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The Nash Bargaining Solution in Vertical Relations With Linear Input Prices

Hamid Aghadadashli† Markus Dertwinkel-Kalt‡ Christian Wey§

June 2016

Abstract

We re-examine the Nash bargaining solution when an upstream and a downstream firm bargain over a linear input price. We show that the profit sharing rule is given by a simple and instructive formula which depends on the parties’ disagreement payoffs, the profit weights in the Nash-product and the elasticity of derived demand. A downstream firm’s profit share increases in the equilibrium derived demand elasticity which in turn depends on the final goods’ demand elasticity. Our simple formula generalizes to bargaining with $N$ downstream firms when bilateral contracts are unobservable.

JEL-Classification: L13

Keywords: Nash Bargaining, Demand Elasticity.

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1 Introduction

We investigate the properties of the Nash bargaining solution when an upstream supplier bargains with a downstream firm over a linear wholesale price. The Nash bargaining solution is given by equating the slopes of the bargaining frontier and the Nash product. The slope of the Nash product depends directly on the parties’ disagreement payoffs and the profit weights. It is well understood that a better disagreement payoff and a higher profit weight in the Nash product improves a party’s bargaining position, and hence, the profit share it gets. Our focus, in contrast, is on the slope of the bargaining frontier which gives the upstream firm’s maximal profit for a given profit level of the downstream firm. Under efficient contracts the slope of the bargaining frontier is $-1$, while it is strictly larger (i.e., between $-1$ and $0$) when bargaining is over a linear input price. This is a direct result of assuming that profits can only be transferred with a linear input price which leads to the well-known double mark-up problem. An increase of the wholesale price (so as to shift profits to the upstream firm) necessarily reduces the overall surplus available. Intuitively, the “steeper” the slope of the bargaining frontier, the harder it is to shift profits to the upstream firm so that the profit share of the downstream firm increases.

Our analysis of the bargaining frontier confirms this basic intuition and we derive a simple and instructive formula which combines all three determinants of parties’ bargaining powers according to the Nash bargaining solution; namely, the disagreement payoffs, the weights in the Nash product, and the slope of the bargaining frontier. The critical step in our analysis is to show that the slope of the bargaining frontier is equal to the total value of 1 plus the derived demand elasticity of the downstream firm for the input. The derived demand elasticity is the elasticity of the optimal input quantity with respect to the price of the input good. Its absolute value must be between zero and one to ensure the existence of a Nash bargaining solution in case of a linear transfer price. It then follows that a more elastic equilibrium derived demand goes hand in hand

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1The concept of Nash bargaining over linear input prices is widely used to solve the bilateral bargaining problem between up- and downstream firms, both theoretically (Horn and Wolinsky, 1988; Dobson and Waterson, 1997; von Ungern-Sternberg 1997; Naylor, 2002; Symeonidis, 2010; Iozzi and Valletti, 2015; Gaudin, 2015, 2016) and empirically (Gowrisankaran et al., 2015; Draganska et al., 2008; Grennan, 2013, 2014). Nash bargaining over linear input prices has been also widely assumed in labor economics where input prices are workers’ wages. For instance, Dowrick (1990) and Conlin and Furusawa (2000) compare inefficient bargaining over wages with efficient bargaining over input prices and employment. They derive conditions such that the employer is better off under inefficient bargaining.
with an increasing share of the total profit the downstream firm gets. This is driven by the fact that the more elastic the derived demand is the less transferable are utilities between the up- and the downstream firms and the larger are the dead-weight losses due to double marginalization. As derived and final demand elasticities are closely related, we can also express our findings in terms of the final demand elasticity.

The paper proceeds as follows. In Section 2 we present the analysis of the bilateral bargaining problem and we derive the central profit sharing formula. In Section 3 we extend our model toward \( N \) downstream firms and provide the analysis both for unobservable and for observable contracts. Section 4 concludes.

