Sequencing Bilateral Negotiations with Externalities

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Discussion Paper #2026/08

DFG Research Unit: “Consumer Preferences, Consumer Mistakes, and Firms’ Response” (FOR 5392)

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July 2026

Abstract

In most bargaining situations, the negotiation sequence is crucial for efficiency and parties' success. However, the determinants of optimal sequencing are poorly understood. We develop a general framework of bilateral negotiations between a principal and two agents with externalities. We establsih that agents' bargaining power is a key driver of sequence choice. Total surplus is maximized when the principal negotiates first with the stronger agent. Under negative externalities, the principal selects this efficient sequence. With positive externalities, however, the principal often negotiates first with the weaker agent, generating potentially substantial efficiency losses. These results remain robust across several extensions.

Keywords: bargaining power, sequential negotiations, externalities, bilateral contracting, endogenous timing

JEL-codes: C72, C78, D62, L14

*We thank Helmut Bester, Felix Bierbrauer, Francis Bloch, Tilman Börgers, Doruk Cetemen, Germain Gaudin, Paul Heidhues, Simon Loertscher, Sebastian Mertesacker, Meg Meyer, Benny Moldovanu, José Luis Moraga-González, Volker Nocke, Régis Renault, Marco Sahm, Odd Rune Straume, Roland Strausz, Thomas Tröger, Eyal Winter, and seminar participants at the Universities of Bamberg, Berlin (Humboldt), Bielefeld, Cergy-Pontoise, Collegio Carlo Alberto (Torino), Cologne, Dusseldorf, Freiburg, Groningen, Mannheim, Oxford, Paris School of Economics, Pompeu Fabra (Barcelona), Porto, the Committee on Economic Theory of the German Economic Asociation (Bonn), the Workshop on Advances in Industrial Organization (Bergamo), the Padova IO Workshop, and the UECE Meetings (Lisbon) for very helpful comments and suggestions. Johannes Münster gratefully acknowledges: Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy −EXC 2126/2− 390838866. Markus Reisinger also gratefully acknowledges financial support from the German Research Foundation (DFG project 462020252). Errors are ours.

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1 Introduction

The question how to sequence bilateral negotiations is ubiquitous. For example, to ensure the delivery of a sufficient number of vaccine doses during the Covid-19 pandemic, the European Commission sequentially secured deals with vaccine manufacturers, such as Johnson and Johnson, BioNTech Pfizer, and AstraZeneca, each consisting of an agreement to deliver several million doses of vaccine.[1] The negotiations were carried out by a team of the European Commission bilaterally with each manufacturer. As manufacturers are competitors in the vaccine market, multilateral negotiations would have been illegal. Although the specific terms of each contract remain confidential, the commission publicly announced the quantity of vaccine doses after an agreement was reached.

A similar problem as the one faced by the European Commission—i.e., how to sequence the deals with the different vaccine manufacturers—also arises e.g. for a supplier who can distribute its product through several retailers, for a landlocked country that negotiates import quotes with neighboring countries which do not have a common border, or an entrepreneur who negotiates with different venture capital firms to raise money for her business. Indeed, as many negotiation consultants (e.g., Wheeler, 2005; Lax and Sebenius, 2012) point out, the proper sequence of the negotiations is one of most crucial choices for negotiators. Sebenius (1996) provides examples of several different situations—ranging from land acquisitions to political endorsements—in which negotiation leaders were confronted with the question whether to approach "harder" or "easier" players first.

An implication of the last statement is that a key variable driving the sequencing choice is the difference in parties' bargaining powers. Several recent studies indeed show that there are substantial asymmetries in the bargaining strengths of (otherwise similar) players. For example, Grennan (2013, 2014), using data from the market for medical devices, finds that the bargaining ability of hospitals in the negotiation with suppliers is responsible for almost 80% of the price variation across hospitals. Karsten et al. (2022) and De los Santos et al. (2026) obtain similar results for negotiations of company acquisitions and for negotiations in the e-book market, respectively. In particular, Karsten et al. (2022) find that better negotiation skills and experience of lawyers lead to favorable target prices in acquisition contracts, and De los Santos et al. (2026) demonstrate that publishers, such as Harper Collins or Penguin Random House, are highly asymmetric in their bargaining power in the

[1] The first agreement was reached with AstraZeneca on August 14, 2020, consisting of 300 million doses, followed by the agreement with Johnson and Johnson on October 8, 2020, consisting of 200 million doses. After further agreements with BioNTech Pfizer and CureVac, the last agreement was reached on November 25, 2020, with Moderna for 80 million doses. See e.g. the document "Questions & Answers on vaccine negotiations" by the European Commission, available at https://ec.europa.eu/commission/presscorner/detail/en/QANDA_21_48.

negotiation with e-book sellers Amazon and Barnes & Noble.

Yet, in the literature on bilateral negotiations, bargaining power plays often only a secondary role.[2] This is due to the fact that, under symmetric information, bargaining power does not affect the efficiency of agreements because the involved parties maximize the joint negotiation surplus regardless of the bargaining strength of each player. The distribution of bargaining power then only determines the share of the surplus that each player obtains.[3]

In this paper, we show that this is no longer true in case the party who negotiates with all others (the *principal* in what follows) can decide about the bargaining sequence. In particular, the chosen sequence is crucially determined by the bargaining strengths of the *agents* (the parties who negotiate with the principal), and different sequences usually lead to different agreements, thereby affecting the total welfare generated by the bargaining process. This result then leads to several interesting questions, such as: How does the sequence chosen by the principal compare to the surplus-maximizing sequence? How do externalities between agents shape the results? Under which conditions does the principal prefer simultaneous negotiations?[4]

To answer these questions, we consider a general framework in which a principal negotiates with two agents who differ in their bargaining power. (Our main insights carry over to a general number of agents, as we show in Section 7.2). Bargaining power of a player is captured by the probability of making a (take-it-or-leave-it) offer to the counter party.[5] The principal chooses the bargaining sequence, that is, whether to negotiate with the weak or the strong agent first. Each negotiation is over a contract that fixes a decision—e.g., a quantity to be traded—and a transfer. While, in general, there may be an incentive to renegotiate a contract signed in the first negotiation after the second negotiation, in practice, requirements of time or legal costs often preclude renegotiation, and we therefore focus on this case.

We allow for positive and negative externalities between agents, and also interaction of the traded quantities in the principal's utility function.[6] For the sake of exposition, we

[2]For instance, the literature on contracting with externalities has mainly focused on situations in which bargaining power is concentrated either on the principal's side (e.g., Segal, 1999; Genicot and Rey, 2006; Möller, 2007) or the agents' side (e.g., Bernheim and Whinston, 1986).

[3]If there is asymmetric information between bargainers, the distribution of bargaining power may affect the outcome. See e.g. Chatterjee and Samuelson (1983) or Sobel and Takahshi (1983).

[4]Understanding how such sequencing decisions are made may also be important for structural empirical analysis of negotiations. For empirical papers explicitly modeling negotiations between parties, see e.g. Crawford and Yurukoglu (2012), Gowrisankaran et al. (2015), or Ho and Lee (2017).

[5]Such a random proposer structure is commonly used to represent bargaining strength in trading situations (e.g., Zingales, 1995; Inderst, 2001; Ali, 2015;) or political processes (e.g., Baron and Ferejohn, 1989; Eraslan and Evdokimov, 2019; Britz and Gersbach, 2026), and is sometimes referred to as the recognition probability of a player. In our game, each bilateral negotiation is also equivalent to asymmetric Nash bargaining with the respective bargaining weights, and can be interpreted as such.

[6]The latter naturally occurs if, for example, the principal is a supplier with a non-linear cost function, or

assume that, if the principal fails to reach an agreement with an agent, this agent's payoff is independent of the outcome in the other negotiation. This assumption seems natural e.g. in the situations described above.[7] To trace out the effect of asymmetric bargaining strengths, we derive our main results under the assumption that agents are symmetric except for bargaining power.

We first show that the sequence that generates a larger surplus is the one in which the principal bargains first with the agent who has *higher bargaining power*. This result holds *regardless* of whether externalities between agents are positive or negative. The intuition is based on the effect that the principal obtains only a fraction of the surplus in the second-stage negotiation. The bargainers in the first stage, therefore, consider the second-stage surplus only partially. However, they do so to a larger extent if the principal has more bargaining power in the second stage. Due to this *partial-surplus effect*, the distortion in the first negotiation relative to the efficient outcome is smaller when negotiating first with the strong agent.

We then study the sequence chosen by the principal. We find that the principal chooses the sequence that generates the larger surplus if externalities are negative, but may choose an inefficient timing when externalities are positive. In addition to the partial-surplus effect, the principal's preferred sequence is driven by an additional effect: the *anticipated-externality effect*.

This effect occurs because the payoff of the agent with whom the principal bargains first depends on the outcome in the second negotiation. The agent anticipates that an agreement will be reached in the second stage. Therefore, if the principal proposes in the first negotiation, she can demand a transfer that depends on this anticipated externality.[8] Instead, when the agent proposes, the principal obtains her outside option, which is the payoff from rejecting the offer of the first agent and negotiating only with the remaining agent. This payoff does not depend on any externality. When the principal bargains with the weak agent first, she proposes with a high probability and, hence, is likely to bear the externality herself. Therefore, bargaining with the weak agent first is attractive for the principal when externalities are positive, and the reverse holds for negative externalities.

The principal's preferred sequence depends on the interplay of the two effects. With negative externalities, both effects favor the sequence of negotiating first with the strong agent. The chosen sequence is therefore the efficient one. The same result occurs if externalities between agents are absent, but the decisions interact in the principal's utility function.

a buyer, whose willingness-to-pay depends on the aggregate quantity.

[7] Our results, however, are robust to such externalities. We show this in the Working Paper version available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=7034238.

[8] The feminine pronoun denotes the principal and the masculine ones the agents.

Then, the anticipated-externality effect is not present, but the partial-surplus effect is at work. This holds regardless of the type of interaction between the decisions (i.e., whether the principal's utility function is super-modular or sub-modular).

