic consumers first-period profits of firm A are low for two reasons. First, firm B prices aggressively to prevent firm A from gaining much customer data. Second, firm A charges a relatively high price to serve less consumers in the first period to make firm B price softer in the second period. On the top of that, with sophisticated consumers firm A suffers from a decrease in the first-period demand, such that the resulting losses cannot be anymore compensated by higher profits in the second period.

Symmetric subgame. The following lemma states consumer demand in the first period in the symmetric subgame.

Lemma 7. *(First period. Symmetric subgame. Sophisticated consumers. Demand.) Assume that both firms invested in customer-tracking technology and consumers are sophisticated. Then* *the demand of firm A in the first period is given by:*

$$\alpha^{1,S}\left(p_A^1, p_B^1\right) = \begin{cases} 0 & \text{if} \quad p_A^1 - p_B^1 > \frac{t(2-\delta)}{2} \\ \frac{1}{2} - \frac{p_A^1 - p_B^1}{t(2-\delta)} & \text{if} \quad -\frac{t(2-\delta)}{2} \leq p_A^1 - p_B^1 \leq \frac{t(2-\delta)}{2} \\ 1 & \text{if} \quad p_A^1 - p_B^1 < -\frac{t(2-\delta)}{2}. \end{cases} \tag{2}$$

Proof. See Appendix.

Similar to the first-period demand in the asymmetric subgame, in the symmetric subgame, demand when consumers are sophisticated is *more* elastic than when consumers are myopic. If $\alpha^1 \leq 1/2$, upon buying at firm A in the first period, the indifferent consumer buys at firm A in the second period at a discriminatory price $p_A^{2,S}\left(\alpha^1\right) = t\left(1 - 2\alpha^1\right)$. If instead she purchases at firm B in the first period, firm A does not learn her preferences, and in the second period the indifferent consumer buys at firm A at the non-discriminatory price $p_A^{2,S} = t\left(1 - 2\alpha^1\right)/2$. Both prices decrease in α^1, but the discriminatory price decreases more because it is targeted at the indifferent consumer directly. Hence, the difference between the two prices becomes smaller, and more consumers switch to firm A in case of a price reduction compared to the case of myopic consumers who do not take into account prices in the second period while making their first-period purchases.[15] In the following lemma we state the equilibrium in the symmetric subgame when consumers are sophisticated.

Lemma 8. *(First period. Symmetric subgame. Sophisticated consumers.) Assume that both firms invested in customer-tracking technology and consumers are sophisticated. Two asymmetric equilibria exist.* $p_i^{1,S}(\delta) = t\left(24 - 26\delta + 5\delta^2\right)/(24 - 10\delta)$ *and* $p_j^{1,S}(\delta) = t\left(24 - 30\delta + 7\delta^2\right)/(24 - 10\delta)$ *are prices in the first period, where the market share of firm i is* $\alpha^{1,S} = (12 - 7\delta)/(24 - 10\delta)$*, with* $i, j = \{A, B\}$ *and* $i \neq j$*. Over two periods firms realize profits* $\Pi_i^{1+2,S}(\delta) = -t\left(6\delta^3 - 61\delta^2 + 168\delta - 144\right)/\left[2\left(5\delta - 12\right)^2\right]$ *and* $\Pi_j^{1+2,S}(\delta) = -t\left(5\delta^3 - 78\delta^2 + 288\delta - 288\right)/$

[15] This result is different from Fudenberg and Tirole (2000), where with a uniform consumer distribution and sophisticated consumers first-period consumer demand is *less* elastic if price discrimination in the second period is banned (in the latter case first-period consumer demand is same as in the case with myopic consumers in our analysis). In Fudenberg and Tirole the indifferent consumer of the first period switches from the firm it bought in the first period. Assume that the address of the indifferent consumer gets larger. Then upon buying at firm A in the first period, the indifferent consumer will buy at firm B in the second period at a *higher* price, because first-period consumers of firm A become on average more loyal to firm B. In contrast, in that case in our model upon buying at firm A in the first period, the indifferent consumer will buy (again) at firm A in the second period at a *lower* price, because she becomes less loyal to firm A. This difference makes first-period demand in Fudenberg and Tirole (2000) less responsive to price changes than in the symmetric subgame with sophisticated consumers in our model.

$\left[4\left(5\delta-12\right)^2\right]$.

Proof. See Appendix.

As we know from Lemma 2, each firm's second-period profit increases in the size of its turf and the amount of data collected about consumers. Similar to the symmetric subgame with myopic consumers, every firm has then an incentive to reduce its first-period price to get more customer data. However, with sophisticated consumers firms face a more elastic demand in the first period leading to a more intense competition. As a result, firms charge lower prices in the first period and get lower profits over two periods compared to the case of myopic consumers. Finally, in the following proposition we characterize firms' equilibrium incentives to invest in customer-tracking technology when consumers are sophisticated. With the subscript "s" we will refer to the equilibrium values when consumers are sophisticated.

Proposition 2. *(Sophisticated consumers. Investment incentives and welfare.) If consumers are sophisticated, there exists the unique equilibrium (in dominant strategies), where neither firm invests in customer-tracking technology. Over two periods each firm realizes the profit* $\Pi_{i,s}^{1+2}\left(\delta\right) = t\left(1+\delta\right)/2$. *Social welfare and consumers surplus over two periods are given by* $SW_s^{1+2}\left(\delta\right) = \left(v - t/4\right)\left(1+\delta\right)$ *and* $CS_s^{1+2}\left(\delta\right) = \left(v - 5t/4\right)\left(1+\delta\right)$.

Proof. See Appendix.

Different from the case of myopic consumers where one of the firms invests in equilibrium, with sophisticated consumers no investment is made in customer-tracking technology. In the latter case a firm does not have a unilateral incentive to invest, because it cannot make advantage of its ability to collect data as consumers anticipate that this data will be used for price discrimination in the second period and reduce their first-period demand respectively. Similarly, a firm does not have an incentive to invest if the rival invests. With sophisticated consumers investment incentives in that case are even weaker than with myopic consumers, because in the symmetric subgame with sophisticated consumers firms face a more elastic demand, which intensifies competition in the first period. The intuition behind our results is similar to the one, which explains the change in monopolist's profits over two periods when it can recognize consumers in the second period compared to the case when recognition is not possible (see, for instance, Fudenberg and Villas-Boas, 2005). The profits increase when consumers are myopic and decrease with sophisticated consumers. In the latter case the monopolist faces a lower demand in the first period as some consumers postpone their purchases to a second period to buy at a lower price. Then if the monopolist could choose whether to recognize consumers, it would prefer to recognize (not to recognize) when consumers are myopic (sophisticated). Parallel to that result, in our model in equilibrium a firm invests (does not invest) in customer-tracking technology if consumers are myopic (sophisticated).

6 Comparison of Equilibria with Myopic and Sophisticated Consumers

In this section we compare the equilibrium results in the two versions of our model and then conclude on firms' incentives to educate consumers. Precisely, we assume that firms could communicate to consumers that customer data collected using customer-tracking technology allows firms to discriminate among them. The following proposition summarizes our results on the equilibrium comparison.

Proposition 3. *(Comparison: Myopic consumers vs. sophisticated consumers.) Compared to the case of sophisticated consumers, when consumers are myopic:*
i) firms realize lower discounted joint profits over two periods,
ii) discounted social welfare over two periods is smaller,
iii) consumers enjoy a larger discounted surplus over two periods if $\delta > \left(3\sqrt{69} - 15\right)/11 \approx 0.9$ *and a (weakly) smaller otherwise.*

Proof. See Appendix.

In the literature on competitive price discrimination with demand-side asymmetries there is a famous prisoners' dilemma result, which states that each firm has an individual incentive to discriminate while both firms jointly are worse-off compared to the no-discrimination case (see, for instance, Thisse and Vives, 1988). In our model firms do not end up in the prisoners' dilemma: In equilibrium at most only one firm invests in the ability to discriminate (when consumers are myopic). Avoiding investment in the asymmetric subgame firm B foregoes the opportunity to learn the preferences of consumers on its turf but benefits from softer competition in both periods.[16,17] Firm A, in contrast, chooses to invest in customer-tracking technology, if consumers are myopic. While it is individually better-off, in the spirit of Thisse and Vives firms' joint profits over two periods decrease compared to the no-investment case. However, with sophisticated consumers the unilateral incentives to invest vanish too. In case of investment the profits of firm A in the first period are too low, because consumers respond with demand reduction in this period. As a result, surprisingly, firms jointly are better-off when consumers are sophisticated.

In each period, social welfare is smaller when consumers are myopic. In that case the asymmetric distribution of consumers between the firms driven by the asymmetric investment decisions, gives rise to the allocative inefficiency. Precisely, in the first period some loyal consumers of firm A buy at firm B, because their preferred firm charges a relatively high price to keep its market share small in order to soften competition in the second period. In the second period, some of the loyal consumers of firm B buy at firm A, because the latter is able to offer them better prices as it holds some customer data.

Over two periods myopic consumers pay less to the firms than sophisticated consumers, but at the same time have to bear higher transportation costs. The first effect dominates if the discount factor is sufficiently large ($\delta > \left(3\sqrt{69} - 15\right)/11$), in which case consumers enjoy a higher surplus when they are myopic. To understand this result, consider the equilibrium (asymmetric) when consumers are myopic. As shown in Table 1, firms' joint profits in the first period decrease when the discount factor becomes larger, because in that case firms value a lot second-period profits and distort more first-period prices. Although this leads to a lower first-period market share of firm A and, hence, to higher transportation costs, consumer surplus in that period gets larger because their payments to the firms decrease. Given a smaller turf of firm A, in the second period firm B prices softer, which results in higher profits for each firm.

[16] When consumers are myopic, in the first period in equilibrium the uniform price of firm A in the asymmetric subgame is higher than any price in the two asymmetric equilibria in the symmetric subgame (compare Lemmas 3 and 4). Also, in the second period the uniform price of firm A on the turf of firm B is higher in the asymmetric subgame compared to the second-period price in any asymmetric equilibrium in the symmetric subgame. (Indeed, compare $p_A^{2,As}\left(x; (24-\delta)/(10\delta+48)\right) = t\left(12+7\delta\right)/\left(5\delta+24\right)$ with $p_A^{2,S}\left(x; (12-\delta)/(24+2\delta)\right) = t\delta/\left(\delta+12\right)$ and $p_A^{2,S}\left(x; 3\left(\delta+4\right)/\left(24+2\delta\right)\right) = 0$.)

[17] Liu and Serfes (2004) also introduce the investment stage where firms decide whether to acquire customer data of an exogenously given quality. They find that if customer data is perfect (the case most similar to ours), firms end up in the prisoners' dilemma. Compared to Liu and Serfes, in our model (with myopic consumers) a firm has weaker incentives to invest in customer-tracking technology given that the rival invests, because in case of investment competition intensifies in both periods (where firms collect and compete using the collected customer data), while in Liu and Serfes competition intensifies only in one period.

In contrast, consumer transportation costs decrease, because firm A gains little customer data and attracts only a few loyal consumers of the rival in the second period. As a result, with an increase in δ, consumer surplus in that period becomes smaller because consumers' payments to the firms increase. When δ is large enough, the gains in consumer surplus in the first period outweigh the losses in the second period, and over two periods myopic consumers enjoy a higher surplus than sophisticated consumers.

Our results imply that if the discount factor is large enough ($\delta > \left(3\sqrt{69}-15\right)/11$), there is a conflict between the maximization of consumer surplus and social welfare. While social welfare is always larger when consumers are sophisticated (and no investment in customer-tracking technology takes place), in that case consumers are better-off when they are myopic (and one of the firms invests). Then governmental intervention aiming at the maximization of social welfare should either educate consumers or prohibit customer-tracking. However, both those policies would decrease consumer surplus.