## 2 Model and Analysis

### 2.1 The Model Setup

We refer to a successive monopoly problem with an upstream firm \( U \) and a downstream firm \( D \). The input is produced at marginal cost \( c = 0 \) and transformed one to one by the downstream firm into the final good. Consumer demand for the final good is given by \( x(p) \), where \( p \) is the final good price, and \( p(x) \) gives the inverse demand. The game proceeds in two stages. In the first stage both firms bargain over a linear wholesale price \( w \). In the second stage, the downstream firm sets the final good price (or, equivalently, the quantity of the final good).

We impose the standard assumption

\[
p'' x + p' < 0, \tag{1}
\]

which guarantees the existence of a unique equilibrium. We abstract from all downstream costs other than the procurement costs \( w \cdot x \), such that the downstream firm’s profit is given by

\[
\pi := p(x)x - w \cdot x.
\]

while the upstream firm maximizes \( L := w \cdot x \). In equilibrium the downstream firm chooses quantity \( x^* \) such that the first-order condition

\[
p'(x^*)x^* + p(x^*) = w \tag{2}
\]
holds. For any given \( w \), Equation (2) determines a well-defined function, the derived demand \( x^*(w) \) of the downstream firm when bargaining with the upstream firm. Taking the total derivative of (2) gives the slope of the derived demand function:

\[
\frac{dx^*}{dw} = \frac{1}{p''x + 2p'}
\]  

(3)

such that \( dx^*/dw < 0 \). Due to (1), the downstream firm’s second-order condition

\[
\frac{d^2\pi}{dx^2} = p''x + 2p' < 0.
\]

holds, which ensures that the derived demand function is strictly downward sloping. We can write the downstream firm’s profit as a function of its derived demand, that is,

\[
\pi(w) = p(x^*(w))x^*(w) - wx^*(w)
\]  

(4)

and the upstream firm’s profit function as

\[
L(w) = wx^*(w).
\]  

(5)

### 2.2 The Bargaining Frontier

As \( d\pi/dw < 0 \) and \( dx^*/dw < 0 \) hold, there is a one-to-one relation between wage levels and profit levels. Thus, the supplier’s profit can be written as a well-defined function of the downstream firm’s profit, \( L = L(\pi(w)) \), which assigns each profit level of the downstream firm the according profit level of the upstream firm. We denote \( L = L(\pi(w)) \) the bargaining frontier. The chain rule yields

\[
\frac{dL(\pi(w))}{dw} = \frac{dL(\pi(w))}{d\pi(w)} \cdot \frac{d\pi(w)}{dw}.
\]

Rearranging gives the slope of the bargaining frontier

\[
\frac{dL(\pi(w))}{d\pi(w)} = \frac{dL(\pi(w))}{dw} \left( \frac{d\pi(w)}{dw} \right)^{-1}.
\]  

(6)

Denote the derived demand elasticity as

\[
\epsilon := \frac{dx^*(w)w}{x^*(w)}.
\]

Using \( dL/dw = x + w \cdot dx/dw \) (which follows from Equation (5)) and \( d\pi/dw = \partial\pi/\partial w = -x \) (which follows from Equation (4) and the Envelope Theorem), the slope of the bargaining
Bargaining frontier can be written as a function of the derived demand elasticity,  
\[
\frac{dL(\pi(w))}{d\pi(w)} = -(1 + \epsilon).  
\]  
(7)

This formula reflects that the transferability of utility between the retailer and the supplier depends crucially on the derived demand elasticity. The more inelastic derived demand is in equilibrium the larger is the loss the retailer has to bear in order to shift one unit of utility to the supplier. We will speak of a bargaining frontier effect when a change in the economic environment changes the derived demand elasticity $\epsilon$ and thus the slope of the bargaining frontier.