With positive externalities, both effects are at work and point in opposite directions. Therefore, the result is no longer clear-cut. If externalties are small and the principal's utility function is additively separable, however, the anticipated-externality effect dominates. The intuition is that the partial-surplus leads to distortions in the first-stage decision, which are of second order if externalities are small. By contrast, the anticipated-externality effect has a first-order effect on the agents' utility. The principal therefore chooses an inefficient sequence, as this allows her to obtain a larger share of a smaller pie. The resulting loss in surplus can be substantial and may even result in an outcome with almost zero surplus, despite the fact that the potential gains from the negotiations are large.

If externalities are positive and large, both effects are sizable. The principal still prefers to bargain with the weaker agent first if e.g. equilibrium decisions are bounded, as the importance of the partial-surplus is then limited. However, the opposite result may also occur, leading to a non-monotonicity in the sequence, as the principal prefers to bargain with the strong agent first if externalities are negative and if they are highly positive, but with the weak agent first if externalities are moderately positive.

In summary, the externalities between agents and the interaction of the decisions in the principal's utility function shape the sequence of negotiations. Whereas the sign of the externalities (i.e., negative or positive) is crucial for the principal's preferred sequence, the type of interaction in the principal's utility function is not. Although, for instance, positive externalities between agents and super-modularity of the principal's utility function both increase the joint surplus, their effect on the equilibrium sequence is highly different: the former favors negotiating with the weak agent first through the anticipated-externality effect, whereas the latter unambiguously favors negotiating with the strong agent first.

Our insights are consistent with practical negotiation guidance, but also go beyond it. In particular, Sebenius (1996) and Lax and Sebenius (2012) point out that the principal should "(s)eek to get the easy parties on board first" in case players benefit from agreements with other players. This is in line with our finding for positive externalities between agents. We also show that this sequence, however, is not efficient from a total surplus perspective. We provide additional guidance by showing that the reverse order is optimal if a player is harmed when the principal reaches an agreement with other players. In addition, we demonstrate that the result for positive externalities no longer holds if the anticipated-externality effect is dominated by the partial-surplus effect.

We also consider simultaneous negotiations. We show that for negative externalities, se-

quential negotiations in which the principal bargains with the stronger agent first dominate simultaneous negotiations. Instead, when externalities are positive, simultaneous negotiations may be optimal for the principal. The intuition is rooted in the fact that, with simultaneous bargaining, agents cannot observe the outcome in the other negotiation. Each agent supposes that an agreement will be reached there. Instead, with sequential negotiations, the agent bargaining at the second stage observes if the bargainers in the first stage failed to reach an agreement. With positive externalities, this effect leads to a higher outside option of the principal with simultaneous negotiations.

Our paper contributes to the literature on contracting with externalities, pioneered by Segal (1999, 2000). He considers the situation in which a principal simultaneously offers contracts to agents in the presence of multi-lateral externalities. In this context, Möller (2007) allows for sequential contracting, and shows that the principal prefers sequential over simultaneous contracting only if externalities on non-traders are sufficiently large.[9] Genicot and Rey (2006) also analyze contracting over time and demonstrate how the principal extracts most surplus from agents by combining simultaneous and sequential offers.[10] These papers consider an *offer game* where the principal has all the bargaining power. Instead, Bernheim and Whinston (1986) and Martimort and Stole (2002, 2003) consider a *bidding game* where the agents simultaneously propose contracts to the principal. Bhaskar and Roketskiy (2021) study the effects of sequential offers in a bidding game. Contrary to these papers, we study a situation with intermediate bargaining power and demonstrate how the bargaining power affects the optimal negotiation sequence.[11]

Several papers consider sequential negotiations in different environments, but take the bargaining sequence as given and assume symmetry between agents. For example, Cai (2000) considers the case in which the principal needs to reach an agreement with all agents to obtain a positive surplus (e.g., a railroad must get permission from all landowners). Chae and Heidhues (2004) consider risk averse buyers and study their incentive to form an alliance to bargain as single entity. Bagwell and Staiger (2010) analyze trade agreement negotiations between countries when most-favored nation clauses are in place. Iaryczower and Oliveros (2017) study collective action problems in which competing principals bargain to seek support

[9] Jehiel and Moldovanu (1995a, 1995b) show that externalities on non-traders can also cause substantial delay in reaching in agreement, both with a finite and an infinite horizon. This result also occurs if renegotiation among agents is possible (Jehiel and Moldovanu, 1999).

[10] Horn and Wolinsky (1988) and Marshall and Merlo (2004) study the difference between simultaneous and sequential negotiations in the context of a union negotiating over wages with two competing firms, and find that sequential bargaining is always preferred by the union.

[11] Galasso (2008) combines the offer and the bidding game in a sequential bargaining model along the lines of Rubinstein (1982). He focuses on negative externalities between symmetric agents and shows that the principal's payoff can be decreasing in her bargaining power, but does not analyze optimal sequencing of negotiations.

of a majority of agents. Abreu and Manea (2026) analyze optimal bargaining of a seller in case it can only serve a subset of buyers.

A few papers analyze the optimal sequencing of negotiations, but focus on different issues than our paper. Noe and Wang (2004) consider a situation in which the principal can keep the order of negotiations confidential, and determine conditions for secrecy being more profitable than public negotiations. Krasteva and Yildirim (2012) analyze a model in which two sellers offer complementary products and the buyer's payoff with only a single agreement is uncertain. They show that the sequence only matters in case of uncertainty, and depends on the degree of complementarity and the extent of asymmetry between buyers.[12] Xiao (2015) endogenizes the bargaining sequence in a model in which a buyer negotiates with several sellers who own complementary goods (similar to Cai, 2000). He shows that the buyer wants to negotiate with small sellers first. None of these papers analyzes how the principal's optimal sequence depends on the direction of the externalities or the interplay between the agents' bargaining strengths and the externalities. In addition, they all consider binary decisions in a negotiation, whereas we allow for continuous decisions.[13]

The remainder of the paper is organized as follows: Section 2 sets out the model. Section 3 presents some preliminary results. Section 4 considers the bargaining sequence that generates the highest surplus for the players. Section 5 analyzes the sequence chosen by the principal. Section 6 considers simultaneous negotiations. Section 7 presents three extensions: first, an analysis in which agents differ not only in their bargaining power, second, the case with a general number of agents, and, finally, an analysis of exclusive dealing contracts. Section 8 concludes. The Appendix contains the proofs of the results in Sections 4 and 5, whereas the Online Appendix contains the proofs of the results in Sections 6 and 7.

2 The Model

Assumptions. There are three players: a principal A ("she") and two agents B and C ("he").[14] A and B negotiate over a decision $b \in \mathcal{B} \subseteq \mathbb{R}_+$, with $0 \in \mathcal{B}$, and a monetary transfer $t_B \in \mathbb{R}$ from B to A (i.e., the transfer can be positive or negative). Similarly, A and C negotiate over a decision $c \in \mathcal{C} \subseteq \mathbb{R}_+$, with $0 \in \mathcal{C}$, and a transfer $t_C \in \mathbb{R}$. The payoff of the principal is $u_A(b,c) + t_B + t_C$, and the payoffs of the agents are $u_B(b,c) - t_B$ and

[12] Marx and Shaffer (2007, 2010) and Raskovich (2007) also analyze a situation with one buyer who bargains with two sellers, but consider the case in which there are no direct externalities between sellers.

[13] Sequencing has also been studied in the literature on agenda formation (e.g., Winter 1997). However, sequencing here refers to the order of different issues.

[14] As mentioned above, in Section 7.2, we analyze the case with N agents and show that our main insights carry over.

$u_C(b,c) - t_C$, respectively. We assume that the sets $\mathcal{B}$ and $\mathcal{C}$ are compact and that the utility functions u_i $(i = A, B, C)$ are continuous on $\mathcal{B} \times \mathcal{C}$.

Negotiations are bilateral, and the sequence is chosen by A. Within each stage, there is random proposer take-it-or-leave-it bargaining. Bargaining power is modelled as the probability of making the offer: B proposes with probability $\beta \in [0,1]$, and C proposes with $\gamma \in [0,1]$.[15] We assume that $\beta > \gamma$, that is, B is the stronger bargainer among the agents.[16] As it is the objective of the paper to analyze which agent the principal will approach first, we follow the literature on sequencing decisions and rule out renegotiation. For instance, Iaryczower and Oliveros (2017) point out that sequential contracting without renegotiation is prevalent in inter-firm bargaining or political endorsements. Möller (2007) and Montez (2014) explain that renegotiation is often infeasible, as it involves high legal costs.

The timing of the game is as follows. In stage 0, A chooses whether to bargain with B first (sequence BC) or with C first (sequence CB). In sequence BC, in stage 1, A bargains with B. With probability β, B proposes a contract $(b, t_B) \in \mathcal{B} \times \mathbb{R}$, and A either accepts or rejects. With probability $1-\beta$, A proposes, and B then accepts or rejects. If A and B reach an agreement on a contract (b, t_B), the decision b is implemented and the transfer t_B is made. In case of rejection, $b = t_B = 0$. In stage 2, C observes the decision of stage 1.[17] Then, A and C bargain. With probability γ, C proposes a contract $(c, t_C) \in \mathcal{C} \times \mathbb{R}$; with probability $1-\gamma$, A proposes. If they reach an agreement on a contract (c, t_C), the decision c is implemented and the transfer t_C is paid. Otherwise, $c = t_C = 0$. Sequence CB is similar, except that A bargains with C in stage 1 and with B in stage 2.

In our bargaining game, the principal negotiates with one agent at a time. This is a highly relevant situation because negotiations often require physical presence of the principal, and it is too costly to communicate with all agents at the same time. However, there can be circumstances in which the principal can delegate the negotiations, which allows for the possibility of simultaneous bargaining. We consider this case in Section 6.