Table 1: Comparison of the Equilibria in the Cases of Myopic and Sophisticated Consumers

	Myopic consumers			*Comparison*	
	$t=1$	$t=2$		$t=1$	$t=2$
$\partial\Pi_A^t/\partial\delta$	$-$	$+$	$\Pi_{A,m}^t - \Pi_{A,s}^t$	$-$	$+$
$\partial\Pi_B^t/\partial\delta$	$-$	$+$	$\Pi_{B,m}^t - \Pi_{B,s}^t$	$-$	$-$
$\partial\left(\Pi_A^t+\Pi_B^t\right)/\partial\delta$	$-$	$+$	$\sum_i \Pi_{i,m}^t - \sum_i \Pi_{i,s}^t$	$-$	$+$
$\partial SW^t/\partial\delta$	$-$	$+$	$\Pi_{A,m}^{1+2} - \Pi_{A,s}^{1+2}$	$+$	
$\partial CS^t/\partial\delta$	$+$	$-$	$\Pi_{B,m}^{1+2} - \Pi_{B,s}^{1+2}$	$-$	
$\alpha^t(\delta)-1/2$	$-$	$+$	$\sum_i \Pi_{i,m}^{1+2} - \sum_i \Pi_{i,s}^{1+2}$	$-$	
$\partial\left\|2\alpha^t(\delta)-1\right\|/\partial\delta$	$+$	$-$	$CS_m^t - CS_s^t$	$+$	$-$
			$CS_m^{1+2} - CS_s^{1+2}$	$+$ if $\delta > \frac{3\sqrt{69}-15}{11}$	$-$ if $\delta < \frac{3\sqrt{69}-15}{11}$
			$SW_m^t - SW_s^t$	$-$	$-$
			$SW_m^{1+2} - SW_s^{1+2}$	$-$	

Note: "+" stays for a positive sign and "−" for a negative sign.
Subscript "m" ("s") denotes the equilibrium value when consumers are myopic (sophisticated).

We next analyze firms' incentives to educate consumers through making it known to them that customer-tracking technology allows firms to collect customer data, which can be used for price discrimination. We will show that firms choose to educate consumers, which is optimal from the social welfare perspective and increases firms' joint profits, but may harm consumers (if $\delta > \left(3\sqrt{69}-15\right)/11$). We assume that initially consumers are myopic and introduce Period 0, where firms decide simultaneously and independently whether to educate consumers. Consumers become sophisticated if at least one of the firms educates them. We relax the assumption that firm A is the firm, which invests in the asymmetric equilibrium when consumers are myopic and assume instead that it is firm A with probability $p \in [0,1]$ and with probability $1-p$ it is firm B. Table 2 presents the discounted sum of firms' profits over two periods depending on their decisions to educate consumers. We search for a subgame-perfect Nash equilibrium. The following proposition summarizes our results on firms' incentives to educate myopic consumers.

Table 2: Firms' Profits over Two Periods Depending on Their Decisions to Educate Consumers

		Firm B	
		Educate	Not Educate
Firm A	Educate	$\frac{t(1+\delta)}{2}$; $\frac{t(1+\delta)}{2}$	$\frac{t(1+\delta)}{2}$; $\frac{t(1+\delta)}{2}$
	Not Educate	$\frac{t(1+\delta)}{2}$; $\frac{t(1+\delta)}{2}$	$f(p)$; $g(p)$

Note: $f(p) := p\Pi^{1+2}_{A,m}(\delta) + (1-p)\,\Pi^{1+2}_{B,m}(\delta)$ and $g(p) := (1-p)\,\Pi^{1+2}_{A,m}(\delta) + p\Pi^{1+2}_{B,m}(\delta)$, where $\Pi^{1+2}_{A,m}(\delta)$ and $\Pi^{1+2}_{B,m}(\delta)$ are stated in Proposition 1.

Proposition 4. *(Firms' incentives to educate myopic consumers.) Assume that consumers are initially myopic. For any $p \in [0,1]$ there exists the equilibrium, where both firms educate consumers. If $p < p_B(\delta)$ ($p > p_A(\delta)$), there exists the other equilibrium, where only firm A (B) educates consumers ($p_A(\delta) := \left[t(1+\delta)/2 - \Pi^{1+2}_{B,m}(\delta)\right] / \left[\Pi^{1+2}_{A,m}(\delta) - \Pi^{1+2}_{B,m}(\delta)\right]$ and $p_B(\delta) := \left[\Pi^{1+2}_{A,m}(\delta) - t(1+\delta)/2\right] / \left[\Pi^{1+2}_{A,m}(\delta) - \Pi^{1+2}_{B,m}(\delta)\right]$). If $p_B(\delta) \leq p \leq p_A(\delta)$, then two other equilibria exist, where one of the firms educates consumers.*

Proof. See Appendix.

In Gabaix and Laibson (2006) the shrouded prices equilibrium where firms hide high add-on prices is sustained because firms do not have an incentive to educate myopic consumers. Educating myopic consumers a firm teaches them how to exploit the existing prices at the market, who prefer in the end the shrouding rival because of its low base-good price. We show, in contrast, that if firms can educate consumers, then in any equilibrium at least one of the firms does that to prevent the individually profitable investment in the customer-tracking technology, which increases firms' joint profits. While firms' choices are efficient from the social welfare perspective, consumers are worse-off if the discount factor is sufficiently large ($\delta > \left(3\sqrt{69} - 15\right)/11$).

7 Conclusions

In this article we considered an industry where firms can invest in customer-tracking technology, which allows a firm to collect data on the brand preferences of its customers. Our article makes four main contributions. *First*, we show that at most one of the firms invests in customer-tracking technology in equilibrium, such that firms never have data on all consumers in the market. Investment by both firms would intensify competition in both periods: In the first period firms would compete for larger market shares and more customer data, and in the second period each firm would compete aggressively for the loyal consumers of the rival. *Second*, in equilibrium investment in customer-tracking technology takes place only if consumers are myopic, which makes both firms worse-off compared to the case when consumers are sophisticated, where firms do not invest. Sophisticated consumers correctly anticipate that the firm holding customer-tracking technology will use the data collected in the first period to price discriminate in the second period and decrease the first-period demand for that firm respectively. *Third*, myopic consumers can be better off than sophisticated consumers. Precisely, if the discount factor is sufficiently large, myopic consumers benefit from lower payments to the firms driven by their investment decisions. As social welfare is always larger when consumers are sophisticated, in that case there is a tension between the maximization of consumer surplus and social welfare. *Fourth*, if firms can educate consumers, they always have an incentive to do that in order to make the investment in customer-tracking technology individually unprofitable for any firm and thus secure higher joint profits. While firms' decisions are efficient, consumers are better off remaining myopic if the discount factor is sufficiently large.

8 Appendix

In this Appendix we provide the proofs omitted in the text.

Proof of Lemma 1. We start with the optimal strategy of firm A on its turf, which is to make any consumer indifferent whenever possible with a non-negative price: $p_A^{2,As}(x;p_B^2) = \max\left\{0, p_B^2 - t(2x-1)\right\}$. If $p_B^2 \leq t\left(2\alpha^1 - 1\right)$, firm A serves consumers with $x \leq 1/2 + p_B^2/\left(2t\right)$ on its turf. If $p_B^2 > t\left(2\alpha^1 - 1\right)$, firm A gains all consumers there. We now turn to the turf of firm B. If firms' prices are not too different, there is an indifferent consumer $x^I(p_A^2, p_B^2) = 1/2 + (p_B^2 - p_A^2)/2t$. Then on firm B's turf firm A maximizes the profit $p_A^2\left[x^I(p_A^2, p_B^2) - \alpha^1\right]$. The optimal strategy of firm A on B' turf is $p_A^{2,As}(p_B^2;\alpha^1|\, p_B^2 \leq t(2\alpha^1-1)) = 0$, $p_A^{2,As}(p_B^2;\alpha^1|\, t(2\alpha^1-1) < p_B^2 < t(3-2\alpha^1)) = \left[p_B^2 + t\left(1-2\alpha^1\right)\right]/2$ and $p_A^{2,As}(p_B^2;\alpha^1|\, p_B^2 \geq t(3-2\alpha^1)) = p_B^2 - t$. Using the optimal strategies of firm A on the two turfs we get the demand of firm B as $D_B^2\left(p_B^2;\alpha^1|\, p_B^2 \leq t(2\alpha^1-1)\right) = 1/2 - p_B^2/\left(2t\right)$, $D_B^2\left(p_B^2;\alpha^1|\, t(2\alpha^1-1) < p_B^2 < t\left(3-2\alpha^1\right)\right) = \left(3-2\alpha^1\right)/4 - p_B^2/\left(4t\right)$ and $D_B^2\left(p_B^2;\alpha^1|\, p_B^2 \geq t(3-2\alpha^1)\right) = 0$. It is straightforward then that we should distinguish between the cases $\alpha^1 \leq 1/2$ and $\alpha^1 > 1/2$ to derive the optimal price of firm B.

Case 1: $\alpha^1 \leq 1/2$. The optimal price of firm B is $p_B^{2,As}(\alpha^1) = t(3-2\alpha^1)/2$, and it serves consumers with $x > (5+2\alpha^1)/8$ and realizes the profit $\Pi_B^{2,As}\left(\alpha^1\right) = t(3-2\alpha^1)^2/16$, while $\Pi_A^{2,As}\left(\alpha^1\right) = t\int_0^{\alpha^1}\left[(5-2\alpha^1-4x)/2\right]dx + t(5-6\alpha^1)^2/32 = t\left[-28\left(\alpha^1\right)^2 + 20\alpha^1 + 25\right]/32$.

Case 2: $\alpha^1 > 1/2$. In equilibrium it can either be $p_B^2 \leq t(2\alpha^1-1)$ or $t(2\alpha^1-1) \leq p_B^2 < t(3-2\alpha^1)$, while $p_B^2 \geq t(3-2\alpha^1)$ is not possible, because $\Pi_B^{2,As}(p_B^2;\alpha^1) = 0$ then. As $\Pi_B^{2,As}(p_B^2;\alpha^1)$ is given by different functions on the two intervals, we first derive the maximal profit on each interval and then compare them to conclude about the optimal price. We start with $p_B^2 \leq t(2\alpha^1-1)$. FOC yields $p_B^2 = t/2$. Comparing $t/2$ and $t(2\alpha^1-1)$ we conclude that the profit-maximizing price of firm B is $p_B^2\left(\alpha^1|\,\alpha^1 \leq 3/4\right) = t(2\alpha^1-1)$ and $p_B^2\left(\alpha^1|\,\alpha^1 > 3/4\right) = t/2$. We get $\Pi_B^{2,As}(t(2\alpha^1-1);\alpha^1) = t(1-\alpha^1)(2\alpha^1-1)$ and $\Pi_B^{2,As}\left(t/2;\alpha^1\right) = t/8$. Hence, on the interval $p_B^2 \leq t(2\alpha^1-1)$ the maximal profit of firm B is $\Pi_B^{2,As}\left(\alpha^1|\,\alpha^1 \leq 3/4\right) = t(1-\alpha^1)(2\alpha^1-1)$ and $\Pi_B^{2,As}\left(\alpha^1|\,\alpha^1 > 3/4\right) = t/8$.

We consider now the interval $t(2\alpha^1-1) \leq p_B^2 < t(3-2\alpha^1)$. FOC yields $p_B^2\left(\alpha^1\right) = t(3-2\alpha^1)/2$. Comparing $t(3-2\alpha^1)/2$ and $t(2\alpha^1-1)$ we conclude that if $\alpha^1 \leq 5/6$, the profit-maximizing price of firm B is $p_B^2(\alpha^1) = t(3-2\alpha^1)/2$ and $\Pi_B^{2,As}(t(3-2\alpha^1)/2;\alpha^1) = t(3-2\alpha^1)^2/16$. If $\alpha^1 > 5/6$, the profit-maximizing price of firm B is $p_B^2\left(\alpha^1\right) = t(2\alpha^1-1)$ and $\Pi_B^{2,As}(t(2\alpha^1-1);\alpha^1) = t(1-\alpha^1)(2\alpha^1-1)$.