Next, we describe the curvature of the bargaining frontier $L(\pi(w))$. A necessary condition for a local maximum of $L(\pi)$ is $dL/d\pi = 0$. With formula (7) it is straightforward to check that there is a unique optimum at $\epsilon = -1$. If derived demand is elastic, $\epsilon < -1$, then $dL/dw = x^*(w) + w \frac{dx^*(w)}{dw} < 0$. As $d\pi/dw < 0$, it follows that $dL/d\pi > 0$, that is, the bargaining frontier is positively sloped. If derived demand is inelastic, $\epsilon > -1$, then $dL/dw = x^*(w) + w \frac{dx^*(w)}{dw} > 0$. As $d\pi/dw < 0$, it follows that $dL/d\pi < 0$, that is, the bargaining frontier is negatively sloped.

Figure 1 depicts the bargaining frontier. If the derived demand is elastic, that is, $\epsilon < -1$, then $dL/dw < 0$ and $d\pi/dw < 0$ hold such that both the supplier and the retailer can obtain a higher payoff with a lower input price $w$. Therefore, due to Pareto-optimality, the Nash bargaining
solution has to lie in the domain where the derived demand is inelastic, that is, \( \epsilon \geq -1 \).

### 2.3 Nash Bargaining

We investigate under which conditions a Nash bargaining solution exists.

**Lemma 1.** A bargaining problem \((X, (\pi_0, L_0))\) is defined by the set of feasible payoff combinations
\[ X = \{ (\pi, L) \in \mathbb{R}^2 | L \leq L(\pi) \} \] and the profits \((\pi_0, L_0)\) obtained if negotiation breaks down. Suppose that (I) \( L(\pi) \) is a concave function and (II) there exist \((\pi, L) \in X \) with \( \pi > \pi_0 \) and \( L > L_0 \). Then there exists a unique solution \((\pi(w^*), L(w^*))\) to the Nash bargaining problem which is given by
\[
\arg\max_{(\pi, L)} \{ (\pi - \pi_0)^\alpha (L - L_0)^{1-\alpha} | L \leq L(\pi) \},
\]
where parameter \( \alpha \in [0, 1] \) gives the downstream firm’s profit weight.\(^2\)

**Proof:** See for instance Eichberger (1993), Theorem 9.2 \(\Box\)

In order to apply Lemma 1, we investigate under which conditions the bargaining frontier is concave.

**Lemma 2.** A necessary condition for the bargaining frontier to be concave is that the upstream firm’s second-order condition
\[
\frac{d^2 L}{dw^2} < 0
\]
holds. A sufficient condition for the bargaining frontier to be concave is that the derived demand is concave, that is,
\[
\frac{d^2 x^*}{dw^2} < 0.
\]

**Proof:** We investigate under which conditions \( \frac{d^2 L}{d\pi^2} < 0 \) holds. The chain rule gives
\[
\frac{d}{dw} \left[ \frac{dL(\pi(w))}{d\pi(w)} \right] = \frac{d}{d\pi(w)} \left[ \frac{dL(\pi(w))}{d\pi(w)} \right] \cdot \frac{d\pi(w)}{dw}
\]