Our assumption on the contract space implies that the contract negotiated in stage 1 cannot condition on actions chosen in stage 2. This is motivated by the fact that such contingent contracts are rare and difficult to enforce (see e.g., Möller, 2007). In addition,

[15] An alternative interpretation is that the outcome of each negotiation is determined via the Nash bargaining solution with bargaining weights β for B and $1-\beta$ for A in the negotiation between A and B, and γ for C and $1-\gamma$ for A in the negotiation between A and C (see, for example, Muthoo 1999).

[16] As outlined in the Introduction, capturing different negotiation strengths through differences in the bargaining power of agents is natural in many situations. We note that it is not equivalent to different outside options by the agents, and explain this at the end of the next section.

[17] In the Woring Paper version (available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=7034238), we consider the situation in which C does not observe this decision, and A can decide whether to disclose it or not. We show that disclosure occurs in equilibrium under a wide array of circumstances.

if A is an upstream firm serving two retailers B and C, a contract between A and B that conditions on c is usually prohibited by competition law (Hart and Tirole, 1990; Rey and Tirole, 2007).[18]

We assume that there are no externalities on non-traders: $u_B(0,c)$ is constant in c, $u_C(b,0)$ is constant in b, and we normalize the utility functions such that $u_A(0,0) = u_B(0,c) = u_C(b,0) = 0$. This is a natural assumption in most of the examples given in the Introduction (e.g., a firm's profit in a market in which the firm is not active, is zero and does not depend on the quantity traded in that market).[19]

Externalities between agents are negative if $u_B(b,c) \leq u_B(b,0)$ and $u_C(b,c) \leq u_C(0,c)$ for all $b > 0$ and $c > 0$. Externalities are positive if the inequalities are reversed. Moreover, externalities are strictly negative or strictly positive if the weak inequality signs are replaced by strict inequality signs, and there are no externalities if the inequality signs are replaced with equality signs. At this point, we do not make assumptions on whether externalities are stronger or weaker if b or c are larger, but only whether they are positive or negative as defined above. Towards the end of Section 5, we will put more structure on the externalities to derive further results.

To isolate the impact of differences in bargaining power, we assume some degree of symmetry between B and C. In particular, agents are symmetric except for bargaining power if (i) $\mathcal{B} = \mathcal{C}$, (ii) u_A is a symmetric function, that is, $u_A(b,c) = u_A(c,b)$, and (iii) $u_C(c,b) = u_B(b,c)$. In Section 7.1, we show that our insights *generalize to agent asymmetry*—i.e., agents' decision spaces and utility functions differ, and b and c affect $u_A(\cdot,\cdot)$ asymmetrically—if β is close to 1.

We define the joint surplus of all three players as $S(b,c) := \sum_{i\in\{A,B,C\}} u_i(b,c)$. Finally, we impose the tie-breaking rule that, if A is indifferent between the two sequences, but surplus is strictly higher in one of them, A selects the sequence that generates the higher surplus.

We point out that our basic set-up is quite general. For example, it does not assume differentiability of the utility functions or monotonicity of the externalities in b or c. Similarly, the sets of possible decisions $\mathcal{B}$ and $\mathcal{C}$ can be discrete or continuous.[20] We will make some minor technical assumptions on the structure of the maximization problems in the next

[18] An exception can be exclusive dealing contracts, which allow the principal to commit to deal with only one agent. In Section 7.3, we enrich the contract space to allow for such contracts and show that our results continue to hold.

[19] In the Working Paper version, we show that our results extend to the case where externalities on non-traders are present, but not too large.

[20] Our results also generalize to the case where decisions b and c are not only one-dimensional, but multi-dimensional. In that case, decisions $\mathbf{b}$ and $\mathbf{c}$ are vectors, with $\mathbf{b} \in \mathcal{B} \subseteq \mathbb{R}^{\ell}_{+}$ and $\mathbf{c} \in \mathcal{C} \subseteq \mathbb{R}^{\ell}_{+}$, $\ell \geq 1$. Externalities are then e.g. negative if $u_B(\mathbf{b},\mathbf{c}) \leq u_B(\mathbf{b},\mathbf{0})$ where at least one entry in $\mathbf{c}$ is positive.

section.

Example with a Supplier and Retailers. To provide a concrete example for our framework, suppose that b and c are quantities of an input good sold by supplier A to retailers B and C who pay fixed amounts t_B and t_C for receiving their respective quantity. Retailers transform the input to output, and their production technology is one-to-one. Retailers compete in quantities in the downstream market. Their payoff functions are then $u_B = bp_B(b, c)$ and $u_C = cp_C(c, b)$, respectively, where p_B and p_C are the prices of the products, which depend on the quantities of both retailers. (For simplicity, we set retail costs to zero.) If retailers sell substitutes, p_B is falling in c (and, similarly, p_C is falling in b); hence, externalities are negative. By contrast, if products are complements, externalities are positive since prices are increasing in the quantity of the other retailer. The function u_A describes the supplier's production costs and could be $u_A = -k(b, c)$.[21]

3 Preliminary Analysis

We start by showing the working of the model, that is, how b and c are determined in equilibrium and the resulting payoff for the principal. We then analyze the sequence that generates the higher surplus for the players and the sequence preferred by the principal.

Within each stage, there is take-it-or-leave-it-bargaining with transferable utilities. Thus, the decision maximizes the joint surplus of the two bargainers. Moreover, the proposer chooses a transfer such that the respondent is just willing to accept.

Consider sequence BC (sequence CB can be analyzed similarly). In stage 2, the decision b and transfer t_B are already fixed. The decision reached in stage 2 maximizes the joint surplus of A and C, given b, because whichever player gets to make the offer can extract the surplus of the other player over and above the respective outside option. We assume that, for any b, there exists a unique

$$c^*(b) := \arg\max_{c \in \mathcal{C}} \{u_A(b, c) + u_C(b, c)\},$$

that is, there is a unique value of c that maximizes the joint surplus in stage 2. The existence of an optimal second-stage decision is ensured by our assumptions on the choice sets and the

[21] Our setting can also accommodate strategic interaction after stage 2. For example, b and c could represent the unit prices constituting the variable parts of a two-part tariff, whereas t_B and t_C constitute the fixed parts. To be consistent with our notation that $b = 0$ denotes no trade, we can use an inverse scale of the unit prices. As long as all subgames after stage 2 have unique subgame equilibrium payoffs that (net of transfers) depend only on the decisions (b, c) taken in stages 1 and 2, these payoffs can be expressed by functions u_A, u_B, and u_C, and our model applies.

utility functions.[22] (A sufficient condition for uniqueness is e.g. that $u_A(b,c)+u_C(b,c)$ is strictly quasiconcave in c and $\mathcal{C}$ is convex.)

The expected payoff of A in stage 2 of sequence BC is

$$(1-\gamma)\left(u_A\left(b,c^*\left(b\right)\right)+u_C\left(b,c^*\left(b\right)\right)\right)+\gamma u_A\left(b,0\right)+t_B$$

because, with probability $1-\gamma$, A proposes and extracts C's utility, whereas, with probability γ, C proposes and holds A down to her reservation utility of $u_A(b,0)$. In case $b=t_B=0$, the expected payoff of A in stage 2 is

$$O_A^{BC}=(1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A\left(0,c\right)+u_C\left(0,c\right)\right\}. \tag{1}$$

This is the expected utility of A if the first-stage negotiation with B fails. It is therefore A's outside option in the first stage.

In the first stage of sequence BC, the joint surplus of A and B consists of B's payoff and A's expected payoff in stage 2 (net of t_B):

$$S_{AB}^{BC}\left(b\right):=u_B\left(b,c^*\left(b\right)\right)+(1-\gamma)\left(u_A\left(b,c^*\left(b\right)\right)+u_C\left(b,c^*\left(b\right)\right)\right)+\gamma u_A\left(b,0\right). \tag{2}$$

In an equilibrium of sequence BC, A and B reach a decision $b^{BC}=\arg\max_{b\in\mathcal{B}}S_{AB}^{BC}\left(b\right)$.[23] For simplicity, we assume that there is a unique b^{BC}. The expected payoff of A is

$$U_A^{BC}=(1-\beta)\,S_{AB}^{BC}\left(b^{BC}\right)+\beta O_A^{BC}.$$

We impose the corresponding assumptions on sequence CB, and denote the equilibrium first-stage decision in sequence CB by c^{CB}. The principal (weakly) prefers sequence BC over CB if and only if $U_A^{BC}\geq U_A^{CB}$.

Before proceeding with the analysis of the optimal sequence (for the joint surplus and for the principal), two remarks are in order. First, if the optimal second-stage decision is zero—e.g., because the bilateral surplus net of A's outside option is negative—then the two bargainers in the second stage will not reach an agreement. This is, for instance, the case if in sequence BC, the equilibrium first-stage decision is $b^{BC}=\arg\max_b\left\{u_A\left(b,0\right)+u_B\left(b,0\right)\right\}$ and the equilibrium second-stage decision is $\arg\max_c\left\{u_A\left(b^{BC},c\right)+u_C\left(b^{BC},c\right)\right\}=0$, and similarly in sequence CB. From the preceding analysis, it is then straightforward to show that

[22] We allow for the possibility that the joint surplus (net of A's outside option) is negative and explain this in more detail towards the end of the section.

[23] Existence of a maximum of $S_{AB}^{BC}\left(b\right)$ is ensured by our assumptions. (If $\mathcal{C}$ is compact, $c^*\left(b\right)$ is continuous by the Maximum Theorem; thus, $S_{AB}^{BC}\left(b\right)$ is continuous, and a solution to $\max_{b\in\mathcal{B}}S_{AB}^{BC}\left(b\right)$ exists by the Weierstrass Theorem.)

the principal is indifferent between both sequences as $U_A^{BC} = U_A^{CB} = (1-\beta\gamma)\{u_A\left(b^{BC}, 0\right) + u_B\left(b^{BC}, 0\right)\}$. In what follows, we focus on the (arguably more interesting) cases in which such situations do not occur—i.e., in at least one sequence, the equilibrium second-stage decision is strictly positive.