Finally, we have to determine the optimal price of firm B. $\Pi_B^{2,As}\left(t(3-2\alpha^1)/2;\alpha^1\right) > \Pi_B^{2,As}\left(t(2\alpha^1-1);\alpha^1\right)$ and $p_B^{2,As}\left(\alpha^1\right) = t(3-2\alpha^1)/2$ if $\alpha^1 \leq 3/4$. Assume $3/4 < \alpha^1 \leq 5/6$. If $\alpha^1 \leq (3-\sqrt{2})/2$, we have $\Pi_B^{2,As}(t(3-2\alpha^1)/2;\alpha^1) \geq \Pi_B^{2,As}\left(t/2;\alpha^1\right)$ and $p_B^{2,As}\left(\alpha^1\right) = t(3-2\alpha^1)/2$. If $\alpha^1 > (3-\sqrt{2})/2$, the opposite inequality holds and $p_B^{2,As}\left(\alpha^1\right) = t/2$. Assume $\alpha^1 > 5/6$, in which case $\Pi_B^{2,As}\left(t/2;\alpha^1\right) > \Pi_B^{2,As}(t(2\alpha^1-1);\alpha^1)$ and $p_B^{2,As}\left(\alpha^1\right) = t/2$.

Combining the cases $\alpha^1 \leq 1/2$ and $\alpha^1 > 1/2$ we conclude that the optimal price of firm B is $p_B^{2,As}(\alpha^1|\,\alpha^1 \leq (3-\sqrt{2})/2) = t\left(3-2\alpha^1\right)/2$, $p_B^{2,As}(\alpha^1|\,\alpha^1 > (3-\sqrt{2})/2) = t/2$. In the former case

$\Pi_A^{2,As}\left(\alpha^1\right) = t\int_0^{\alpha^1}\left[\left(5-2\alpha^1-4x\right)/2\right]dx + t\left(5-6\alpha^1\right)^2/32 = t\left[-28\left(\alpha^1\right)^2+20\alpha^1+25\right]/32$ and $\Pi_B^{2,As}(\alpha^1) = t(3-2\alpha^1)^2/16$. In the latter case $\Pi_A^{2,As}\left(\alpha^1\right) = t\int_0^{3/4}(3/2-2x)\,dx = 9t/16$ and $\Pi_B^{2,As}(\alpha^1) = t/8$. *Q.E.D.*

Proof of Lemma 2. Consider first the turf of firm A, where its optimal strategy is to make any consumer indifferent whenever possible with a non-negative price: $p_A^{2,S}(x;p_B^2) = \max\left\{0, p_B^2 - t(2x-1)\right\}$. Firm A serves consumers with $x \le \min\left\{1/2 + p_B^2/(2t);\alpha^1\right\}$. Maximizing the profit $\left[\alpha^1 - \min\left\{1/2+p_B^2/(2t);\alpha^1\right\}\right]p_B$ with respect to p_B yields the optimal price of firm B: $p_B^{2,S}\left(\alpha^1\middle|\,\alpha^1 \le 1/2\right) = 0$ and $p_B^{2,S}\left(\alpha^1\middle|\,\alpha^1 > 1/2\right) = t\left(2\alpha^1-1\right)/2$. In the former (latter) case firm A serves all consumers (consumers with $x \le \left(1+2\alpha^1\right)/4$) on its turf.

We now turn to the turf of firm B, where its optimal strategy is to make any consumer indifferent whenever possible with a non-negative price: $p_B^{2,S}(x;p_A^2) = \max\left\{0, p_A^2 + t(2x-1)\right\}$. Firm B serves consumers with $x \ge \max\left\{1/2 - p_A^2/(2t);\alpha^1\right\}$. Maximizing with respect to p_A the profit $\left[\max\left\{1/2-p_A^2/(2t);\alpha^1\right\}-\alpha\right]p_A$ yields the optimal price of firm A: $p_A^{2,S}\left(\alpha^1\middle|\,\alpha^1 > 1/2\right) = 0$ and $p_A^{2,S}\left(\alpha^1\middle|\,\alpha^1 \le 1/2\right) = t\left(1-2\alpha^1\right)/2$. In the former (latter) case firm B serves all consumers (consumers with $x \ge \left(1+2\alpha^1\right)/4$) on its turf.

$\Pi_A^{2,S}\left(\alpha^1\right) = t\int_0^{\alpha^1}(1-2x)\,dx + t\left[\left(2\alpha^1+1\right)/4-\alpha^1\right]\left(1-2\alpha^1\right)/2 = t\left[-4\left(\alpha^1\right)^2+4\alpha^1+1\right]/8$ and $\Pi_B^{2,S}\left(\alpha^1\right) = t\int_{(2\alpha^1+1)/4}^{1}\left[\left(1-2\alpha^1\right)/2+2x-1\right]dx = t\left[4\left(\alpha^1\right)^2-12\alpha^1+9\right]/16$ are profits if $\alpha^1 \le 1/2$. $\Pi_A^{2,S}\left(\alpha^1\right) = t\int_0^{(2\alpha^1+1)/4}\left[\left(2\alpha^1-1\right)/2+1-2x\right]dx = t\left[4\left(\alpha^1\right)^2+4\alpha^1+1\right]/16$ and $\Pi_B^{2,S}\left(\alpha^1\right) = t\left[\alpha^1-\left(2\alpha^1+1\right)/4\right]\left(2\alpha^1-1\right)/2 + t\int_{\alpha^1}^{1}(2x-1)\,dx = t\left[-4\left(\alpha^1\right)^2+4\alpha+1\right]/8$ are firms' profits if $\alpha^1 > 1/2$. *Q.E.D.*

Proof of Lemma 3. As follows from Lemma 1, we have to distinguish between the cases $\alpha^1 \le (3-\sqrt{2})/2$ and $\alpha^1 > (3-\sqrt{2})/2$. Assume first that in equilibrium $\alpha^1 \le (3-\sqrt{2})/2$. Then p_A^1 and p_B^1 have to maximize the profits $\Pi_A^{1+2,As}\left(p_A^1,p_B^1\right) = \alpha^1 p_A^1 + \delta t\left[-28\left(\alpha^1\right)^2+20\alpha^1+25\right]/32$ and $\Pi_B^{1+2,As}\left(p_B^1,p_A^1\right) = \left(1-\alpha^1\right)p_B^1 + \delta t\left[3-2\left(\alpha^1\right)\right]^2/16$, respectively, where $\alpha^1 = 1/2-(p_A^1-p_B^1)/2t$ (if p_A^1 and p_B^1 are not very different). Solving simultaneously FOCs we get the prices $p_A^{1,As}(\delta) = t\left(24+5\delta-4\delta^2\right)/(5\delta+24) > 0$ and $p_B^{1,As}(\delta) = t\left(24-\delta-4\delta^2\right)/(5\delta+24) > 0$ for any δ, and $\alpha^{1,As}(\delta) = (24-\delta)/(10\delta+48)$. Note that $\alpha^{1,As}(\delta)$ decreases in δ, while $\alpha^{1,As}(0) = 1/2$ and $\alpha^{1,As}(1) = 23/58$, such that $0 < \alpha^{1,As}(\delta) < (3-\sqrt{2})/2$ for any δ. SOCs are also fulfilled. We get $\Pi_A^{1+2,As}(\delta) = t\left(79\delta^3+710\delta^2+2208\delta+1152\right)/\left[4(5\delta+24)^2\right]$ and $\Pi_B^{1+2,As}(\delta) = t\left(-12\delta^3+85\delta^2+528\delta+576\right)/\left[2(5\delta+24)^2\right]$.

To prove that the prices $p_A^{1,As}(\delta)$ and $p_B^{1,As}(\delta)$ constitute the equilibrium, we have to show that none of the firms has an incentive to deviate such that $\alpha^1\left(p_A^1, p_B^1\right) > (3-\sqrt{2})/2$ holds. Assume that firm A deviates on $p_B^{1,As}(\delta) - t \le p_A^1 < p_B^{1,As}(\delta) - t(2-\sqrt{2})$, in which case its profit is $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\delta)) = \alpha^1 p_A^1 + 9\delta t/16$, where $\alpha^1 = 1/2 - (p_A^1 - p_B^{1,As}(\delta))/2t$. FOC yields $p_A^1(\delta) = 2t(-\delta^2+\delta+12)/(5\delta+24) > p_B^{1,As}(\delta) - t(2-\sqrt{2})$. Hence, $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\delta))$ gets its maximum at $p_A^1 = p_B^{1,As}(\delta) - t(2-\sqrt{2}) - \epsilon$, where $\epsilon > 0$ and $\epsilon \to 0$. We get $\Pi_A^{1+2,As}(p_B^{1,As}(\delta) - t(2-\sqrt{2}), p_B^{1,As}(\delta)) = t\left[\left(32\sqrt{2}-51\right)\delta^2 + \left(208\sqrt{2}-128\right)\delta + 768\sqrt{2} - 960\right]/\left[16(5\delta+24)\right]$ and $\Pi_A^{1+2,As}(\delta) > \Pi_A^{1+2,As}(p_B^{1,As}(\delta) - t(2-\sqrt{2}), p_B^{1,As}(\delta))$ for any δ, such that firm A does not have an incentive to deviate on $p_B^{1,As}(\delta) - t \le p_A^1 < p_B^{1,As}(\delta) - t(2-\sqrt{2})$. Note that firm A does not have an incentive to deviate on $p_A^1 < p_B^{1,As}(\delta) - t$ either, because in that case it realizes strictly lower profits than at the price $p_A^1 = p_B^{1,As}(\delta) - t$.

Assume now that firm B deviates on $t(2-\sqrt{2}) + p_A^{1,As}(\delta) \le p_B^1 \le t + p_A^{1,As}(\delta)$, in which case its profit is $\Pi_B^{1+2,As}(p_B^1, p_A^{1,As}(\delta)) = \left(1-\alpha^1\right)p_B^1 + \delta t/8$, where $\alpha^1 = 1/2 - (p_A^{1,As}(\delta) - p_B^1)/2t$. FOC yields $p_B^1(\delta) = t\left(-2\delta^2+5\delta+24\right)/(5\delta+24) < t(2-\sqrt{2}) + p_A^{1,As}(\delta)$, such that for any p_B^1 it holds $\Pi_B^{1+2,As}(p_B^1, p_A^{1,As}(\delta)) \le \Pi_B^{1+2,As}(t(2-\sqrt{2}) + p_A^{1,As}(\delta), p_A^{1,As}(\delta))$, where $\Pi_B^{1+2,As}(t(2-\sqrt{2}) + p_A^{1,As}(\delta), p_A^{1,As}(\delta)) = t\left[\delta^2(21-16\sqrt{2}) + \left(80\sqrt{2}-76\right)\delta + 384\sqrt{2} - 480\right]/\left[8(5\delta+24)\right]$. For any δ it holds that $\Pi_B^{1+2,As}(\delta) > \Pi_B^{1+2,As}(t(2-\sqrt{2}) + p_A^{1,As}(\delta), p_A^{1,As}(\delta))$, such that firm B does not have an incentive to deviate on $t(2-\sqrt{2}) + p_A^{1,As}(\delta) \le p_B^1 \le t + p_A^{1,As}(\delta)$. Firm B does not have an incentive to deviate on $p_B^1 > t + p_A^{1,As}(\delta)$ either, because in that case it realizes the same profits as at the price $p_B^1 = t + p_A^{1,As}(\delta)$.