\(^2\)Strictly speaking, \( \alpha \) denotes the weight on the downstream firm’s gain from trade.
such that
\[
\frac{d^2 L(\pi(w))}{d\pi^2} = \frac{d}{d\pi \frac{dL(\pi(w))}{dw}} = \frac{d\left[ \frac{dL(\pi(w))}{d\pi} \right]}{dw} \left( \frac{d\pi(w)}{dw} \right)^{-1} = \frac{d}{dw} \left[ \frac{dL}{dw} \left( \frac{d\pi}{dw} \right)^{-1} \right] \left( \frac{d\pi}{dw} \right)^{-1} = \left[ \frac{d^2 L}{dw^2} \left( \frac{d\pi}{dw} \right)^{-1} + \frac{dL}{dw} (-1) \left( \frac{d\pi}{dw} \right)^{-2} \frac{d^2 \pi}{dw^2} \right] \left( \frac{d\pi}{dw} \right)^{-1}
\]
\[
= \left[ \frac{d^2 L}{dw^2} - \frac{dL}{dw} \left( \frac{d\pi}{dw} \right) \left( \frac{d^2 \pi}{dw^2} \right) \left( \frac{d\pi}{dw} \right) \right] \left( \frac{d\pi}{dw} \right)^{-2},
\]
where
\[
\frac{d^2 \pi}{dw^2} = \frac{d(d\pi/dw)}{dw} = \frac{d(-x)}{dw} = \frac{-dx}{dw} > 0.
\]
Thus, \(d^2 L/d\pi^2 < 0\) is a necessary condition for the bargaining frontier to be concave. To derive condition (9) note that \(d^2 L/d\pi^2 < 0\) is equivalent to
\[
2 \frac{dx}{dw} + w \frac{d^2 x}{dw^2} < \left( x + w \frac{dx}{dw} \right) \frac{1}{x} \frac{dx}{dw} = (1 + \epsilon) \left( \frac{dx}{dw} \right),
\]
or
\[
\frac{d^2 x}{dw^2} < \frac{dx}{w} \frac{\epsilon - 1}{w}
\]
This holds if the derived demand is not too convex, and, in particular, if \(d^2 x/dw^2 < 0\).

\[\square\]

**Assumption.** The derived demand is concave, that is, \(d^2 x^*/dw^2 < 0\) holds.

Given the preceding Assumption holds (and given that the outside options for both firms are sufficiently small) the Nash bargaining solution is given by the maximum of the Nash product
\[
N = (\pi - \pi_0)^\alpha \cdot (L - L_0)^{1 - \alpha}
\]
subject to \(L \leq L(\pi)\).

The slope of the iso-Nash-product lines is given by the total differential of the Nash product
\[
dN = \alpha(\pi - \pi_0)^{\alpha - 1} \cdot (L - L_0)^{1 - \alpha} d\pi + (\pi - \pi_0)^\alpha \cdot (1 - \alpha)(L - L_0)^{-\alpha} dL = 0.
\]

Rearranging gives the slope of the objective function as
\[
\frac{dL}{d\pi} = \frac{\alpha}{(1 - \alpha) (\pi - \pi_0)} (L - L_0)
\]

Pareto-optimality implies \(L = L(\pi)\) and thus \(dL/d\pi = dL(\pi)/d\pi\). Using (11) and (7), we can equate the slopes of the objective function and of the bargaining frontier and obtain
\[
L - L_0 = \frac{1 - \alpha}{\alpha} \cdot (1 + \epsilon) \cdot (\pi - \pi_0).
\]

7
**Theorem.** Suppose a retailer and a supplier bargain over a linear input price via Nash bargaining while the retailer sets quantities in the final goods market. Then, in equilibrium the relation between profit shares, profit weights and the derived demand elasticity is given by (12).

Note that this formula gives non-negative profits for the upstream and the downstream firm as \( \epsilon \geq -1 \). It gives an equilibrium condition and states that the higher the derived demand elasticity in equilibrium is, the larger is the profit share of the downstream firm. In principle, it can be used to estimate empirically the profit weights of the different parties: if the firms’ profits can be observed and if the derived demand elasticity was known, then the parties’ bargaining power could be estimated. The derived demand elasticity, however, is typically unknown. Therefore, we show in the following that it is closely related to the final good demand elasticity which is often determined in empirical studies.\(^3\)

### 2.4 Demand Elasticity and Derived Demand Elasticity

In order to derive a relationship between the demand elasticity and the derived demand elasticity, we distinguish between the *demand function* \( x(p) \) and the *derived demand function* \( x(w) \).

Demand elasticity is defined by

\[
\eta = \left( \frac{dp}{dx} \right)^{-1} \frac{p}{x}.
\]

We say that (derived) demand elasticity increases if \( \eta (\epsilon, \text{resp.) } \) increases in absolute value.