Second, if agents were symmetric and had the same bargaining power but differed in their outside options, the principal would be also indifferent between both sequences. To see this, denote the outside option of agent $i \in \{B, C\}$ by u_i^0, with $u_B^0 \geq u_C^0$. Then, in the expressions for U_A^{BC} and U_A^{CB}, besides setting $\beta = \gamma$, $u_i(\cdot,\cdot)$ needs to be replaced by $u_i(\cdot,\cdot) - u_i^0$ because the principal, when being selected as the proposer, can only extract the agent's payoff minus his respective outside option. It is then straightforward to check that the outside options enter U_A^{BC} and U_A^{CB} in the same way. If agents are symmetric otherwise, this implies $U_A^{BC} = U_A^{CB}$. As we will show in the next sections, this is not true if agents differ in their bargaining power; hence, differences in bargaining power are not equivalent to differences in outside options.

4 The Sequence that Generates the Higher Surplus

We first determine whether sequence BC or CB generates a higher joint surplus. The first-best surplus is

$$S^{FB} = \max_{b \in \mathcal{B},\, c \in \mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\}.$$

In general, there are two reasons why the equilibrium decisions in a given sequence are not surplus-maximizing. The first is that the negotiation in the second stage maximizes the surplus of the two players involved, but does not take into account the effect of the decision on the agent with whom A has negotiated in the first stage. This effect works through the externality of c on B in sequence BC (and through the externality of b on C in sequence CB). For instance, in the supplier-retailers example with substitute products, negotiating a larger quantity in the second stage has a negative effect on the agent with whom A bargained first.

The second reason why equilibrium decisions are not surplus maximizing is due to the fact that A only receives a fraction of the surplus in the second-stage negotiation. This implies that, in the first stage, the two bargainers only partially consider the second-stage surplus. Therefore, first-stage decisions may be distorted away from the surplus-maximizing outcome. This *partial-surplus effect* works through two channels. First, through the externality of b on C in sequence BC (and through the externality of c on B in sequence CB). In the

supplier-retailer example, if A signs a contract with a large quantity in the first stage, the surplus that A and her negotiation partner can achieve in the second stage is lower if goods are substitutes. Second, it works through the interaction of b and c in A's utility function. This occurs because the agent with whom A bargains in the second stage extracts A's utility with some probability. In the example, suppose that A's cost is given by a convex function of the total quantity $b+c$ and the negotiation sequence is BC. Then, the first-stage decision b will be chosen too high from a joint-surplus perspective because, with some probability, A will not be the proposer in the second stage, implying that C has to bear this higher cost.

The first proposition shows that there is a clear-cut result with respect to the sequence that leads to a higher surplus for the three players. Denote the joint surplus in sequence BC by S^{BC} and in sequence CB by S^{CB}.

Proposition 1 *Sequence BC always leads to a weakly higher surplus than sequence CB (i.e., $S^{BC} \geq S^{CB}$). The inequality is strict if first-stage decisions differ in the two sequences.*

If agents only differ in their bargaining strength, sequence BC dominates sequence CB from a surplus perspective, that is, surplus is higher when the principal bargains with the *stronger agent first*. It is important to note that this result holds *irrespective* of whether externalities are negative or positive.[24] Moreover, it also does not matter how the decisions interact in the principal's utility function u_A.

As stated in the proposition, the surplus is strictly higher in sequence BC compared to CB (i.e., $S^{BC} > S^{CB}$) if first-stage decisions are different in the two sequences. Sufficient, but not necessary, conditions for this are that (i) either externalities are strict or b and c interact in A's utility function, and (ii) the bargaining problems are smooth in the sense that all utility functions are differentiable and equilibrium decisions are interior and also differentiable.

The intuition for the result is rooted in the partial-surplus effect. If the principal negotiates with the weaker agent in the second stage, she receives a larger share of the surplus in this stage. Therefore, the utility of the agent with whom the principal bargains in the second stage is taken into consideration to a larger extent in the first-stage negotiation. The first-stage decision therefore leads to a higher surplus than in the case in which the principal bargains with the weaker agent first. Instead, the other effect described above (i.e., the second-stage decision ignores the utility of the agent with whom the principal bargained first) plays out similarly in the two sequences. As a consequence, the joint surplus is higher in sequence BC than in CB.

[24] Interestingly, it also does not matter whether the principal has more or less bargaining power than the agents, or one of them.

This intuition explains why the surplus is larger with sequence BC than with CB, *regardless* of the externalities. By contrast, the sequence preferred by the principal crucially depends on the externalities, as we show next.

5 The Sequence Preferred by the Principal

We start the section with a simple example in which decisions are binary. This occurs, for instance, in political processes where decisions are often either "Yes" or "No", or when objects are indivisible and buyers either choose to purchase or not. This case shows in a particularly transparent way why and how the principal's preference over the bargaining sequences depends on the externalities.

Example with binary decisions. Suppose that $\mathcal{B} = \mathcal{C} = \{0, 1\}$, with the possible interpretation that $b = 1$ ($c = 1$) indicates that B (C) participates in a joint project, or the sale of an indivisible object between A and B (C). Moreover, suppose that participation is optimal in every subgame, that is, regardless of the decision made in the other negotiation, the bilateral surplus in each negotiation is larger with $b = 1$ than with $b = 0$ (resp. with $c = 1$ than with $c = 0$).

The latter implies that the equilibrium decisions are independent of the sequence chosen by the principal (i.e., they are $b = 1$ and $c = 1$ in both sequences). Perhaps surprisingly, despite this fact, the principal has a clear preference for one sequence over the other, dependent on the externalities.

Proposition 2 *In the case with binary decisions, A strictly prefers sequence CB if externalities are strictly positive, whereas A strictly prefers sequence BC if they are strictly negative. If there are no externalities, A is indifferent between both sequences.*

The proposition can be easily shown by using the results of Section 3. Since $b^{BC} = c^*(b^{BC}) = 1$, we can plug these values into (1) and (2) to determine U_A^{BC}. In the same way, (i.e, by using $c^{CB} = b^*(c^{CB}) = 1$) we can determine U_A^{CB}. Taking the difference between U_A^{BC} and U_A^{CB} yields (as agents are symmetric except for bargaining power)

$$U_A^{BC} - U_A^{CB} = (\beta - \gamma)\left(u_B(1,0) - u_B(1,1)\right).$$

Because $u_B(1,1) > (<)\, u_B(1,0)$ if externalities are strictly positive (negative), the result follows. If there are no externalties, then $u_B(1,1) = u_B(1,0)$, which implies that the principal is indifferent.

The intuition behind the result of Proposition 2 is as follows: First, because the equilibrium first-stage decision is to participate in either sequence (i.e., $b^{BC} = c^{CB} = 1$), there is no distortion in this decision. This implies that the partial-surplus effect, which was the driver of the result in the previous section, is not present. Instead, the principal's incentives with respect to the sequence choice are not driven by surplus considerations but by a different effect. We denote this effect the *anticipated-externality effect*.

To see this effect, we note first that, because $\beta > \gamma$, the principal receives a larger (expected) payoff in the negotiation with C than with B. The main question is therefore whether A's payoff from C, in case she makes a proposal, is larger in timing BC or in CB. Consider first timing CB. When A negotiates with C in the first stage, the two bargainers anticipate the decision $b^* = 1$ taken between A and B in the second stage. Therefore, in case A proposes in the first stage, she can demand a payment of $u_C(1,1)$—i.e., an amount that depends on the externality that A and B exert on C by reaching an agreement in the second stage. This payment represents A's outside option in the second-stage negotiation with B, that is, in case B is the proposer in the second stage, he must leave A this amount as otherwise A would reject.[25]

Consider next timing BC. As agent C observes the first-stage decision $b^{BC} = 1$, the principal can also demand a payoff of $u_C(1,1)$ from C in this timing. However, if B proposes the contract in the first stage, he will extract the expected payment that A receives from C, that is, B drives the principal down to her outside option in the first stage. This outside option is equal to the expected payoff that A receives in the negotiation with C, given that she rejected B's offer in the first stage, which implies $b = t_B = 0$. Therefore, the outside option is $u_C(1,0)$, which is independent of the externality. It follows that the principal prefers sequence CB if externalities are positive because in this sequence she can (with positive probability) gain the positive external effect that reaching an agreement with agent B has on agent C. By contrast, if externalities are negative, she prefers sequence BC, which insulates her from the externality. Finally, if there are no externalities, the principal is indifferent.

General model. We now turn to the analysis of the general model of Section 2. In particular, we determine whether and, if yes, how the conclusions of the example with binary decisions extend to the general case.

In the general case, the partial-surplus effect and the anticipated-externality effect are both at work. If externalities are weakly negative, we nevertheless obtain a clear-cut result:

[25] We are ignoring here the direct utility of A—i.e., $u_A(\cdot,\cdot)$—as is enters A's total payoff in the same way in both sequences. Similarly, if A proposes to both B and C, she obtains the full surplus in either sequence.

Proposition 3 *If there are negative or no externalities, then* $U_A^{BC} \geq U_A^{CB}$. *The inequality is strict if either (i) externalities are strictly negative and all equilibrium decisions are non-zero, or (ii) first-stage decisions differ in the two sequences.*

With strictly negative externalities, both effects point in the same direction. First, as explained in Section 4, the partial-surplus effect favors sequence BC, as, in this sequence, the second-stage surplus is considered to a larger extent in the first negotiation than in sequence CB. Second, the anticipated-externality effect favors sequence BC as well, because the principal's outside option in the negotiation with agent B is larger in timing BC than in CB. As a consequence, with strictly negative externalities, the result of Proposition 2 fully extends to the general model.

Interestingly, Proposition 3 shows that even without externalities, the principal may have a clear preference for sequence BC in the general model. This occurs if the equilibrium decision of the first stage is different in both timings. The intuition is that even if there are no externalities between agents, the decisions b and c may interact in the principal's utility function. Although the anticipated-externality effect is then not at work, the partial-surplus effect is, and it favors sequence BC compared to sequence CB. This is in contrast to the case with binary decisions where the optimal decisions were the same in both sequences. Therefore, it was not relevant for the optimal sequence how b and c interact in the principal's utility function. Instead, in the general model, this interaction can induce a non-equivalence of the sequences, even if externalities are absent.