Hence, the prices $p_A^{1,As}(\delta)$ and $p_B^{1,As}(\delta)$ constitute the equilibrium. Finally, we show that an equilibrium does not exist with $\alpha^1 > (3-\sqrt{2})/2$, in which case $p_B^1 > t(2-\sqrt{2}) + p_A^1$ and $\Pi_B^{1+2,As}(p_B^1, p_A^1) = \left(1-\alpha^1\right)p_B^1 + \delta t/8$. FOC yields $p_B^1\left(p_A^1\right) = \left(p_A^1 + t\right)/2 < p_A^1 + t(2-\sqrt{2})$ for any p_A^1, such that $\Pi_B^{1+2,As}(p_B^1, p_A^1)$ gets its maximum at $p_B^1 \to t(2-\sqrt{2}) + p_A^1$ and $\lim_{p_B^1 \to t(2-\sqrt{2})+p_A^1} \Pi_B^{1+2,As}(p_B^1, p_A^1) = \left[t(2-\sqrt{2}) + p_A^1\right]\left(\sqrt{2}-1\right)/2 + \delta t/8$. However, if firm B charges some $\left(p_A^1 + t\right)/2 < p_B^1 < p_A^1 + t(2-\sqrt{2})$, then both its first-period and second-period profits increase. The former increase because they are maximized at $p_B^1\left(p_A^1\right) = \left(p_A^1 + t\right)/2 < p_A^1 + t(2-\sqrt{2})$ for any p_A^1. The latter increase because, as follows from Lemma 1, $\Pi_B^{2,As}(\alpha^1 | \alpha^1 \le (3-\sqrt{2})/2) = t(3-2\alpha^1)^2/16$ and $\Pi_B^{2,As}(\alpha^1 | \alpha^1 > (3-\sqrt{2})/2) = t/8$, while $t(3-2\alpha^1)^2/16 > t/8$ for any $\alpha^1 < (3-\sqrt{2})/2$. Hence, no equilibrium exists with $\alpha^1 > (3-\sqrt{2})/2$. *Q.E.D.*

Proof of Lemma 4. As follows from Lemma 2, we have to distinguish between the cases $\alpha^1 \le 1/2$ and $\alpha^1 > 1/2$. Assume that in equilibrium $\alpha^1 \le 1/2$ holds, in which case $\Pi_A^{1+2,S}\left(p_A^1, p_B^1\right) =$

$\alpha^1 p_A^1 + \delta t \left[-4\left(\alpha^1\right)^2 + 4\alpha^1 + 1\right]/8$, $\Pi_B^{1+2,S}\left(p_B^1, p_A^1\right) = \left(1-\alpha^1\right) p_B^1 + \delta t \left[4\left(\alpha^1\right)^2 - 12\alpha^1 + 9\right]/16$, where $\alpha^1 = 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$. We get $p_A^{1,S}(\delta) = t\left(12 - \delta - \delta^2\right)/(12+\delta)$, $p_B^{1,S}(\delta) = t\left(12 - 3\delta - \delta^2\right)/(12+\delta)$ and $\alpha^{1,S}(\delta) = (12-\delta)/[2(12+\delta)] \leq 1/2$ for any δ solving simultaneously FOCs. Firms realize profits $\Pi_A^{1+2,S}(\delta) = t\left(\delta^3 + 2\delta^2 + 96\delta + 288\right)/\left[4(\delta+12)^2\right]$ and $\Pi_B^{1+2,S}(\delta) = t\left(-\delta^3 + 3\delta^2 + 72\delta + 144\right)/\left[2(\delta+12)^2\right]$. SOCs are fulfilled. For $p_A^{1,S}(\delta)$ and $p_B^{1,S}(\delta)$ to constitute the equilibrium, none of the firms should have an incentive to deviate such that $\alpha^1 > 1/2$ holds. Assume that firm A deviates on $p_A^1 < p_B^{1,S}(\delta)$, in which case $\Pi_A^{1+2,S}\left(p_A^1, p_B^{1,S}(\delta)\right) = \alpha^1 p_A^1 + \delta t\left[4\left(\alpha^1\right)^2 + 4\alpha^1 + 1\right]/16$, where $\alpha^1 = 1/2 - \left[p_A^1 - p_B^{1,S}(\delta)\right]/(2t)$. FOC yields $p_A^1\left(p_B^{1,S}(\delta)\right) = \left[p_B^{1,S}(\delta)(4-\delta) + 2t(2-\delta)\right]/(8-\delta)$. We get $p_A^1\left(p_B^{1,S}(\delta)\right) - p_B^{1,S}(\delta) = -2t\delta(4-\delta)/[(8-\delta)(\delta+12)] < 0$ and if $p_A^1 < p_B^{1,S}(\delta)$, $\max_{p_A^1} \Pi_A^{1+2,S}\left(p_A^1, p_B^{1,S}(\delta)\right) = \Pi_A^{1+2,S}\left(p_A^1\left(p_B^{1,S}(\delta)\right), p_B^{1,S}(\delta)\right) = t\left(\delta^4 - 5\delta^3 - 64\delta^2 + 240\delta + 1152\right)/\left[2(8-\delta)(\delta+12)^2\right]$. We get $\Pi_A^{1+2,S}(\delta) - \Pi_A^{1+2,S}\left(p_A^1\left(p_B^{1,S}(\delta)\right), p_B^{1,S}(\delta)\right) = t\delta^2\left(48 + 16\delta - 3\delta^2\right)/\left[4(8-\delta)(\delta+12)^2\right] > 0$ for any δ, such that firm A does not have an incentive to deviate.

Assume that firm B deviates on $p_B^1 > p_A^{1,S}(\delta)$. Then $\Pi_B^{1+2,S}\left(p_B^1, p_A^{1,S}(\delta)\right) = \left(1-\alpha^1\right)p_B^1 + \delta t\left[-4\left(\alpha^1\right)^2 + 4\alpha^1 + 1\right]/8$, where $\alpha^1 = 1/2 - \left[p_A^{1,S}(\delta) - p_B^1\right]/(2t)$. FOC yields $p_B^1\left(p_A^{1,S}(\delta)\right) = \left[p_A^{1,S}(\delta)(2+\delta) + 2t\right]/(\delta+4)$. We get $p_B^1\left(p_A^{1,S}(\delta)\right) - p_A^{1,S}(\delta) = 2t\delta(\delta+2)/\left(\delta^2 + 16\delta + 48\right) > 0$, such that if $p_B^1 > p_A^{1,S}(\delta)$, $\Pi_B^{1+2,S}\left(p_B^1, p_A^{1,S}(\delta)\right)$ gets its maximum at $p_B^1 = p_B^1\left(p_A^{1,S}(\delta)\right)$ and $\Pi_B^{1+2}\left(p_B^1\left(p_A^{1,S}(\delta)\right), p_A^{1,S}(\delta)\right) = t\left(\delta^4 + 2\delta^3 + 144\delta^2 + 864\delta + 1152\right)/\left[4(\delta+4)(\delta+12)^2\right]$. We get $\Pi_B^{1+2,S}(\delta) - \Pi_B^{1+2,S}\left(p_B^1\left(p_A^{1,S}(\delta)\right), p_A^{1,S}(\delta)\right) = t\delta^2\left(24 - 4\delta - 3\delta^2\right)/\left[4(\delta+4)(\delta+12)^2\right] > 0$ for any δ, such that firm B does not have an incentive to deviate. Hence, the prices $p_A^{1,S}(\delta)$ and $p_B^{1,S}(\delta)$ constitute the equilibrium. Symmetrically, there exists the equilibrium, where $p_A^1 = p_B^{1,S}(\delta)$ and $p_B^1 = p_A^{1,S}(\delta)$. *Q.E.D.*

Proof of Proposition 1. We start with the unilateral incentives to invest. Assume that firm B does not invest. If firm A does not invest either, over two periods it realizes the profit $\Pi_A^{1+2}(\delta) = t(1+\delta)/2$. If it invests, it gets $\Pi_A^{1+2,As}(\delta) = t\left(79\delta^3 + 710\delta^2 + 2208\delta + 1152\right)/\left[4(5\delta+24)^2\right]$. For any δ we have $\Pi_A^{1+2,As}(\delta) - \Pi_A^{1+2}(\delta) = t\delta\left(29\delta^2 + 180\delta + 576\right)/\left[4(5\delta+24)^2\right] > 0$, such that firm A always invests.

Assume next that firm A invests. If firm B does not invest, then over two periods it realizes the profit $\Pi_B^{1+2,As}(\delta) = t\left(-12\delta^3 + 85\delta^2 + 528\delta + 576\right)/\left[2(5\delta+24)^2\right]$. If firm B invests, then the profit is $\Pi_i^{1+2,S}(\delta) = t\left(\delta^3 + 2\delta^2 + 96\delta + 288\right)/\left[4(\delta+12)^2\right]$ or $\Pi_j^{1+2,S}(\delta) = t\left(-\delta^3 + 3\delta^2 + 72\delta + 144\right)/\left[2(\delta+12)^2\right]$, depending on the equilibrium. For any δ it holds

$\max\left\{\Pi_i^{1+2,S}(\delta), \Pi_j^{1+2,S}(\delta)\right\} = \Pi_j^{1+2,S}(\delta)$. We get that for any δ, $\Pi_B^{1+2,As}(\delta) - \Pi_j^{1+2,S}(\delta) = t\delta\left(13\delta^4 - 38\delta^3 - 1104\delta^2 + 2880\delta + 13\,824\right)/2\left(5\delta^2 + 84\delta + 288\right)^2 > 0$, such that firm B does not invest. Combining the incentives of firms A and B to invest we conclude that there are two asymmetric equilibria where only one of the firms invests.

The equilibrium social welfare over two periods is $SW_m^{1+2}(\delta) = v(1+\delta) - t\int_0^{(24-\delta)/(10\delta+48)} x dx - t\int_{(24-\delta)/(10\delta+48)}^1 (1-x)dx - \delta t\int_0^{(3\delta+18)/(5\delta+24)} x dx - \delta t\int_{(3\delta+18)/(5\delta+24)}^1 (1-x)dx = v(1+\delta) - t\left(26\delta^3 + 325\delta^2 + 960\delta + 576\right)/\left[4\left(5\delta+24\right)^2\right]$. Consumer surplus is $CS_m^{1+2}(\delta) = SW_m^{1+2}(\delta) - \Pi_A^{1+2,As}(\delta) - \Pi_B^{1+2,As}(\delta) = v(1+\delta) - t\left(81\delta^3 + 1205\delta^2 + 4224\delta + 2880\right)/\left[4\left(5\delta+24\right)^2\right]$. *Q.E.D.*

Proof of Lemma 5. While making their first-period decisions, sophisticated consumers take into account the equilibrium of the second period. As follows from Lemma 1, we have to distinguish between the cases $\alpha^1 \leq (3-\sqrt{2})/2$ and $\alpha^1 > (3-\sqrt{2})/2$. With $U_i^{1+2}(x, p_i^1; \alpha^1)$ we denote the utility of a consumer x over two periods if in the first period she buys from firm $i = A, B$ and the share of consumers who bought from firm A in the first period is α^1.

Case 1: $(3-\sqrt{2})/2 < \alpha^1 < 1$. We show that prices must satisfy $p_B^1 - t < p_A^1 < p_B^1 - t(2-\sqrt{2})$ and $\alpha^1(p_A^1, p_B^1) = 1/2 + \left(p_B^1 - p_A^1\right)/2t$. We start with consumers with $x \leq 3/4$, for whom $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - tx - t(3/2 - 2x)\right]$. If a consumer buys at firm B in the first period, firm A does not learn her preference and has to charge $p_A^{2,As}\left(\alpha^1\right) = 0$. Then $U_A^2(x, p_A^{2,As}\left(\alpha^1\right)) = v - tx \geq U_B^2(x, p_B^{2,As}\left(\alpha^1\right)) = v - t(1-x) - t/2$, and a consumer buys at firm A in the second period upon buying at firm B in the first period. Hence, $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1-x) - p_B^1 + \delta(v - tx)$. All consumers with $x \leq 3/4$ must buy at firm A in the first period, because $\alpha^1 > (3-\sqrt{2})/2 > 3/4$ holds by assumption. The inequality $U_A^{1+2}(x, p_A^1; \alpha^1) \geq U_B^{1+2}(x, p_B^1; \alpha^1)$ yields the condition $p_A^1 \leq t\left[1 - 3\delta/2 - 2x(1-\delta)\right] + p_B^1$. Plugging $x = 3/4$ into the latter inequality we get $p_A^1 \leq p_B^1 - t/2$.