Equations (2) and (3) yield the following relationship between the demand and the derived demand elasticity:

\[
\epsilon = \frac{1 + \eta}{\frac{p^2}{p^2} x + 2}.
\]

For instance, for linear final demand, the relation between the elasticities is linear, such that with known profits and known equilibrium final demand elasticity, (12) and (13) allow to estimate the parties’ bargaining power.

**Symmetric Nash Bargaining.** Under symmetric Nash bargaining where both parties have no outside option we can derive a more explicit relation between \( \epsilon \) and \( \eta \). Under symmetric Nash

\(^3\)Other papers such as Grennan (2013, 2014) have derived similar interpretations of the Nash bargaining solution (see also Gaudin, 2016). None of these papers, however, stresses the relation to the derived demand elasticity which we focus on.
bargaining, the Nash product

\[ [(p(x(w)) - w)x(w)] \cdot [w \cdot x(w)]. \] (14)

is maximized. Considering the first-order conditions and applying equations (2) and (3) gives

\[ 3p'p + xp''p + x(p')^2 = -2xp' \cdot (p''x + 2p'). \]

Using this as well as the equations (2) and (3), we can re-write the sum of the two elasticities as

\[ \epsilon + \eta = \frac{3p'p + xp''p + x(p')^2}{xp'(p''x + 2p')} = -2. \] (15)

Thus, under symmetric bargaining the two elasticities add up in equilibrium to -2. As the derived demand elasticity lies between -1 and 0, final demand is always elastic in equilibrium, that is, \( \eta \in (-2, -1) \).

**Example: Linear Demand**  Suppose that the downstream firm faces a linear inverse demand function \( p(x) = a - bx \). The outside options are zero for both firms, \( \pi_0 = L_0 = 0 \). The downstream firm’s profit is equal to \( \pi = (a - bx - w)x \), while the supplier gets \( L = wx \). In equilibrium, the downstream firm’s first-order condition \( a - 2bx - w = 0 \) holds, which gives rise to the derived demand

\[ x^*(w) = \frac{a - w}{2b}. \] (16)

As Condition (9) holds, the bargaining frontier is concave and a unique Nash bargaining solution exists. Substituting (16) into the downstream firm’s and the supplier’s profit functions gives \( \pi(w) = (a - w)^2/(4b) \), and \( L(w) = w(a - w)/(2b) \), respectively. Moreover, from (16) we obtain the derived demand elasticity

\[ \epsilon = -\frac{w}{a - w}. \]

Using (12), we obtain the bargaining solution \( w^* = a(1 - \alpha)/2 \) and \( x^* = a(1 + \alpha)/(4b) \). In particular, both the derived demand and the demand elasticity depend only on firms’ bargaining power and equal

\[ \epsilon = -\frac{1 - \alpha}{1 + \alpha}, \]

\[ \eta = -\frac{3 - \alpha}{1 + \alpha}. \]
In equilibrium, the downstream firm’s share is also independent of the demand function’s parameters $a$ and $b$ as

$$\frac{\pi}{\pi + L} = \frac{1 + \alpha}{2(1 - \alpha)}$$

holds. Thus, the sharing rule between the up- and the downstream firm is independent from the exact specification of the linear demand function. Note that, under symmetric bargaining, in particular Equation (15) is satisfied, that is, the demand elasticities sum up to $-2$ in equilibrium.

3 Extensions

We show that our equilibrium condition holds also in more general setups, for instance, if there are $N > 1$ downstream firms. We provide the analysis both for the case with unobservable and with observable contracts.

3.1 $N$ Downstream Firms and Unobservable Contracts

We extend our model toward $N$ downstream firms facing a single upstream firm $U$. As in the previous section, we normalize $U$’s marginal production cost to zero and assume that all firms have the same production technology which transforms one unit of input to one unit of output. Firm $i \in \{1, \ldots, N\}$ produces quantity $x_i$ of a homogeneous product. Demand is given by the inverse demand function $p(x_1, \ldots, x_N)$ for which the $N$-firm analogon of condition (1) is assumed to hold.