As we show in the proof of Proposition 3, without externalities a sufficient condition for first-stage decisions to differ in the two sequences is that (i) u_A *is* strictly super-modular or strictly sub-modular, (ii) equilibrium decisions are interior, and (iii) u_i $(i = A, B, C)$ and $c^*(b)$ are differentiable. Condition (i) is to ensure interaction between the bargaining problems. The condition holds in many economic applications—e.g., it is satisfied in our supplier-retailers example if A has strictly increasing marginal costs. Condition (i) alone is not sufficient to rule out the possibility that first-stage decisions might be identical in the two sequences, be it because they occur at a boundary of the feasible set, or because the utility functions are not differentiable. Conditions (ii) and (iii) serve to rule out these possibilities.[26]

It is important to note that the way how b and c interact in the principal's utility function does not drive the result; instead, the *mere* fact that the utility function is not additively separable is enough for the principal to prefer sequence BC. This implies—e.g., in the supplier-retailers example—that not only strictly increasing marginal costs, but also strictly

[26] Similarly, Edlin and Shannon (1998) rely on interiority and differentiability assumptions for strictly monotone comparative statics.

decreasing marginal costs lead to the same result.[27] Therefore, whereas the *direction* of the externalities is crucial for the sequencing decision, this does not hold for the *type of interaction* in u_A. For example, suppose there are complementarities in the production of the joint surplus. How this affects the sequence preferred by the principal depends on whether these complementarities stem from the principal's utility function u_A, or from externalities between agents. Super-modularity of u_A (e.g., the supplier's cost is a concave function of the sum $b + c$) favors sequence BC through the partial-surplus effect. In contrast, positive externalities between agents may favor sequence CB through the anticipated-externality effect, as was shown in Proposition 2.

We next turn to the analysis of positive externalities. In the example with binary decisions, only the anticipated-externality effect was present, which yielded the result that the principal unambiguously prefers sequence CB. In general, however, both effects are present. It is evident from the preceding discussion that the two effects work in opposite directions with respect to the sequence preferred by the principal. Therefore, with positive externalities, *both* sequences BC and CB can emerge in equilibrium.

To obtain insights under which circumstances the principal prefers one or the other sequence, we put more structure on the agents' utility functions and parametrize the strength of the externalities.

Parametric externalities. Assume that the agents' utility functions include a parameter $k \in \mathbb{R}$, which captures the importance of the externalities, but does not affect the principal's utility function. The parameter k affects the agents' utility functions in the following way:
(i) if $k = 0$, there are no externalities;
(ii) if $c = 0$ ($b = 0$), then k has no effect on u_B (u_C);
(iii) if $b > 0$ and $c > 0$, u_B and u_C are strictly increasing in k with $\lim_{k\to\infty} u_B(b, c; k) = \lim_{k\to\infty} u_C(b, c; k) = \infty$.
In addition, we assume that the bargaining problems are smooth in the sense that all utility functions are differentiable and equilibium decisions are interior and differentiable, and that equilibrium decisions converge to strictly positive limits as $k \to \infty$.[28]

We focus on the case in which u_A is additively separable in b and c. From the previous analysis, we know that, if this was not the case, the principal would prefer sequence BC when externalities are absent; by continuity, she may also prefer sequence BC with small

[27] The principal's preference for BC is strict as long as the cost function is not highly concave, as otherwise the solution would be at the boundary.

[28] Since $\mathcal{B}$ and $\mathcal{C}$ are compact by assumption, these sets are bounded above. Hence, equilibrium decisions cannot diverge to infinity.

positive externalities. Therefore, to determine how the strength of the externalities affect the optimal sequence in case externalties are positive, we restrict attention to the case of u_A being additively separable. The next proposition shows that in case the bargaining problems are smooth, we obtain the clear result that the principal prefers sequence CB if externalities are positive but small.

Proposition 4 *Consider the case of parametric externalities and suppose that u_A is additively separable.*
(i) There exists a $\hat{k} > 0$ such that $U_A^{CB} > U_A^{BC}$ for all $k \in (0, \hat{k})$.
(ii) For intermediate k, either sequence can be optimal.
(iii) There exists $\check{k} > \hat{k}$ such that $U_A^{CB} > U_A^{BC}$ for all $k > \check{k}$.

The intuition behind the first statement is as follows: From Section 4, the partial-surplus effect leads to a sub-optimal choice of the first-stage decision, which may differ in the two sequences. This effect arises because the second-stage surplus is considered only partially in the first stage, which distorts the first-stage decision compared to the surplus-maximizing one. Suppose now that $k = 0$. Because u_A is additively separable, the bargaining problems do not interact, which implies that all decisions are chosen in an optimal way. When increasing k slightly (i.e., moving from no to small positive externalities), due to the Envelope Theorem, the effect of the changing first-stage decisions on A's payoff is only of second order. Therefore, the partial-surplus is only of second order. By contrast, the anticipated-externality effect, which does not depend on the sub-optimality of the first-state decisions, but on the direction of the externalities, is still of first order. This effect is strictly positive because decisions are strictly positive. It follows that, for small positive externalities, the anticipated-externality dominates. Therefore, the principal strictly prefers to bargain with the weaker agent first.

If externalities are not small, both effects are of first order and either sequence can be optimal. The second statement represents this result. However, if externalities become very large (i.e., grow beyond bounds), then we are able to show that the optimal sequence is still CB (third statement). The intuition for the result is that the anticipated-externality effect is stronger than the partial-surplus effect. Although the latter is present, its size is bounded. Instead, the size of the anticipated-externality grows without bounds, which implies that the principal prefers sequence CB over BC if k is large enough.

Overall, we find that the sequence chosen by the principal coincides with the sequence that generates a higher surplus if externalities are negative. Instead, with positive externalities, this is no longer true—i.e., whether the sequence that is optimal for the principal is the same as the one that generates the higher surplus or not depends on the extent of the externalities.

Surplus loss that results from an inefficient sequence. Because the principal may select an inefficient bargaining sequence if externalities are positive, we next study the size of the surplus loss resulting from the choice of an inefficient sequence. Returning to the main model—i.e., agents are symmetric except for bargaining power—the next proposition shows that this loss can be substantial.

Proposition 5 *If A chooses sequence CB, then the resulting loss in surplus can be up to (almost) 100% of the first-best surplus.*

The proposition states that the choice of the inefficient sequence can lead to an outcome in which in equilibrium almost no surplus is generated.[29] This can hold even if the first-best surplus is large. To show this result we make use of the following example:

Agents can invest in a club good where the investments have positive externalities. For simplicity, $\mathcal{B} = \mathcal{C} \in \{0, 1, 2\}$, that is, each agent can stay out, or participate with either a low investment of 1 or a high investment of 2. The high investment has a cost of $z > 0$, whereas the low investment is costless. To simplify the exposition, suppose $u_A(b, c) = 0$ for all (b, c). The benefit gross of investment costs for each participating agent is $\phi(b + c)$, with $\phi(\cdot)$ strictly increasing, that is, $0 < \phi(1) < \phi(2) < \phi(3) < \phi(4)$.[30] We make the following assumptions on the investment costs:

$$z > \phi(x + 1) - \phi(x) \quad \forall x \in \{1, 2, 3\} \tag{3}$$

and

$$z < \min\left\{2\left(\phi(3) - \phi(2)\right), \phi\left(3\right) - \phi\left(1\right)\right\}. \tag{4}$$

Assumption (3) implies that the investment costs are large enough so that choosing the high investment does not pay off in terms of one agent's utility. Instead, assumption (4) ensures that the investment costs are sufficiently small so that, if agent $-j$ chooses the low investment, then choosing the high investment, first, increases total surplus, and, second, is better for agent j than if agent j was the only participant in the club and would be choosing the low investment.

Given this set-up, the first-best decisions are that both agents choose high investments if $\phi(4) \geq \phi(3) + z/2$, and that one agent chooses the high and the other agent the low

[29] The surplus loss can also be zero or anywhere between zero and almost 100% of the first-best surplus. A simple example in which it is zero, is the one with binary decisions as the equilibrium decisions are the same in either sequence and coincide with the surplus-maximizing ones.

[30] An agent who does not participate obtains an outside option of zero.

investment if $\phi(4) < \phi(3) + z/2$. The resulting first-best surplus is

$$S^{FB} = \begin{cases} 2\left(\phi(4) - z\right), & \text{if } \phi(4) \geq \phi(3) + z/2, \\ 2\phi(3) - z, & \text{if } \phi(4) < \phi(3) + z/2. \end{cases}$$

In equilibrium, the second-stage decision is always the low investment: this leads to a higher surplus in the second stage compared non-participation of the agent because the low investment is costless, and compared to the high investment because the latter is too costly by (3). Similarly, (3) implies that the first-stage decision is the low investment if the agent with whom the principal bargains in the second stage, has high bargaining power. The reason is that the bargainers in the first stage then take that agent's utility into account only to a small extent. On the other hand, if the second-stage agent has low bargaining power, (4) implies the first-stage decision is the high investment. The relevant cutoff for the bargaining power of the agent in the second stage is[31]

$$\kappa := 2 - \frac{z}{\phi(3) - \phi(2)}.$$

We focus on the case $\gamma < \kappa < \beta$, where the first-stage decisions differ across the bargaining sequences: in BC, it is $b^{BC} = 2$ whereas in CB, it is $c^{CB} = 1$.

Turning to the sequence chosen by the principal, we show in the proof of Proposition 5 that under assumption (4), A strictly prefers sequence CB for $\gamma < \kappa < \beta$. The low investment is then chosen in both stages, and total surplus is $S^{CB} = 2\phi(2)$. For β sufficiently close to 1, S^{CB} can be close to zero, regardless of the size of S^{FB}, which implies that the surplus loss can be arbitrarily close to 100%, while A still has a strict preference for CB. To see this, suppose, for instance, that $\beta = 1$. Then, $U_A^{BC} = (1-\gamma)\,\phi(1) < (1-\gamma)\,\phi(2) = U_A^{CB}$. No matter how small $\phi(2)$ and, since $\phi(1) < \phi(2)$, therefore also $\phi(1)$, A strictly prefers CB.[32] The reason is that in sequence BC, agent B appropriates all additional surplus, which induces A the choose the inefficient sequence.