Consider now consumers with $3/4 < x \leq \alpha^1$, for whom $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - t(1-x) - t/2\right]$. If a consumer buys at firm B in the first period, then $U_A^2(x, p_A^{2,As}\left(\alpha^1\right)) = v - tx < U_B^2(x, p_B^{2,As}\left(\alpha^1\right)) = v - t(1-x) - t/2$, such that a consumer buys at firm B in the second period too and $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1-x) - p_B^1 + \delta\left[v - t(1-x) - t/2\right]$. Comparing $U_A^{1+2}(x, p_A^1; \alpha^1)$ and $U_B^{1+2}(x, p_B^1; \alpha^1)$ we conclude that consumers with $x \leq 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$ buy at firm A and consumers with $x > 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$ buy at firm B in the first period, such that $\alpha^1(p_A^1, p_B^1) = 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$. The condition $(3-\sqrt{2})/2 < \alpha^1 < 1$ is fulfilled if prices are such that $p_B^1 - t < p_A^1 < p_B^1 - t(2-\sqrt{2})$.

Case 2: $0 < \alpha^1 \leq \left(3 - \sqrt{2}\right)/2$. We will show that in this case prices satisfy $p_B^1 - t(2-\sqrt{2}) - \delta t(3\sqrt{2}-4)/4 \leq p_A^1 < p_B^1 + t(4-5\delta)/4$ and $\alpha^1(p_A^1, p_B^1) = \left[t(4-5\delta) + 4(p_B^1 - p_A^1)\right]/\left[2t(4-3\delta)\right]$. Consider first consumers with $x \leq \alpha^1$, for whom $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - tx - t(3-2\alpha^1)/2 - t(1-2x)\right]$. If a consumer buys at firm B in the first period, then $U_A^2(x, p_A^{2,As}\left(\alpha^1\right)) = v - tx - t\left[5 - 6\alpha^1\right]/4 \geq U_B^2(x, p_B^{2,As}\left(\alpha^1\right)) = v - t\left(1-x\right) - t\left[3 - 2\alpha^1\right]/2$ for any $x \leq \left(5 + 2\alpha^1\right)/8$. Hence, upon buying at firm B in the first period a consumer buys at firm A in the second period and $U_B^{1+2}\left(x, p_B^1; \alpha^1\right) = v - t(1-x) - p_B^1 + \delta\left[v - tx - t\left(5 - 6\alpha^1\right)/4\right]$. Consumers with $x \leq \alpha^1$ buy at firm A in the first period if $U_A^{1+2}(x, p_A^1; \alpha^1) \geq U_B^{1+2}\left(x, p_B^1; \alpha^1\right)$ holds, which yields the condition

$$p_A^1 \leq p_B^1 + t\left(1 - 2\alpha^1\right)(1-\delta) - \delta t\left(1 + 2\alpha^1\right)/4. \tag{3}$$

Consider now consumers with $\alpha^1 \leq x \leq \left(5 + 2\alpha^1\right)/8$, for whom $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1-x) - p_B^1 + \delta\left[v - tx - t(5 - 6\alpha^1)/4\right]$. If a consumer buys at firm A in the first period, it learns her preference and makes her indifferent in the second period whenever possible with a non-negative price, such that a consumer's utility is always given by $U_B^2(x, p_B^{2,As}\left(\alpha^1\right)) = v - t\left(1-x\right) - t\left(3 - 2\alpha^1\right)/2$ independently of the firm it buys. Hence, $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx + \delta\left[v - t\left(1-x\right) - t\left(3 - 2\alpha^1\right)/2\right]$. Comparing $U_B^{1+2}(x, p_B^1; \alpha^1)$ and $U_A^{1+2}(x, p_A^1; \alpha^1)$ we conclude that consumers with $\alpha^1 \leq x \leq \left(5 + 2\alpha^1\right)/8$ buy at firm B in the first period if prices are such that

$$p_A^1 \geq p_B^1 + t\left(1 - 2\alpha^1\right)(1-\delta) - \delta t\left(1 + 2\alpha^1\right)/4. \tag{4}$$

Combining (3) and (4) we conclude that only the price $p_A^1 = p_B^1 + t(1-2\alpha^1)(1-\delta) - \delta t\left(1 + 2\alpha^1\right)/4$ satisfies both conditions. This price yields $\alpha^1(p_A^1, p_B^1) = \left[4(p_B^1 - p_A^1) + t(4-5\delta)\right]/\left[2t(4-3\delta)\right]$. Imposing the constraint $0 < \alpha^1 \leq \left(3 - \sqrt{2}\right)/2$ we get $p_B^1 - t(2-\sqrt{2}) - \delta t(3\sqrt{2}-4)/4 \leq p_A^1 < p_B^1 + t(4-5\delta)/4$.

We finally show that consumers $x \geq \left(5 + 2\alpha^1\right)/8$ buy at firm B. We have $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1-x) - p_B^1 + \delta\left[v - t(1-x) - t(3-2\alpha^1)/2\right]$. If a consumer buys at firm A in the first period, it learns her preference and makes her indifferent in the second period whenever possible with a non-negative price, such that a consumer's utility is always given by $U_B^2(x, p_B^{2,As}\left(\alpha^1\right)) = v - t(1-x) - t(3-2\alpha^1)/2$ independently of the firm it buys. Hence, $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - t(1-x) - t(3-2\alpha^1)/2\right]$. Comparing $U_B^{1+2}(x, p_B^1; \alpha^1)$ and $U_A^{1+2}(x, p_A^1; \alpha^1)$ we conclude that a consumer buys at firm B in the first period if $p_A^1 \geq p_B^1 + t(1-2x)$. It is straightforward to show that $p_A^1 = p_B^1 + t(1-2\alpha^1)(1-\delta) - \delta t\left(1+2\alpha^1\right)/4$ satisfies the latter inequality for any $x \geq \left(5+2\alpha^1\right)/8$, because $\alpha^1 < 5/6$ holds by assumption. Hence, consumers with $x \geq \left(5+2\alpha^1\right)/8$ buy at firm B.

It is straightforward to show that $\alpha^1(p_A^1, p_B^1) = 1$ if $p_A^1 \leq p_B^1 - t$ and $\alpha^1(p_A^1, p_B^1) = 0$ if $p_A^1 \geq p_B^1 + t(4-5\delta)/4$. To complete the derivation of the demand we note that for any $\delta \in [0,1]$ it holds that $-1 < \sqrt{2} - 2 - \delta(3\sqrt{2}-4)/4 \leq \sqrt{2} - 2 < (4-5\delta)/4$, such that the demand is given by a correspondence. *Q.E.D.*

Proof of Lemma 6. In the following we will use $a(p_B^1) := p_B^1 - t(2-\sqrt{2}) - \delta t(3\sqrt{2}-4)/4$, $b(p_A^1) := p_A^1 + t(2-\sqrt{2}) + \delta t(3\sqrt{2}-4)/4$, $c(p_B^1) := p_B^1 - t(2-\sqrt{2})$ and $d(p_A^1) := p_A^1 + t(2-\sqrt{2})$. We know from Lemma 5 that consumer demand in the first period is given by a correspondence. We will consider two cases depending on $\alpha^1(p_A^1, p_B^1)$ on the interval $a(p_B^1) \leq p_A^1 < c(p_B^1)$ and will show that both cases yield the same equilibrium.

Case 1. Assume that $\alpha^1(p_A^1, p_B^1) = \left[t\,(4-5\delta) + 4\left(p_B^1 - p_A^1\right)\right] / \left[2t\,(4-3\delta)\right]$ if $a(p_B^1) \leq p_A^1 < c(p_B^1)$. In the following we will consider three cases depending on δ. In each case we will show that the equilibrium exists where $\alpha^1 \leq \left(3-\sqrt{2}\right)/2$, such that $p_A^{1,As} \geq a(p_B^{1,As})$. We will then show that this is the unique equilibrium.

Case 1.a). Let $\delta < \left(70 - 2\sqrt{721}\right)/21$. Assume that in equilibrium $p_A^{1,As} \geq a(p_B^1)$ holds, in which case firms maximize the profits $\Pi_A^{1+2,As}\left(p_A^1, p_B^1\right) = p_A^1\alpha^1 + \delta t\left[-28\left(\alpha^1\right)^2 + 20\alpha^1 + 25\right]/32$ and $\Pi_B^{1+2,As}\left(p_A^1, p_B^1\right) = p_B^1\left(1-\alpha^1\right) + \delta t\left(3-2\alpha^1\right)^2/16$, where $\alpha^1(p_A^1, p_B^1) =$
$\left[t\,(4-5\delta) + 4\left(p_B^1 - p_A^1\right)\right] / \left[2t\,(4-3\delta)\right]$. FOCs yield the prices

$$p_A^{1,As}(\delta) = \frac{t\left(96 - 140\delta + 21\delta^2\right)}{96 - 52\delta} \text{ and } p_B^{1,As}(\delta) = \frac{t\left(96 - 132\delta + 25\delta^2\right)}{96 - 52\delta} \tag{5}$$

and $\alpha^{1,As}(\delta) = (24-23\delta)/(48-26\delta)$. For any δ it holds $\alpha^{1,As}(\delta) < \left(3-\sqrt{2}\right)/2$, such that $p_A^{1,As} \geq a(p_B^1)$ is true. $p_A^{1,As}(\delta) > 0$ and $p_B^{1,As}(\delta) > 0$ hold if $\delta < \left(70 - 2\sqrt{721}\right)/21$. SOCs are also fulfilled. Firms' profits in the first period are

$$\Pi_A^{1,As}(\delta) = \frac{t\,(24-23\delta)\left(21\delta^2 - 140\delta + 96\right)}{(26\delta - 48)(52\delta - 96)},\ \Pi_B^{1,As}(\delta) = \frac{t\left(-75\delta^3 + 996\delta^2 - 3456\delta + 2304\right)}{1352\delta^2 - 4992\delta + 4608}.$$

Firms' profits over two periods are

$$\Pi_A^{1+2,As}(\delta) = \frac{t\left(395\delta^3 - 404\delta^2 - 1536\delta + 2304\right)}{8\left(13\delta - 24\right)^2},\ \Pi_B^{1+2,As}(\delta) = \frac{t\left(53\delta^3 + 228\delta^2 - 2304\delta + 2304\right)}{8\left(13\delta - 24\right)^2}.$$

To prove that the prices in (5) constitute the equilibrium, we have to show that firm A (B) does not have an incentive to deviate on $p_A^1 < a(p_B^{1,As}(\delta))$ ($p_B^1 > b(p_A^{1,As}(\delta))$), where $\alpha^1 > \left(3 - \sqrt{2}\right)/2$ holds.

Assume that $p_B^1 = p_B^{1,As}(\delta)$ and firm A deviates on $p_B^{1,As}(\delta) - t \leq p_A^1 < a(p_B^{1,As}(\delta))$, where $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\cdot)) = p_A^1\alpha^1(p_A^1, p_B^{1,As}(\cdot)) + \delta 9t/16$, $\alpha^1(p_A^1, p_B^{1,As}(\cdot)) = 1/2 + \left[p_B^{1,As}(\cdot) - p_A^1\right]/(2t)$. It must be that $a(p_B^{1,As}(\delta)) > 0$, because if $a(p_B^{1,As}(\delta)) \leq 0$, deviation is not possible. FOC yields $p_A^1(p_B^{1,As}(\delta)) = t\left(25\delta^2 - 184\delta + 192\right)/[8(24 - 13\delta)]$. We have $p_A^1(p_B^{1,As}(\delta)) > a(p_B^{1,As}(\delta))$ for any δ, such that $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\delta))$ is maximized at $p_A^1 \to a(p_B^{1,As}(\delta))$. We get that $\Pi_A^{1+2,As}(a(p_B^{1,As}(\delta)), p_B^{1,As}(\delta)) < \Pi_A^{1+2,As}(p_A^{1,As}(\delta), p_B^{1,As}(\delta))$ for any δ, such that firm A does not have an incentive to deviate on $p_B^{1,As}(\delta) - t \leq p_A^1 < a(p_B^{1,As}(\delta))$. Firm A does not have an incentive to deviate on $p_A^1 < p_B^{1,As}(\delta) - t$ either, because in that case it realizes a strictly lower profit than at $p_A^1 = p_B^{1,As}(\delta) - t$.