In the first stage of the game, $U$ bargains simultaneously with the $N$ downstream firms. We follow the literature on simultaneous Nash bargaining (see, for instance, Inderst and Wey, 2003) and assume that $U$ bargains which the downstream firms through sales agents, that is, for each downstream firm there is one sales agent representing firm $U$ in the negotiation. In the second stage, downstream firms compete à la Cournot.

As contracts are unobservable, the sales agents and the downstream firms cannot observe outcomes in the other negotiations and therefore have to form beliefs on them. Most common in the economic literature on multilateral contracting are “passive beliefs” according to which it is assumed that all unobservable bargaining outcomes are equilibrium outcomes, even if it receives an out-of-equilibrium offer (see Hart and Tirole, 1990; O’Brien and Shaffer, 1992; Inderst
and Ottaviani, 2012).\(^4\) In order to guarantee that the Nash product is well-defined, we assume that a sales agent and the downstream firm he bargains with have the same beliefs on the outcomes of all simultaneous negotiations. Denote \(\hat{w}_j\) firm \(i\)'s and the respective sales agent’s belief about firm \(j\)'s negotiated input price.

We solve the game via backward induction. If downstream firm \(i\) has negotiated input price \(w_i\), it expects to get a profit of

\[
\pi_i(w_i) = [p(x_i^*(\hat{w}_1), \ldots, x_{i-1}^*(\hat{w}_{i-1}), x_i^*(w_i), x_{i+1}^*(\hat{w}_{i+1}), \ldots, x_N^*(\hat{w}_N)) - w_i]x_i^*(w_i),
\]

while the upstream firm \(U\) expects to get

\[
L(w_i) = w_i x_i^*(w_i) + \sum_{i \neq j} \hat{w}_j x_j^*(\hat{w}_j).
\]

The best-response function of firm \(i\) solves the first-order condition

\[
p - w_i = -\frac{\partial p}{\partial x_i} x_i.
\]

Firm \(i\)'s equilibrium quantity choice can be written as

\[
x_i^*(w_i) = x_i(\hat{w}_1, \ldots, \hat{w}_{i-1}, w_i, \hat{w}_{i+1}, \ldots, \hat{w}_N).
\]

Note that, in particular, firm \(i\)'s equilibrium quantity \(x_i^*\) depends only on its own and not on its rivals’ input prices.

When bargaining with firm \(i\), \(U\)'s outside option is \(L_{i,0} = L(\hat{w}_1, \ldots, \hat{w}_{i-1}, 0, \hat{w}_{i+1}, \ldots, \hat{w}_N)\) while we set the disagreement point of the downstream firms to zero. Thus, we can write the Nash bargaining problem between the supplier and firm \(i\) as

\[
N_i(w_i) = (\pi_i(w_i))^\alpha \cdot (L(w_i) - L_{i,0})^{1-\alpha}.
\]

If the Nash product is maximized, the first-order condition

\[
\frac{dL(w_i)}{dw_i} \frac{d\pi_i(w_i)}{dw_i} = -\frac{\alpha}{1-\alpha} \frac{(L(w_i) - L_{i,0})}{\pi_i(w_i)}
\]