It is important to note that this results can even hold if sequence BC would lead to the first-best surplus. In particular, if $\phi(3) + z/2 \geq \phi(4)$, the first-best surplus is generated by an overall investment of $b + c = 3$, which is achieved in sequence BC. As the principal's preference for sequence CB holds regardless of the size of $\phi(3)$, this implies that $S^{BC} = S^{FB}$ may be very high, but the principal still chooses sequence CB for β close to 1, despite the fact that the overall surplus in sequence CB is close to 0. Therefore, the surplus loss then

[31] Note that $\kappa > 0$ by (4) and $\kappa < 1$ by (3).

[32] For example, if $\phi(b+c)$ approaches a step function with the step at $b = 3$ such that $\phi(b+c) \to 0$ for $b + c < 3$, then $S^{CB} \to 0$.

results *only* from the choice of the inefficient sequence.

Example with a supplier and two retailers in Cournot competition. Finally, we illustrate our results in the example outlined in Section 2, in which A is a supplier contracting with two retailers, B and C. Suppose that retailers compete in quantities and that their utility functions are $u_B(b,c) = (1-b+kc)b$ and $u_C(b,c) = (1-c+kb)c$, respectively, with $k \in [-1,1]$. Therefore, if $k=-1$, the retailers sell perfect substitutes, whereas if $k=1$, the two goods are perfect complements. If $k=0$, the profit functions are independent of each other (i.e., there are no externalities). The supplier's utility function is $u_A(b,c) = -y(b+c) - x(b+c)^2/2$, with $0 \leq y \leq 1$ to ensure that, in equilibrium, $b, c > 0$, and $x \geq 0$. This implies that the supplier's cost function has a linear and a convex term. For $x=0$, u_A is additively separable.

After solving for the optimal quantities and the respective utilities in both sequences, we obtain that the utility of the principal in sequence BC is

$$U_A^{BC} = \frac{(1-\beta)(1-y)^2\left[\gamma x^2 + 2(2(1+k)-\gamma k)(2+x) + k^2\right]}{2(2+x)\left[\gamma x^2 + 2(2(1+k)-\gamma k)x + 4 - (3-\gamma)k^2\right]} + \frac{\beta(1-\gamma)(1-y)^2}{2(2+x)}, \tag{5}$$

whereas her utility in sequence CB is

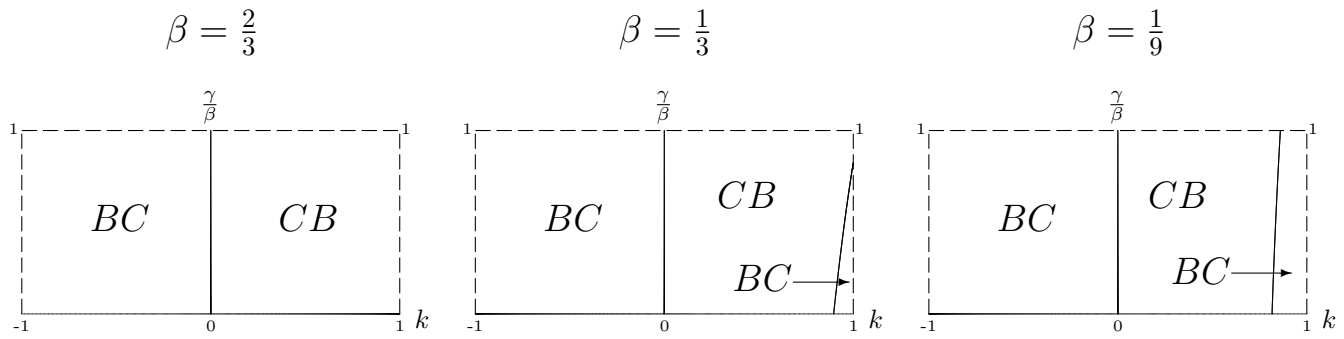
$$U_A^{CB} = \frac{(1-\gamma)(1-y)^2\left[\beta x^2 + 2(2(1+k)-\beta k)(2+x) + k^2\right]}{2(2+x)\left[\beta x^2 + 2(2(1+k)-\beta k)x + 4 - (3-\beta)k^2\right]} + \frac{\gamma(1-\beta)(1-y)^2}{2(2+x)}. \tag{6}$$

Comparing U_A^{BC} with U_A^{CB}, it is easy to check that for all $k \leq 0$ and $x \geq 0$, the principal prefers sequence BC over CB, strictly so if k is strictly negative and/or x is strictly positive, following Proposition 3. For $k > 0$ and $x = 0$, the principal prefers sequence CB for k close to 0 (as stated by Proposition 4). Instead, for $k=1$, we obtain that $\text{sign}\left\{U_A^{BC} - U_A^{CB}\right\} = \text{sign}\left\{9(1+\beta\gamma) - 16(\beta+\gamma)\right\}$, which implies that the principal prefers sequence BC if $\beta \leq (9-16\gamma)/(16-9\gamma)$. Because $\beta \geq \gamma$, this inequality can be fulfilled only if $\gamma \leq \left(16-5\sqrt{7}\right)/9 \approx 0.308$. If the inequality holds, there is a unique threshold value of k between 0 and 1, such that the principal prefers sequence CB for k below this threshold and sequence BC for k above this threshold. Instead, if the inequality does not hold, sequence CB is optimal for all $k \in (0,1]$. These results confirm that either sequence can be optimal if externalities are positive. We also note that, while in general it is possible that the principal's preferred sequence switches between CB and BC multiple times if externalities are positive, in this example, this occurs at most once.[33]

Figure 1 displays the different equilibrium regions for $x=0$ and three values of β (i.e., $\beta = 2/3$, $\beta = 1/3$, and $\beta = 1/9$). It is evident from the figure that the range in which

[33] Because $k \leq 1$, Statement (iii) of Proposition 4, which holds for $k \to \infty$, does not apply in this example.

sequence BC is optimal for positive externalities is larger if β is lower. Indeed, the partial-surplus effect, which favors sequence BC, is the more substantial, the lower the agents' bargaining powers.



Each diagram shows the equilibrium timing for the respective value of β.
In all diagrams, the horizontal axis displays $k \in [-1, 1]$ and the vertical axis $\gamma/\beta \in [0, 1]$.

Figure 1: Equilibrium timing in the supplier-retailers example

We can also determine the surplus loss in the supplier-retailer example. We focus on the range $x = 0$, $k \in (0, 1)$, and $\beta > (9 - 16\gamma)/(16 - 9\gamma)$, as the principal strictly prefers sequence CB then. Determining the first-best surplus and the surplus in the two sequences yields that the surplus loss resulting from sequence BC is

$$S^{FB} - S^{BC} = \frac{k^2(1-y)^2\left((4-3k^2)(1+k) + 2k^2\gamma(1+k) - 2\gamma^2(2-k^2)\right)}{4(1-k)\left(4-k^2(3-\gamma)\right)^2},$$

and the additional surplus loss from choosing the inefficient sequence CB is

$$S^{BC} - S^{CB} = \frac{(\beta-\gamma)k^2(1-y)^2\left((\beta+\gamma)(4-3k^2) + 2\beta\gamma k^2\right)}{4\left(4-k^2(3-\beta)\right)^2\left(4-k^2(3-\gamma)\right)^2}.$$

One can show that $S^{FB} - S^{BC}$ is larger than $S^{BC} - S^{CB}$. However, the latter is substantial relative to the former if the difference between the bargaining powers is sizable. E.g., for $k = 0.5$, $\beta = 0.9$, and $\gamma = 0.2$, we obtain $\left(S^{BC} - S^{CB}\right) / \left(S^{FB} - S^{BC}\right) \approx 0.464$.

We conclude this section by summarizing three main insights that result from our analysis in this and the previous section: (i) The sequence that leads to a higher surplus is the one in which the principal bargains with the stronger agent first, *regardless* of the externalities, whereas the sequence that is optimal for the principal depends on the externalities. (ii) If externalities are negative, the sequence chosen by the principal is *efficient*, whereas the equilibrium sequence may be *inefficient* if externalities are positive, and the surplus loss due this inefficiency can be *substantial*. (iii) The effect of the externalities on the equilibrium

sequence can be *non-monotonic:* as externalities change from negative to positive, the optimal sequence may switch from BC to CB, but can switch back to BC for stronger positive externalities.

6 Simultaneous Negotiations

We so far focused on the optimal timing of sequential negotiations. Under some circumstances, simultaneous bilateral negotiations with the two agents are also possible. This occurs, if, for instance, the principal can outsource the negotiations to delegates (or representatives) who act on her behalf. In this section, we analyze whether the principal may prefer such simultaneous to sequential negotiations. We, thus, extend our game by allowing the principal, in stage 0, to choose between simultaneous and sequential negotiations.

In simultaneous negotiations, the two delegates bargain independently of each other, which implies that they do not observe the outcome in the other negotiation. The same holds for the agents. The optimal decisions in each negotiation therefore depend on the beliefs about the agreed outcome in the other negotiation. Because beliefs are arbitrary in a (weak) Perfect Bayesian Equilibrium in case of out-of-equilibrium offers, multiple equilibria exist. We follow the literature (e.g., Horn and Wolinsky, 1988; McAfee and Schwartz, 1994; Segal, 1999; Marshall and Merlo, 2004; Nocke and Rey, 2018) and focus on "passive beliefs": even after an out-of-equilibrium offer, a player believes that the pair of bargainers in the other negotiation play as on the equilibrium path. These beliefs are often referred to as Nash-in-Nash conjectures (Collard-Wexler et al., 2019); they are the most reasonable ones in our case because the two delegates act independently.