Assume now that $p_A^1 = p_A^{1,As}(\delta)$ and firm B deviates on $b(p_A^{1,As}(\delta)) < p_B^1 \leq p_A^{1,As}(\delta) + t$, such that its profit is $\Pi_B^{1+2,As}(p_B^1, p_A^{1,As}(\delta)) = p_B^1\left[1 - \alpha^1(p_A^{1,As}(\delta), p_B^1)\right] + \delta t/8$, where $\alpha^1(p_A^{1,As}(\delta), p_B^1) = 1/2 + \left[p_B^1 - p_A^{1,As}(\delta)\right]/(2t)$. FOC yields $p_B^1(p_A^{1,As}(\delta)) = 3t\left(7\delta^2 - 64\delta + 64\right)/[8\left(24 - 13\delta\right)] > 0$ for any δ. For any $\delta < \left(70 - 2\sqrt{721}\right)/21$ we have that $p_B^1(p_A^{1,As}(\delta)) < b(p_A^{1,As}(\delta))$, such that $\Pi_B^{1+2,As}(p_B^1, p_A^{1,As}(\delta))$ is maximized at $p_B^1 \to b(p_A^{1,As}(\delta))$. We get $\Pi_B^{1+2,As}\left(b(p_A^{1,As}(\delta)), p_A^{1,As}(\delta)\right) < \Pi_B^{1+2,As}\left(p_B^{1,As}(\delta), p_A^{1,As}(\delta)\right)$ for any δ, such that firm B does not have an incentive to deviate on $b(p_A^{1,As}(\delta)) < p_B^1 \leq p_A^{1,As}(\delta) + t$. Note finally that firm B does not have an incentive to deviate on $p_B^1 > p_A^{1,As}(\delta) + t$ either, because in that case firm B realizes the same profits as under the price $p_B^1 = p_A^{1,As}(\delta) + t$.

We conclude that the prices $p_A^{1,As}(\delta)$ and $p_B^{1,As}(\delta)$ in (5) constitute the equilibrium, where $\alpha^1 \leq \left(3 - \sqrt{2}\right)/2$ holds. We will show next that this is the unique equilibrium. Assume that an equilibrium exists with $\alpha^1 > \left(3 - \sqrt{2}\right)/2$. Note first that in equilibrium it cannot be that $p_A^1 < p_B^1 - t$, because firm A realizes then a strictly lower profit than at $p_A^1 = p_B^1 - t$. Hence, prices must satisfy $b(p_A^1) < p_B^1 \leq p_A^1 + t$, in which case $\Pi_B^{1+2,As}(p_B^1, p_A^1) = p_B^1\left[1 - \alpha^1(p_A^1, p_B^1)\right] + \delta t/8$, where $\alpha^1(p_A^1, p_B^1) = 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$. FOC yields $p_B^1\left(p_A^1\right) = \left(p_A^1 + t\right)/2$. As $p_B^1\left(p_A^1\right) < b(p_A^1)$ for any δ and any p_A^1, $\Pi_B^{1+2}(p_B^1, p_A^1)$ is maximized at $p_B^1 = b(p_A^1) + \epsilon$, where $\epsilon > 0$ and

$\epsilon \to 0$. However, it is always profitable for firm B to deviate to the price $p_B^1 = b(p_A^1) - \epsilon$, in which case both its first-period and second-period profits increase. Hence, an equilibrium with $\alpha^1 > \left(3 - \sqrt{2}\right)/2$ does not exist.

Case 1.b). Let $\left(70 - 2\sqrt{721}\right)/21 \leq \delta < 6/7$. Assume that in equilibrium $p_A^{1,As} \geq a(p_B^{1,As})$ holds. We showed in *Case 1.a)* that at least one of the prices in (5) is non-positive. Assume that in equilibrium $p_A^{1,As}(\delta) = 0$. Maximizing $\Pi_B^{1+2,As}\left(p_A^1, p_B^1\right) = p_B^1\left(1 - \alpha^1\right) + \delta t\left(3 - 2\alpha^1\right)^2/16$ with respect to p_B^1 we get

$$p_B^{1,As}(\delta) = t\left(7\delta^2 - 24\delta + 16\right)/(32 - 28\delta) \text{ and} \tag{6}$$

$$\alpha^{1,As}(\delta) = (6 - 7\delta)/(8 - 7\delta)\,. \tag{7}$$

$p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (6) satisfy $p_A^{1,As}(\delta) \geq a(p_B^{1,As}(\delta))$. SOC is also fulfilled. For any δ we have $\alpha^{1,As}(\delta) < 1$, while $p_B^{1,As}(\delta) > 0$ and $\alpha^{1,As}(\delta) > 0$ hold if $\left(70 - 2\sqrt{721}\right)/21 \leq \alpha < 6/7$. Firms' profits in the first period are $\Pi_B^{1,As}(\delta) = t\left(7\delta^2 - 24\delta + 16\right)/\left(98\delta^2 - 224\delta + 128\right)$ and $\Pi_A^{1,As}(\delta) = 0$. $\Pi_A^{1+2,As}(\delta) = t\delta\left(833\delta^2 - 2408\delta + 1552\right)/\left(1568\delta^2 - 3584\delta + 2048\right)$ and $\Pi_B^{1+2,As}(\delta) = t\left(-7\delta^2 + 8\delta + 16\right)/\left[16\left(8 - 7\delta\right)\right]$ are profits over two periods.

To prove that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (6) constitute the equilibrium, we have to show that none of the firms has an incentive to deviate on $p_A^1 < a(p_B^1)$. As $a(p_B^{1,As}) < 0$, firm A cannot deviate on $p_A^1 < a(p_B^{1,As})$. Assume that firm B deviates on $b(0) < p_B^1 \leq t$, in which case its profit is $\Pi_B^{1+2,As}(p_B^1, 0) = p_B^1\left[1 - \alpha^1(0, p_B^1)\right] + \delta t/8$ and $\alpha^1(0, p_B^1) = 1/2 + p_B^1/(2t)$. FOC yields $p_B^1(0) = t/2 < b(0)$, such that $\Pi_B^{1+2,As}(p_B^1, 0)$ is maximized at $p_B^1 \to b(0)$ and $\Pi_B^{1+2}(b(0), 0) < \Pi_B^{1+2,As}(\delta)$ implying that firm B does not have an incentive to deviate on $b(0) < p_B^1 \leq t$. Firm B does not have an incentive to deviate on $p_B^1 > t$ either, because in that case it gets the same profits as under $p_B^1 = t$. Hence, we showed that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (6) constitute the equilibrium.

We next show that if $p_A^{1,As} \geq a(p_B^{1,As})$ holds, then no equilibrium exists with $p_B^{1,As} = 0$. Assume that $p_B^{1,As} = 0$. Then as $a(0) < 0$, for any $p_A^{1,As}$ we have $\alpha^1\left(p_A^{1,As}, 0\right) \leq \left(3 - \sqrt{2}\right)/2 < 1$, such that by increasing its price slightly firm B gets a positive profit in the first period, while its second-period profit decreases slightly. We conclude that if $p_A^{1,As} \geq a(p_B^{1,As})$ holds, then the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (6) constitute the unique equilibrium. As we showed in *Case 1.a)* that an equilibrium does not exist where $p_A^{1,As} < a(p_B^{1,As})$ ($\alpha^{1,As} > \left(3 - \sqrt{2}\right)/2$), we conclude that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (6) constitute the unique equilibrium.

Case 1.c). Let $\delta \geq 6/7$. Assume that in equilibrium $p_A^{1,As} \geq a(p_B^{1,As})$ holds. We showed in *Case 1.a)* that at least one of the prices in (5) is non-positive. Assume that in equilibrium $p_A^{1,As}(\delta) = 0$. For any $\delta \geq 6/7$, $\alpha^{1,As}(\delta)$ in (7) is non-positive. Hence, we get in equilibrium $\alpha^{1,As}(\delta) = 0$, which yields

$$p_B^{1,As}(\delta) = t(5\delta - 4)/4 > 0. \tag{8}$$

Firms' first-period profits are $\Pi_A^{1,As}(\delta) = 0$ and $\Pi_B^{1,As}(\delta) = t(5\delta - 4)/4$. Over two periods firms realize profits $\Pi_A^{1+2,As}(\delta) = 25\delta t/32$ and $\Pi_B^{1+2,As}(\delta) = t(29\delta - 16)/16$. To prove that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (8) constitute the equilibrium, we have to show that none of the firms has an incentive to deviate on $p_A^{1,As} < a(p_B^{1,As})$. As $a(p_B^{1,As}) < 0$, firm A cannot deviate. Assume now that $p_A^1 = 0$ and that firm B deviates on $b(0) < p_B^1 \leq t$, in which case its profit is $\Pi_B^{1+2,As}(p_B^1, 0) = p_B^1\left[1 - \alpha^1(0, p_B^1)\right] + \delta t/8$ and $\alpha^1(0, p_B^1) = 1/2 + p_B^1/(2t)$. FOC yields $p_B^1(0) = t/2 < b(0)$, such that $\Pi_B^{1+2,As}(p_B^1, 0)$ is maximized at $p_B^1 \to b(0)$. We get $\Pi_B^{1+2,As}(b(0), 0) < \Pi_B^{1+2,As}(\delta)$ implying that firm B does not have an incentive to deviate on $b(0) < p_B^1 \leq t$. Firm B does not have an incentive to deviate on $p_B^1 > t$ either, because in that case it gets the same profits as under $p_B^1 = t$. Hence, we showed that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (8) constitute the equilibrium. Similarly as in *Case 1.b)*, one can show that no equilibrium with $p_B^{1,As}(\delta) = 0$ exists, which implies that this is the unique equilibrium when $p_A^{1,As} \geq a(p_B^{1,As})$ holds. As we showed in *Case 1.a)* that an equilibrium does not exist with $p_A^{1,As} < a(p_B^{1,As})$ ($\alpha^{1,As} > \left(3 - \sqrt{2}\right)/2$), we conclude that the prices $p_A^{1,As}(\delta) = 0$ and $p_B^{1,As}(\delta)$ in (8) constitute the unique equilibrium.

Case 2. Assume now that if $a(p_B^1) \leq p_A^1 < c(p_B^1)$, then $\alpha^1(p_A^1, p_B^1) = 1/2 + \left(p_B^1 - p_A^1\right)/(2t)$. We first show that in equilibrium it must hold $p_A^{1,As} \geq c(p_B^{1,As})$, such that $\alpha^{1,As} < \left(3 - \sqrt{2}\right)/2$. Assume that $p_A^{1,As} < c(p_B^{1,As})$ and $\alpha^{1,As} > \left(3 - \sqrt{2}\right)/2$ in equilibrium. Note first that in equilibrium it cannot be that $p_A^1 < p_B^1 - t$, because firm A realizes then a strictly lower profit than at $p_A^1 = p_B^1 - t$. Hence, prices must satisfy $d(p_A^1) < p_B^1 \leq p_A^1 + t$, in which case $\Pi_B^{1+2,As}(p_B^1, p_A^1) = p_B^1\left(1 - \alpha^1\right) + \delta t/8$, where $\alpha^1 = 1/2 - \left(p_A^1 - p_B^1\right)/(2t)$. FOC yields $p_B^1\left(p_A^1\right) = \left(p_A^1 + t\right)/2$. As $p_B^1\left(p_A^1\right) < d(p_A^1)$ for any δ and any p_A^1, $\Pi_B^{1+2,As}(p_B^1, p_A^1)$ is maximized at $p_B^1 = d(p_A^1) + \epsilon$, where $\epsilon > 0$ and $\epsilon \to 0$. However, it is always profitable for firm B to deviate to the price $p_B^1 = d(p_A^1) - \epsilon$, in which case both its first-period and second-period profits increase. Hence, an equilibrium with $\alpha^{1,As} > \left(3 - \sqrt{2}\right)/2$ does not exist.

Assume that in equilibrium $p_A^{1,As} \geq c(p_B^{1,As})$, such that $\alpha^{1,As} < \left(3-\sqrt{2}\right)/2$. We first show that no equilibrium exists with $p_B^{1,As} = 0$. Assume that $p_B^{1,As} = 0$. Then for any $p_A^{1,As}$ it holds that $p_A^{1,As} \geq c(0)$ as $c(0) < 0$, hence, $\alpha^{1,As} < \left(3-\sqrt{2}\right)/2 < 1$. Then if firm B increases its price slightly, it gets a positive profit in the first period, while its second-period profit decreases slightly. Hence, an equilibrium with $p_B^{1,As} = 0$ does not exist.