\(^4\)Besides passive beliefs, also symmetric and wary beliefs are analyzed in the literature (Rey and Vergé, 2004). If firm \(i\) has symmetric beliefs, negotiated prices are assumed to be identical for all firms, that is, \(w_j(w_i) = w_i\). With wary beliefs, if firm \(i\) negotiates an input price \(w_i \neq w^*\) with the upstream firm, then firm \(i\) believes that \(w_j(w_i)\) maximizes the Nash product of \(U\) bargaining with firm \(j\), conditional on \(U\) knowing \(w_i\).
holds. From (18) we obtain
\[
\frac{dL(w_i)}{dw_i} = x^*_i(w_i) + \frac{dx^*_i(w_i)}{dw_i} - w_i. \tag{21}
\]
Using firm $i$'s derived demand elasticity $\epsilon_i = \frac{dx^*_i(w_i)}{dw_i} \frac{w_i}{x^*_i(w_i)}$, we can rewrite (21) as
\[
\frac{dL(w_i)}{dw_i} = x^*_i(w_i) (1 + \epsilon_i). \tag{22}
\]
Similarly, (17) yields
\[
\frac{d\pi_i(w_i)}{dw_i} = \frac{\partial p}{\partial x_i} \frac{dx^*_i(w_i)}{dw_i} x^*_i(w_i) - x^*_i(w_i) + \frac{dx^*_i(w_i)}{dw_i} (p - w_i).
\]
Using (19) we then obtain
\[
\frac{d\pi_i(w_i)}{dw_i} = -x^*_i(w_i). \tag{23}
\]
Inserting (22) and (23) into (20) yields the equilibrium profit of the downstream firm $i$ as
\[
\pi^*_i = (L^* - L_{i,0}) \frac{\alpha}{(1 - \alpha)} \frac{1}{(1 + \epsilon_i)}. \tag{24}
\]
where, with passive beliefs, $L_{i,0} = \sum_{j \neq i} w_j^* x_j^*$ and $L^* - L_{i,0} = w_i^* x_i^*$. Thus, the equilibrium condition (12) derived for the bilateral monopoly case generalizes to $N$ downstream firms if contracts are not observable. Note, however, that $\epsilon$ in (12) stands for the overall demand’s elasticity with respect to input prices while $\epsilon_i$ denotes the elasticity of firm $i$'s derived demand with respect to its input price.

### 3.2 $N$ Downstream Firms and Observable Contracts

We repeat the preceding analysis for the case in which contracts are observable. Profits are given by $\pi_i := p(x_1, ..., x_N)x_i - w_i x_i$ and $L := \sum_{i=1}^N w_i x_i$, and the first-order condition (19) holds. As quantities are observable, firm $i$'s equilibrium quantity choice can be written as $x^*_i(w_1, ..., w_N)$. We assume that the second order condition holds.

We can write the downstream firm $i$'s profit as
\[
\pi_i(w_1, ..., w_N) = [p(x_1^*(w_1, ..., w_N), ..., x_N^*(w_1, ..., w_N)) - w_i]x_i^*(w_1, ..., w_N), \tag{25}
\]
while the upstream firm's profit equals
\[
L(w_1, ..., w_N) = \sum_{i=1}^N w_i x_i^*(w_1, ..., w_N). \tag{26}
\]
The general Nash bargaining problem between the supplier and firm $i$ is given by

$$N_i(w_i) = (\pi_i(w_1, ..., w_N) - \pi_{i,0})^\alpha \cdot (L(w_1, ..., w_N) - L_{i,0}(w_1, w_{i-1}, w_{i+1}, ..., w_N))^{1-\alpha}, \quad (27)$$

where $\pi_{i,0}$ is firm $i$’s the outside option and $L_{i,0}(w_j)$ is the outside option of the upstream firm when it bargains with firm $i$. As before, $\pi_{i,0} = 0$.

If the Nash product is maximized, the first-order condition

$$\frac{dL(w_1, ..., w_N)}{dw_i} \frac{d\pi_i(w_1, ..., w_N)}{dw_i} = -\frac{\alpha}{(1 - \alpha)} \frac{L - L_{i,0}}{\pi_i}. \quad (28)$$

holds. Formula (26) gives

$$\frac{dL(w_1, ..., w_N)}{dw_i} = x_i + \frac{dx_i(w_1, ..., w_N)}{dw_i} w_i + \sum_{j \neq i} \frac{dx_j(w_1, ..., w_N)}{dw_i} w_j. \quad (29)$$