With passive beliefs, in the negotiation between A and B, the solution b^* is given by

$$b^*\left(c^e\right) := \arg\max_{b\in\mathcal{B}} \left\{u_A\left(b, c^e\right) + u_B\left(b, c^e\right)\right\},$$

where c^e is the belief (expectation) about the outcome in the other negotiation.[34] As in case of sequential negotiations, we assume that b^* is unique for any c. Similarly, in the negotiation between A and C, c^* is given by

$$c^*\left(b^e\right) := \arg\max_{c\in\mathcal{B}} \left\{u_A\left(b^e, c\right) + u_C\left(b^e, c\right)\right\},$$

where b^e is the belief about the outcome in the other negotiation. In equilibrium, expectations are correct, that is, $b^e = b^\star$ and $c^e = c^\star$. They are jointly determined by the two equations

[34] Note that passive beliefs imply that c^e is independent of b.

above.[35]

Turning to the transfers, if A's delegate is drawn as the proposer in the negotiation with B, she sets $t_B = u_B(b^\star, c^\star)$. Similarly, in the negotiation with C, the delegate sets $t_C = u_C(b^\star, c^\star)$. By contrast, if B is selected as the proposer in the negotiation with A, he offers $t_B = -u_A(b^\star, c^\star) + u_A(0, c^\star)$. This occurs because if the delegate accepts B's proposal, her belief about A's payoff is $u_A(b^\star, c^\star) + t_B + t_C$. Instead, if the delegate rejects, her belief about A's payoff is $u_A(0, c^\star) + t_C$. The outcome in the other negotiation, i.e., $(c^\star, t_C)$, is the same in either case. Since B proposes, he brings the delegate down to her outside option by proposing $t_B = -u_A(b^\star, c^\star) + u_A(0, c^\star)$. By the same argument, if C is selected as the proposer in the negotiation with A, he sets $t_C = -u_A(b^\star, c^\star) + u_A(b^\star, 0)$.

The expected payoff of the principal with simultaneous negotiations can then be written as

$$U_A^{sim} = (1-\beta)(1-\gamma)\left\{u_A(b^\star, c^\star) + u_B(b^\star, c^\star) + u_C(b^\star, c^\star)\right\} + (1-\beta)\gamma\left\{u_A(b^\star, 0) + u_B(b^\star, c^\star)\right\}$$

$$+ \beta(1-\gamma)\left\{u_A(0, c^\star) + u_C(b^\star, c^\star)\right\} + \beta\gamma\left\{u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)\right\}. \qquad (7)$$

We can now compare the principal's payoff in case of simultaneous negotiations with that in the sequential timing. We start with the cases of negative or no externalities where we know from Proposition 3 that sequence BC dominates CB.

Proposition 6

(i) If externalities are negative and u_A is additively separable, then $U_A^{BC} \geq U_A^{sim}$.
(ii) In case of no externalities, $U_A^{BC} \geq U_A^{sim}$ if u_A is super-modular. Instead, if u_A is sub-modular, then there exists a threshold $\hat{\gamma} \geq 0$ such that $U_A^{BC} \geq U_A^{sim}$ if $\gamma \leq \hat{\gamma}$. Moreover, there exists a threshold $\tilde{\gamma} \leq 1$ such that $U_A^{sim} \geq U_A^{BC}$ if $\gamma \geq \tilde{\gamma}$.

The intuition behind Statement (i) of Proposition 6 is driven by two effects. The first one is related to the effect that drives the optimality of sequence BC with respect to the total surplus. In that timing, the two bargainers in the first stage take the utility of agent C partially into account because the principal receives in expectation a share of it. By contrast, with simultaneous negotiations, the delegate of the principal and agent B do not consider the utility of agent C when negotiating with respect to b because a change in b will not affect the outcome in the negotiation between the principal and agent C. As a consequence,

[35] We note that even when focusing on passive beliefs, multiple (weak) Perfect Bayesian Equilibria may exist. This can occur, for instance, if the decisions b and c are strategic complements. The results that we derive in Propositions 6 and 7 below, however, hold regardless of the equilibrium that is selected in the simultaneous game, as the proofs apply to any such equilibrium.

the decision made by A and B leads to a (weakly) smaller overall cake in the simultaneous negotiations.

The second effect is inherent in the simultaneous timing. It is rooted in the fact that the bargainers in each negotiation cannot observe the outcome of the other negotiation (because negotiations take place simultaneously). In particular, agent C cannot observe if an agreement was reached between A and B. She supposes—correctly so on the equilibrium path—that the decision in the other negotiation was $b^\star > 0$. In the sequence BC, agent C instead observes whether there was an agreement in the negotiation between A and B. This difference affects the expected transfer that A obtains: with simultaneous negotiations, in case A and B failed to reach an agreement, the principal can demand a transfer from C that equals C's utility given that $b = b^\star$, whereas in the sequence BC, she can demand a transfer from C that equals C's utility given that $b = 0$. In case of negative externalities, the latter is larger than the former, thereby favoring the sequential negotiation.

Next, we turn to the intuition behind Statement (ii) of the proposition. In contrast to sequential negotiations only, if simultaneous negotiations are possible, the way how b and c interact in the principal's utility function (i.e., whether u_A is super-modular or sub-modular) matters for the optimal timing. This can be seen from the last term of (7): with probability $\beta\gamma$—i.e., the probability with which both agents are selected as the proposers—A's payoff is $u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)$. This occurs because in the negotiation with, say, agent B, the principal's outside option is to reject B's offer and obtain a payoff of $u_A(0, c^\star)$. Therefore, the agent can claim a payment from the principal equal to $u_A(b^\star, c^\star) - u_A(0, c^\star)$. The same reasoning holds for agent C. It is evident that if u_A is super-modular, A's payoff is negative, which works in favor of timing BC. In addition, the first effect above, which also favors sequence BC, is at work as well. Therefore, the principal prefers sequence BC if u_A is super-modular.[36]

Instead, of u_A is sub-modular, the result depends on the bargaining power of the agents. If the principal has a lot of bargaining power, (e.g., γ is relatively small), the event that both agents make the offer is unlikely. The effect described above is then dominant, which implies that the principal prefers sequential negotiations. By contrast, if both agents have high bargaining power (i.e., γ is relatively large, which also implies that β is relatively large due to the fact that $\beta > \gamma$), the sub-modularity of u_A is the dominating effect, and the principal favors simultaneous negotiations.

We note that the inequalities of Proposition 6 are strict if, in (i), externalities are strictly negative or, in (ii), the super- or sub-modularity of u_A is strict, provided that equilibrium

[36]For the same reason, BC is still optimal if externalities are negative, but u_A was not additively separable and super-modular instead.

decisions differ in the sequential versus the simultaneous timing.

The result for the sub-modularity case can be illustrated with the help of the supplier-retailers example considered in the previous section. Recall that for all $x > 0$, u_A is sub-modular. If there are no externalities (i.e., $k = 0$), the utility of the principal in sequence BC is

$$\frac{(1-y)^2\left[(1-\beta\gamma)(4+\gamma x^2)+4(2-\beta\gamma)\right]}{2(2+x)(4(1+x)+\gamma x^2)}. \tag{8}$$

With simultaneous negotiations, the principal's utility is

$$\frac{(1-y)^2\left[4(1+x)-(\beta+\gamma)(2+x)\right]}{8(1+x)^2}. \tag{9}$$

Subtracting (9) from (8) and letting $\gamma \to 0$ yields $\left((1-y)^2\beta x^2\right) / \left(8(2+x)(1+x)^2\right)$, which is strictly positive. Hence, sequential negotiations are preferred over simultaneous ones. By contrast, if $\gamma \to 1$, which implies that also $\beta \to 1$, the difference between (8) and (9), becomes $-\left((1-y)^2 x\right) / \left(4(1+x)^2\right)$, which is strictly negative. It is easy to show that there is a unique threshold value of γ, such that simultaneous negotiations are preferred for γ above this threshold.

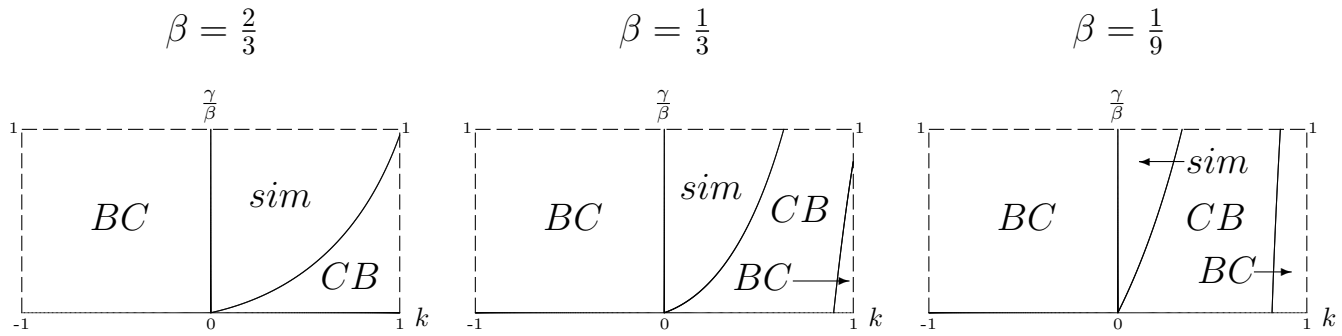
Finally, we turn to the case of positive externalities and derive a result for small externalities. We consider the case of parametric externalities, as in Proposition 4 above, and focus on sequence CB (since it dominates BC by Proposition 4).