Note next that the equilibrium prices $p_A^{1,As}$ and $p_B^{1,As}$ from *Case 1.a)-Case 1.c)* satisfy $p_A^{1,As} \geq c(p_B^{1,As})$, which together with the results derived above implies that those prices are the only candidate equilibria. To prove that those prices constitute the equilibrium, we have to show that none of the firms has an incentive to deviate on $p_A^1 < c(p_B^1)$, where $\alpha^1 > \left(3-\sqrt{2}\right)/2$. We showed in *Case 1* that firm B does not have an incentive to deviate under any δ. This implies that firm B does not have an incentive to deviate in *Case 2* too, because on $d\left(p_A^{1,As}\right) < p_B^1 \leq b\left(p_A^{1,As}\right)$ firm B gets a smaller market share in the first period than in *Case 1* leading to lower first-period profits (while second-period profits do not change due to $\alpha^1 > \left(3-\sqrt{2}\right)/2$).

We show next that firm A does not have an incentive to deviate either. If $\left(70-2\sqrt{721}\right)/21 \leq \delta < 6/7$, we showed in *Case 1.b)* that $p_B^{1,As}(\delta) = t\left(7\delta^2 - 24\delta + 16\right)/(32-28\delta)$. For any δ it holds that $c(p_B^{1,As}(\delta)) < 0$, such that firm A cannot deviate on $p_A^1 < c(p_B^{1,As}(\delta))$. If $\delta \geq 6/7$, we showed in *Case 1.c)* that $p_B^{1,As}(\delta) = t(5\delta-4)/4$. For any δ it holds that $c(p_B^{1,As}(\delta)) < 0$, such that firm A cannot deviate on $p_A^1 < c(p_B^{1,As}(\delta))$ either. Consider finally $\delta < \left(70-2\sqrt{721}\right)/21$, where the candidate equilibrium is given by the prices (5). Assume that firm A deviates on $p_B^{1,As}(\delta) - t \leq p_A^1 < c(p_B^{1,As}(\delta))$, in which case its profit is $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\delta)) = p_A^1\alpha^1 + \delta 9t/16$, where $\alpha^1 = 1/2 + \left[p_B^{1,As}(\delta) - p_A^1\right]/(2t)$. The FOC yields $p_A^1(p_B^{1,As}(\delta)) = t\left(25\delta^2 - 184\delta + 192\right)/[8(24-13\delta)]$. We have $p_A^1(p_B^{1,As}(\delta)) > c(p_B^{1,As}(\delta))$ for any δ, such that $\Pi_A^{1+2,As}(p_A^1, p_B^{1,As}(\delta))$ is maximized at $p_A^1 \to c(p_B^{1,As}(\delta))$. We get that for any δ, $\Pi_A^{1+2}(c(p_B^{1,As}(\delta)), p_B^{1,As}(\delta)) < \Pi_A^{1+2}(p_A^{1,As}(\delta), p_B^{1,As}(\delta))$, such that firm A does not have an incentive to deviate on $p_B^{1,As}(\delta) - t \leq p_A^1 < c(p_B^{1,As}(\delta))$. Firm A does not have an incentive to deviate on $p_A^1 < p_B^{1,As}(\delta) - t$ either, because in that case it gets a strictly lower profit than at $p_A^1 = p_B^{1,As}(\delta) - t$. We conclude that the prices $p_A^{1,As}$ and $p_B^{1,As}$ from *Case 1.a)-Case 1.c)* constitute the unique equilibria for the respective δ. *Q.E.D.*

Proof of Lemma 7. As follows from Lemma 2, we have to distinguish between the cases $\alpha^1 \leq 1/2$ and $\alpha^1 > 1/2$. With $U_i^{1+2}(x, p_i^1; \alpha^1)$ we will denote the utility of a consumer x over two periods if in the first period she buys from firm $i = A, B$.

Case 1: $\alpha^1 \leq 1/2$. We first assume that that there is an indifferent consumer in the first period, and p_A^1 and p_B^1 are such that consumers with $x \leq \alpha^1 \left(p_A^1, p_B^1\right)$ (weakly) prefer firm A. Consider first consumers with $x \leq \alpha^1$, for whom $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - tx - t\left(1 - 2x\right)\right]$. If a consumer buys at firm B, firm A does not learn her preference and charges $p_A^{2,S}\left(x; \alpha^1\right) = t\left(1 - 2\alpha^1\right)/2$, such that $U_A^2(x, p_A^{2,S}\left(x; \alpha^1\right)) = v - tx - t\left(1 - 2\alpha^1\right)/2$. Firm B, in contrast, learns her preference and makes her indifferent in the second period whenever possible with a non-negative price, such that consumer's utility in the second period is always given by $U_A^2(x, p_A^{2,S}\left(\alpha^1\right))$ and $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t\left(1 - x\right) - p_B^1 + \delta\left[v - tx - t\left(1 - 2\alpha^1\right)/2\right]$. It must hold $U_A^{1+2}(x, p_A^1; \alpha^1) \geq U_B^{1+2}(x, p_B^1; \alpha^1)$ for any $x \leq \alpha^1$, which yields

$$p_A^1 - p_B^1 \leq t\left(1 - 2\alpha^1\right)\left(2 - \delta\right)/2. \tag{9}$$

Consider now consumers with $\alpha^1 \leq x < \left(2\alpha^1 + 1\right)/4$, for whom $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1 - x) - p_B^1 + \delta\left[v - tx - t\left(1 - 2\alpha^1\right)/2\right]$. If a consumer buys at firm A, then firm B does not learn her preference and charges $p_B^{2,S}\left(x; \alpha^1\right) = 0$, such that $U_B^2(x, p_B^{2,S}\left(x; \alpha^1\right)) = v - t\left(1 - x\right)$. Firm A, in contrast, learns her preference and makes her indifferent in the second period whenever possible with a non-negative price, such that consumer's utility in the second period is always given by $U_B^2(x, p_B^{2,S}\left(x; \alpha^1\right))$ and $U_A^{1+2}(x, p_A^1; \alpha^1) = v - tx - p_A^1 + \delta\left[v - t\left(1 - x\right)\right]$. It must hold $U_B^{1+2}(x, p_B^1; \alpha^1) \geq U_A^{1+2}(x, p_A^1; \alpha^1)$ for any $\alpha^1 \leq x < \left(2\alpha^1 + 1\right)/4$, which yields

$$p_A^1 - p_B^1 \geq t\left(1 - 2\alpha^1\right)\left(2 - \delta\right)/2. \tag{10}$$

Only the prices $p_A^1 - p_B^1 = t\left(1 - 2\alpha^1\right)\left(2 - \delta\right)/2$ satisfy (9) and (10) simultaneously, which yields

$$\alpha^{1,S}\left(p_A^1, p_B^1\right) = 1/2 - \left(p_A^1 - p_B^1\right)/\left[t\left(2 - \delta\right)\right], \tag{11}$$

if p_A^1 and p_B^1 are such that

$$0 \leq p_A^1 - p_B^1 \leq t\left(2 - \delta\right)/2. \tag{12}$$

It is left to check that at the prices (12) consumers with $x \geq \left(2\alpha^1 + 1\right)/4$ buy at firm B in the first period. We have $U_B^{1+2}(x, p_B^1; \alpha^1) = v - t(1 - x) - p_B^1 + \delta\left[v - t\left(1 - 2\alpha^1\right)/2 - tx\right]$. If a consumer buys at firm A, firm B does not learn her preference and charges $p_B^{2,S}\left(x; \alpha^1\right) = 0$ in the second period, such that $U_B^2(x, p_B^{2,S}\left(x; \alpha^1\right)) = v - t\left(1 - x\right)$. Firm A, in contrast, learns a consumer's preference and makes her indifferent in the second period whenever possible with a non-negative price, such that the second-period utility is always given by $U_B^2(x, p_B^{2,S}(x;\alpha^1))$ and $U_A^{1+2}(x, p_A^1;\alpha^1) = v - tx - p_A^1 - \delta\left[v - t\left(1-x\right)\right]$. It must hold $U_B^{1+2}(x, p_B^1;\alpha^1) \geq U_A^{1+2}(x, p_A^1;\alpha^1)$ for any $x \geq \left(2\alpha^1+1\right)/4$, which yields $p_A^1 - p_B^1 \geq t\left(1-2\alpha^1\right)/2$. The latter inequality is fulfilled for any δ as $p_A^1 - p_B^1 = t\left(1-2\alpha^1\right)\left(2-\delta\right)/2$, and consumers with $x \geq \left(2\alpha^1+1\right)/4$ buy at firm B in the first period.

It is straightforward to check that if in the first period none of the consumers (weakly) prefers firm A, then p_A^1 and p_B^1 are such that $p_A^1 - p_B^1 > t\left(2-\delta\right)/2$.

Case 2: $\alpha^1 > 1/2$. We first assume that there is an indifferent consumer in the first period, and p_A^1 and p_B^1 are such that consumers with $x \geq \alpha^1\left(p_A^1, p_B^1\right)$ (weakly) prefer firm B. Note that this case is symmetric to *Case 1*, such that using (11), we get the market share of firm B as $1 - \alpha^{1,S}\left(p_A^1, p_B^1\right) = 1/2 - \left(p_B^1 - p_A^1\right)/\left[t\left(2-\delta\right)\right]$, which yields again (11) for $\alpha^{1,S}\left(p_A^1, p_B^1\right)$ provided

$$-t\left(2-\delta\right)/2 \leq p_A^1 - p_B^1 < 0. \tag{13}$$

Combining (12) and (13) we conclude that $\alpha^1\left(p_A^1, p_B^1\right)$ is given by (11) if $-t\left(2-\delta\right)/2 \leq p_A^1 - p_B^1 \leq t\left(2-\delta\right)/2$.

It is straightforward to check that if in the first period none of the consumers (weakly) prefers firm B, then p_A^1 and p_B^1 are such that $p_A^1 - p_B^1 < -t\left(2-\delta\right)/2$. *Q.E.D.*

Proof of Lemma 8. Assume first that $\alpha^1\left(p_A^1, p_B^1\right) \leq 1/2$. Then p_A^1 and p_B^1 have to maximize the profits $\Pi_A^{1+2,S}\left(p_A^1, p_B^1\right) = \alpha^1 p_A^1 + \delta t\left[-4(\alpha^1)^2 + 4\alpha^1 + 1\right]/8$ and $\Pi_B^{1+2,S}\left(p_B^1, p_A^1\right) = \left(1-\alpha^1\right)p_B^1 + \delta t\left[4(\alpha^1)^2 - 12\alpha^1 + 9\right]/16$, where $\alpha^1 = 1/2 - \left(p_A^1 - p_B^1\right)/\left[t\left(2-\delta\right)\right]$. FOCs yield $p_A^{1,S}\left(\delta\right) = t\left(24 - 26\delta + 5\delta^2\right)/\left(24 - 10\delta\right) > 0$, $p_B^{1,S}\left(\delta\right) = t\left(24 - 30\delta + 7\delta^2\right)/\left(24 - 10\delta\right) > 0$ and $0 < \alpha^{1,S}\left(\delta\right) = \left(12 - 7\delta\right)/\left(24 - 10\delta\right) \leq 1/2$ for any δ. SOCs are fulfilled. Firms realize profits $\Pi_A^{1+2,S}\left(\delta\right) = -t\left(6\delta^3 - 61\delta^2 + 168\delta - 144\right)/\left[2\left(5\delta - 12\right)^2\right]$ and $\Pi_B^{1+2,S}\left(\delta\right) =$
$-t\left(5\delta^3 - 78\delta^2 + 288\delta - 288\right)/\left[4\left(5\delta - 12\right)^2\right]$.