Using firm $i$’s elasticity of derived demand, $\epsilon_i = \frac{dx_i w_i}{dw_i x_i}$, and the cross-price elasticity of derived demand, $\epsilon_{ji} = \frac{dx_j w_j}{dw_i x_i}$, we can rewrite (29) as

$$\frac{dL(w_1, ..., w_N)}{dw_i} = x_i \left(1 + \epsilon_i + \sum_{j \neq i} \epsilon_{ji} \frac{x_j w_j}{x_i w_i} \right). \quad (30)$$

Similarly, (25) yields

$$\frac{d\pi_i(w_1, ..., w_N)}{dw_i} = \frac{\partial p(x_1, ..., x_N)}{\partial x_i} \frac{dx_i}{dw_i} x_i + \sum_{j \neq i} \frac{\partial p(x_1, ..., x_N)}{\partial x_j} \frac{dx_j}{dw_i} x_i - x_i + \frac{dx_i}{dw_i} (p - w_i). \quad (31)$$

Inserting (19) into the preceding equation gives

$$\frac{d\pi_i(w_1, ..., w_N)}{dw_i} = \sum_{j \neq i} \frac{\partial p(x_1, ..., x_N)}{\partial x_j} \frac{dx_j}{dw_i} x_i - x_i, \quad (32)$$

or,

$$\frac{\partial \pi_i(w_1, ..., w_N)}{\partial w_i} = -x_i \left(1 - \sum_{j \neq i} \frac{p}{w_i} \eta_j \epsilon_{ji} \right), \quad (33)$$

where $\eta_j = \frac{\partial p(x_1, ..., x_N)}{\partial x_j} \frac{x_j}{p}$ gives firm $j$’s elasticity of demand. Inserting (30) and (33) into (28) yields

$$\frac{\alpha}{(1 - \alpha)} \frac{L - L_{i,0}}{\pi_i} = \left(1 + \epsilon_i + \sum_{j \neq i} \epsilon_{ji} \frac{x_j w_j}{x_i w_i} \right) \left(1 - \sum_{j \neq i} \frac{p}{w_i} \eta_j \epsilon_{ji} \right). \quad (34)$$
As the downstream firms are symmetric, we assume a symmetric Nash solution in which \( w_i^* = w_j^* \) and \( x_i^* = x_j^* \), for any \( i, j \in 1, \ldots, N \). We can write equilibrium profit of the downstream firm \( i \) as

\[
\pi_i^* = (L - L_i,0) \alpha \left( 1 - (N - 1) \frac{L_{ij}}{w_i \eta_j \epsilon_{ji}} \right) \left( 1 + \epsilon_i + (N - 1) \epsilon_{ji} \right). \tag{35}
\]

As a consequence, with observable contracts, the profit sharing rule does not only depend on the elasticity of derived demand, but also on the cross-price elasticities of the derived demand.

4 Conclusion

We have established a novel link between the profit shares and the demand elasticity in vertical relations if up- and downstream firms bargain over linear input prices. Besides the disagreement payoffs and the weights of firms’ profits in the Nash product, our formula singles out the slope of the bargaining frontier as an additional determinant of bargaining power. The slope of the bargaining frontier is equal to the total value of one plus the downstream firm’s derived demand elasticity. We have provided various examples in which a more elastic equilibrium demand benefits the downstream firm through a change of the slope of the bargaining frontier. Our model should be instructive also for empirical studies which seek to determine the bargaining power of the different parties based on observables such as absolute profit levels and equilibrium demand elasticity.

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<table>
<thead>
<tr>
<th>No.</th>
<th>Authors</th>
<th>Title and Additional Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>219</td>
<td>Schain, Jan Philip and Stiebale, Joel</td>
<td>Innovation, Institutional Ownership, and Financial Constraints, April 2016.</td>
</tr>
<tr>
<td>214</td>
<td>Dertwinkel-Kalt, Markus and Riener, Gerhard</td>
<td>A First Test of Focusing Theory, February 2016.</td>
</tr>
</tbody>
</table>


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