To prove that the prices $p_A^{1,S}\left(\delta\right)$ and $p_B^{1,S}\left(\delta\right)$ constitute the equilibrium, we have to show that none of the firms has an incentive to deviate on $p_A^1 < p_B^1$, where $\alpha^1\left(p_A^1, p_B^1\right) > 1/2$. Assume that firm A deviates on $p_A^1 < p_B^{1,S}\left(\delta\right)$, in which case its profit is $\Pi_A^{1+2,S}\left(p_A^1, p_B^{1,S}\left(\delta\right)\right) = \alpha^1 p_A^1 + \delta t\left[4\left(\alpha^1\right)^2 + 4\alpha^1 + 1\right]/16$, where $\alpha^1 = 1/2 - \left(p_A^1 - p_B^{1,S}\left(\delta\right)\right)/\left[t\left(2-\delta\right)\right]$. FOC yields $p_A^1\left(p_B^{1,S}\left(\delta\right)\right) = \left[4p_B^{1,S}\left(\delta\right) + t\left(4 - 4\delta + \delta^2\right)\right]/\left(8 - 2\delta\right)$. We get $p_A^1\left(p_B^{1,S}\left(\delta\right)\right) - p_B^{1,S}\left(\delta\right) =$
$t\delta\left(2-\delta\right)/\left(12 - 5\delta\right) > 0$, such that on $p_A^1 < p_B^{1,S}\left(\delta\right)$, $\Pi_A^{1+2,S}\left(p_A^1, p_B^{1,S}\left(\delta\right)\right)$ gets its maximum

at $p_A^1 \to p_B^{1,S}(\delta)$ and $\Pi_A^{1+2,S}\left(p_B^{1,S}(\delta), p_B^{1,S}(\delta)\right) = t\left(\delta^2 - 9\delta + 12\right)/[2(12-5\delta)]$. We have $\Pi_A^{1+2,S}(\delta) - \Pi_A^{1+2,S}\left(p_B^{1,S}(\delta), p_B^{1,S}(\delta)\right) = t\delta^2(4-\delta)/\left[2(5\delta-12)^2\right] > 0$, such that firm A does not have an incentive to deviate. Assume that firm B deviates on $p_B^1 > p_A^{1,S}(\delta)$, in which case its profit is $\Pi_B^{1+2,S}\left(p_B^1, p_A^{1,S}(\delta)\right) = \left(1-\alpha^1\right)p_B^1 + \delta t\left[-4\left(\alpha^1\right)^2 + 4\alpha^1 + 1\right]/8$, where $\alpha^1 = 1/2 - \left(p_A^{1,S}(\delta) - p_B^1\right)/[t(2-\delta)]$. FOC yields $p_B^1\left(p_A^{1,S}(\delta)\right) = \left[4p_A^{1,S}(\delta) + t\left(4 - 4\delta + \delta^2\right)\right]/(8-2\delta)$. We get $p_B^1\left(p_A^{1,S}(\delta)\right) - p_A^{1,S}(\delta) = 4t\delta(2-\delta)/[(4-\delta)(24-10\delta)] > 0$, such that on $p_B^1 > p_A^{1,S}(\delta)$ the profit of firm B gets maximum at $p_B^1 = p_B^1\left(p_A^{1,S}(\delta)\right)$ and $\Pi_B^{1+2,S}\left(p_B^1\left(p_A^{1,S}(\delta)\right), p_A^{1,S}(\delta)\right) = -t\left(35\delta^3 - 288\delta^2 + 720\delta - 576\right)/\left[2(4-\delta)(5\delta-12)^2\right]$. We have $\Pi_B^{1+2,S}(\delta) - \Pi_B^{1+2,S}\left(p_B^1\left(p_A^{1,S}(\delta)\right), p_A^{1,S}(\delta)\right) = t\delta^2\left(5\delta^2 - 28\delta + 24\right)/\left[4(4-\delta)(5\delta-12)^2\right] > 0$, such that firm B does not have an incentive to deviate either. We conclude that the prices $p_A^1 = p_A^{1,S}(\delta)$ and $p_B^1 = p_B^{1,S}(\delta)$ constitute the equilibrium. Symmetrically, there is the equilibrium with $p_A^1 = p_B^{1,S}(\delta)$ and $p_B^1 = p_A^{1,S}(\delta)$. *Q.E.D.*

Proof of Proposition 2. We first analyze the unilateral incentives of a firm to invest. Assume that firm B does not invest. If firm A does not invest either, then over two periods it realizes the profit $\Pi_A^{1+2}(\delta) = t(1+\delta)/2$. If firm A does invest, then $\Pi_A^{1+2,As}(\delta) = t\left(395\delta^3 - 404\delta^2 - 1536\delta + 2304\right)/\left[8(13\delta-24)^2\right]$ if $\delta \le (70 - 2\sqrt{721})/21$. We get $\Pi_A^{1+2,As}(\delta) - \Pi_A^{1+2}(\delta) = -t\delta\left(281\delta^2 - 1416\delta + 1344\right)/\left[8(13\delta-24)^2\right] < 0$ for any δ, such that firm A does not invest. $\Pi_A^{1+2,As}(\delta) = t\delta\left[833\delta^2 - 2408\delta + 1552\right]/\left[1568\delta^2 - 3584\delta + 2048\right]$ if $(70 - 2\sqrt{721})/21 < \delta < 6/7$. We have $\Pi_A^{1+2,As}(\delta) - \Pi_A^{1+2}(\delta) = t\left(49\delta^3 - 1400\delta^2 + 2320\delta - 1024\right)/\left[32(7\delta-8)^2\right] < 0$ for any δ, such that firm A does not invest either. If $\delta \ge 6/7$, then $\Pi_A^{1+2,As}(\delta) = 25\delta t/32$. We get $\Pi_A^{1+2,As}(\delta) - \Pi_A^{1+2}(\delta) = (9\delta - 16)/32 < 0$ for any δ, such that firm A does not invest either. We conclude that there are no unilateral incentives to invest when consumers are sophisticated.

We next assume that firm A invests and analyze the incentives of firm B to invest too. Consider first $\delta \le (70 - 2\sqrt{721})/21$. If firm B does not invest, its profit is $\Pi_B^{1+2,As}(\delta) = t\left(53\delta^3 + 228\delta^2 - 2304\delta + 2304\right)/\left[8(13\delta-24)^2\right]$. If firm B invests, then its profit is either $\Pi_i^{1+2,S}(\delta) = -t\left(6\delta^3 - 61\delta^2 + 168\delta - 144\right)/\left[2(5\delta-12)^2\right]$ or $\Pi_j^{1+2,S}(\delta) = -t\left(5\delta^3 - 78\delta^2 + 288\delta - 288\right)/\left[4(5\delta-12)^2\right]$ depending on the equilibrium. It holds $\max\left\{\Pi_i^{1+2,S}(\delta), \Pi_j^{1+2,S}(\delta)\right\} = \Pi_j^{1+2,S}(\delta)$. We get that $\Pi_j^{1+2,S}(\delta) - \Pi_B^{1+2,As}(\delta) = -9\delta t\left(335\delta^4 - 3696\delta^3 + 13\,680\delta^2 - 19\,968\delta + 9216\right)/\left[8\left(65\delta^2 - 276\delta + 288\right)^2\right] < 0$ for any $\delta \le (70 - 2\sqrt{721})/21$, such that firm B does not invest. Consider now $(70 - 2\sqrt{721})/21 < \delta < 6/7$. $\Pi_B^{1+2,As} = t\left(-7\delta^2 + 8\delta + 16\right)/[16(8-7\delta)]$ if firm B does not invest. We have $\Pi_j^{1+2,S}(\delta) -$

$\Pi_B^{1+2,As}(\delta) = t\left(315\delta^4 - 3384\delta^3 + 12\,128\delta^2 - 16\,512\delta + 6912\right)/\left[16\left(8-7\delta\right)\left(5\delta-12\right)^2\right] < 0$ for any $(70-2\sqrt{721})/21 < \delta < 6/7$, such that firm B does not invest either. Assume finally that $\delta \geq 6/7$. If firm B does not invest, then $\Pi_B^{1+2,As}(\delta) = t\left(29\delta-16\right)/16$. We get $\Pi_j^{1+2,S}(\delta) - \Pi_B^{1+2,As}(\delta) = -t\left(745\delta^3 - 4192\delta^2 + 7248\delta - 3456\right)/\left[16\left(5\delta-12\right)^2\right] < 0$ for any $\delta \geq 6/7$, such that firm B does not invest either. We conclude that for any δ firm B does not invest given that the rival invests.

Combining the optimal strategies of firms A and B we conclude that there is the unique equilibrium (in dominant strategies), where none of the firms invests. Then in each period each firm charges the price t and serves half of consumers. Over two periods firms realize profits $\Pi_{i,s}^{1+2}(\delta) = t\left(1+\delta\right)/2$ $(i=\{A,B\})$, and social welfare and consumer surplus are $SW_s^{1+2}(\delta) = (1+\delta)\left[v - 2t\int_0^{1/2} x dx\right] = (1+\delta)\left(v - t/4\right)$ and $CS_s^{1+2}(\delta) = SW^{1+2}(\delta) - t\left(1+\delta\right) = (1+\delta)\left(v-5t/4\right)$, respectively. *Q.E.D.*

Proof of Proposition 3. The comparison of firms' joint profits over two periods yields $\Pi_{A,m}^{1+2}(\delta) + \Pi_{B,m}^{1+2}(\delta) - 2\Pi_{i,s}^{1+2}(\delta) = -45t\delta^2\left(\delta+4\right)/\left[4\left(5\delta+24\right)^2\right] < 0$, such that firms are worse off when consumers are myopic. The comparison of the discounted sum of consumer surplus in both cases yields $CS_m^{1+2}(\delta) - CS_s^{1+2}(\delta) = t\delta\left(11\delta^2 + 30\delta - 36\right)/\left(5\delta+24\right)^2$. The latter expression is positive if $\delta > \left(3\sqrt{69}-15\right)/11$ and (weakly) negative otherwise. The comparison of the discounted sum of social welfare in both cases yields $SW_m^{1+2}(\delta) - SW_s^{1+2}(\delta) = -t\delta\left(\delta^2 + 60\delta + 144\right)/\left[4\left(5\delta+24\right)^2\right] < 0$, such that social welfare is lower when consumers are myopic.

Proof of Proposition 4. We first note that if one firm educates consumers, the other firm is indifferent between the two strategies. We now derive the best response of a firm if the rival does not educate. We know that $\Pi_{A,m}^{1+2}(\delta) > t(1+\delta)/2 > \Pi_{B,m}^{1+2}(\delta)$ and $\Pi_{A,m}^{1+2}(\delta) + \Pi_{B,m}^{1+2}(\delta) < t(1+\delta)$. We introduce the function $f(p,\delta) := p\Pi_{A,m}^{1+2}(\delta) + (1-p)\,\Pi_{B,m}^{1+2}(\delta)$ and $1/2 < p_A(\delta) < 1$, such that $f(p_A(\delta),\delta) = t(1+\delta)/2$ for any δ. It holds that $f(p,\delta) > t(1+\delta)/2$ if $p > p_A(\delta)$, with the opposite inequality otherwise. Hence, given that firm B does not educate, the best response of firm A is *i)* not educate if $p > p_A(\delta)$, *ii)* educate if $p < p_A(\delta)$, *iii)* if $p = p_A(\delta)$, firm A is indifferent.

We introduce next the function $g(p,\delta) := (1-p)\,\Pi_{A,m}^{1+2}(\delta) + p\Pi_{B,m}^{1+2}(\delta)$ and $0 < p_B(\delta) < 1/2$, such that $g(p_B(\delta),\delta) = t(1+\delta)/2$ for any δ. It holds that $g(p,\delta) > t(1+\delta)/2$ if $p < p_B(\delta)$, with the opposite inequality otherwise. Hence, given that firm A does not educate, the best response of firm B is *i)* not educate if $p < p_B(\delta)$, *ii)* educate if $p > p_B(\delta)$, *iii)* if $p = p_B(\delta)$, then firm B is indifferent.

As $p_A(\delta) > p_B(\delta)$, combining firms' best responses we get the equilibria as stated in Proposition 4. *Q.E.D.*

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