Proposition 7 *Consider the case of parametric externalities. Suppose that u_A is additively separable. Then, there exists a $\underline{k} > 0$ such that $U_A^{sim} > U_A^{CB}$ for all $k \in (0, \underline{k})$.*

The intuition why simultaneous negotiations dominate sequential negotiations if externalities are positive, but small, follows from the logic of the second effect described after Proposition 6. In any sequential negotiation, the bargainers in the second stage know the outcome of the first stage. If there was no agreement reached in the first stage, the principal, when being selected as the proposer, can extract from the agent in the second stage an amount that equals his payoff, given that the decision in the first stage is 0. By contrast, in the simultaneous negotiations, an agent does not observe the outcome of the other bargaining game and supposes that an agreement was reached there. If externalities are positive, this implies that the principal can demand more from the agent. Although rejections do not happen on the equilibrium path, this effect increases the outside option of the principal.[37]

[37]The first effect explained after Proposition 6 is only of second order if externalities are small and u_A is additively separable.

Finally, we illustrate the result with positive externalities in our example with a supplier and two retailers. If $x = 0$, so that u_A is additively separable, the supplier-retailers example fulfills the requirements of Proposition 7. As Figure 2 shows, there exists a threshold $\underline{k}$, such that $U_A^{sim} > U_A^{CB}$ if and only if $k < \underline{k}$. Moreover, $\underline{k}$ is always below the threshold value of k at which the sequence switches from CB to BC (in case such a switching point exists). Therefore, if externalities are positive, three different timings may be optimal for the principal, dependent on the level of k: if k is small, the principal chooses simultaneous negotiations, for intermediate values of k, she chooses the sequence CB, and for large values of k, she chooses the sequence BC.



Each diagram shows the equilibrium timing for the respective β, including simultaneous negotiations. In all diagrams, the horizontal axis displays $k \in [-1, 1]$ and the vertical axis $\gamma/\beta \in [0, 1]$.

Figure 2: Equilibrium timing with simultaneous negotiations

7 Extensions

In this section, we consider three extensions of our model. First, in Subsection 7.1, we study the case in which agents are fully asymmetric—i.e., not only with respect to bargaining power. Second, in Subsection 7.2, we consider the case with more than 2 agents. Finally, in Subsection 7.3, we analyze the situation in which exclusivity contracts are possible, that is, the bargainers can negotiate an exclusive dealing contract in which the principal commits not to contract with the the other agent. To keep the section concise, we focus on our main question whether the principal prefers sequence BC or CB.

7.1 General Asymmetry of Agents

To single out the effect of different bargaining powers on the optimal sequence, our main model assumed that agents are symmetric except for bargaining power. While our results

are robust to some simple types of asymmetries,[38] in general, if agents would differ not only in their bargaining powers but also had different utility functions and/or their respective decisions would affect the principal's utility differently, then additional effect might come into play, which may affect A's preference over bargaining sequences.

This subsection aims at shedding some light on the issues that arise with such asymmetries. We start by reconsidering the case of binary decisions studied above and determine the additional considerations that affect the principal's optimal sequence. We then present the main result of this subsection: allowing for any asymmetries between the agents, for sufficiently high β, A's preference over bargaining sequences depends *only on the sign of the externalities*.

Proposition 8 *Suppose that $\mathcal{B} = \mathcal{C} = \{0, 1\}$ and participation is optimal in any subgame. For general utility functions u_i, $i \in \{A, B, C\}$, A strictly prefers sequence BC if and only if*

$$(\beta - \gamma)\left(u_B(1,0) - u_B(1,1)\right) > \beta(1-\gamma)\left[u_C(1,1) - u_C(0,1) - \left(u_B(1,1) - u_B(1,0)\right)\right]. \quad (10)$$

Proposition 8 generalizes Proposition 2 above. The left-hand side of (10) is identical to the corresponding term under symmetry and represents the anticipated-externality effect. Instead, the right-hand side of (10) is zero under symmetry. However, if agents are asymmetric in their utility functions, this term can be positive or negative, dependent on which agent is affected more strongly by the participation of the other agent.

To gain intuition on the additional effect at work, it is instructive to consider the case where agents have the same bargaining power ($\beta = \gamma$). Then, the left-hand side of (10) is zero, whereas the right-hand side is

$$\beta(1-\beta)\left[u_C(1,1) - u_C(0,1) - \left(u_B(1,1) - u_B(1,0)\right)\right].$$

Thus, A strictly prefers sequence BC if and only if $u_B(1,1) - u_B(1,0) > u_C(1,1) - u_C(0,1)$. This s equivalent to B being more positively or less negatively affected by C's participation, than vice versa. The intuition is as follows: A bears the externality on the agent with whom she bargains first, herself only in case she proposes in stage 1. Instead, she bears the externality on the agent with whom she bargains second only if she proposes in *both* stages. Hence, A bears the externality on the first agent with higher probability. The difference in these probabilities is the factor $\beta(1-\beta)$.[39] Thus, if agents have the same bargaining

[38] For instance, suppose that $u_B(b,c) = u_C(c,b) + k$ for some constant k whenever $b > 0$ and $c > 0$, and participation is optimal in any subgame. Then all our results above carry over.

[39] Assuming $\beta = \gamma$, A proposes in stage 1 with probability $(1-\beta)$, but proposes in both stages with probability $(1-\beta)^2$. The difference is $(1-\beta) - (1-\beta)^2 = \beta(1-\beta)$.

power, A prefers to start negotiating with the agent who is benefits more or is hurt less from participation of the other agent.

This result shows how Proposition 2 above needs to be qualified when allowing for general utility functions. If externalities are negative and B's participation is more detrimental to C than vice versa, the sequence BC is still optimal for the principal. Similarly, if externalities are positive and B's participation benefits C more than C's participation benefits B, the sequence CB is still the principal's preferred sequence. In the other cases, there are two opposing forces at work, and the optimal sequence is driven by whether the anticipated-externality effect dominates the effect that agents are differently affected by the other agent's participation. However, the model is continuous in the sense that if the asymmetry in the agents' utility functions is sufficiently small, then the result of Proposition 2 above still holds.

We also note that in case of binary decisions, asymmetries in the function u_A are irrelevant for A's preference over the sequences. This is due to the fact that A obtains the respective payoff from her own utility with the same probability in either sequence, which implies that u_A cancels out in the comparison.

Proposition 8 illustrates that asymmetries can introduce additional considerations in determining A's preference over the bargaining sequences. Despite this fact, we can show a very general statement for the case where B's bargaining power is sufficiently large.

Proposition 9 *For general utility functions $u_i(b,c)$, $i \in \{A,B,C\}$, and decision spaces $\mathcal{B}$ and $\mathcal{C}$, there exists a $\hat{\beta} \leq 1$ such that for $\beta \geq \hat{\beta}$, the following holds:*
If b has negative externalities on C then $U_A^{BC} \geq U_A^{CB}$. Instead, if b has positive externalities on C then $U_A^{CB} \geq U_A^{BC}$. If b has no externalities on C, then $U_A^{BC} = U_A^{CB}$.

Proposition 9 shows that if β is sufficiently large, the principal's preference over bargaining sequences is solely driven by the sign of the externalities, even when agents are asymmetric and may have different utility functions and decision spaces. The intuition for why A prefers BC with negative externalities, and CB with positive externalities, is again the expected-externality effect highlighted above. In fact, for β sufficiently close to 1, it is mainly the externality of b on C that matters for the principal, because B has most of the bargaining power and bears the externality of c to a large extent by himself. In this respect, for the principal, the surplus in the negotiation with C is of first-order importance, whereas the surplus in the negotiation with B is only of second order, which allows for the general statement.

We finally note that, as in the cases above, the inequalities in Proposition 9 are strict if externalities are strictly negative or strictly positive, and equilibrium decisions are not

zero. In this case, the threshold value $\hat{\beta}$ is strictly below 1 if the bargaining problems are sufficiently smooth.

7.2 N Agents

In this section, we show that our insights are robust to the case with a larger number of agents. Suppose that there are $N \geq 2$ agents. We assume that all agents differ in their bargaining power (but are otherwise symmetric). As in the main model, the principal decides about the sequence of the negotiations (i.e., with which agent to bargain first, second, etc.). To simplify the exposition, we consider the case in which the principal commits to a sequence before the negotiations start.[40] Moreover, we assume that each negotiation subgame has a unique equilibrium, after breaking ties, such that a player accepts when indifferent between accepting and rejecting an offer.

The analysis with N agents is considerably more involved than with two agents because changing the sequence in any two time periods may affect the decisions taken in other stages of the game. To avoid this complication, we first consider the example of binary decisions, following the description in Section 5. With binary decisions, $d_i \in \mathcal{D} = \{0, 1\}$ for all $i = 1, ..., N$, and participation of all agents is always optimal. The latter implies that a change in the sequence does not change equilibrium decisions. This case allows us to bring out the role of externalities on the bargaining sequence in the clearest way, as the optimal sequence is then only driven by the direction of the externalities. As the next proposition shows, we obtain a clear-cut result that provides a generalization of Proposition 2 to N agents:

Proposition 10 *Consider the case with binary decisions. With negative externalities, A optimally negotiates with agents in decreasing order of their bargaining power. By contrast, with positive externalities, A optimally negotiates with agents in increasing order of their bargaining power.*

The result shows that one of the main insights of the case with two agents carries over to the case with a larger number of agents: negative externalities imply that the principal prefers to bargain with weaker agents later, whereas with positive externalities, the opposite holds true. As in the two-agent case, the result is driven by the anticipated-externality effect.

Although the intuition is based on the same effects as in Section 5, the proof is more complicated. We show the result by an "adjacent pairwise interchange" argument (see, for

[40] All our results extend to the case without commitment (i.e., the principal chooses with which agent to bargain after each round of negotiations): then there exists an equilibrium where the principal approaches the agents in the order given in Propositions 10 and 11.

example, Baker and Trietsch, 2009): this involves—starting from an arbitrary bargaining sequence—to check whether A prefers to change the position of agents in two consecutive time periods. To prove the proposition, we present an algorithm that sequentially assigns decisions and transfers to all agents, such that A has the same payoff from the algorithm as in the equilibrium of the bargaining game. Our algorithm uses only the equilibrium decisions, but not the equilibrium transfers, as an input, which makes it comparatively simple and tractable and may make the method useful for the analysis of other dynamic bargaining games as well.

Second, we analyze the case without externalities between agents, but where the bargaining decisions interact in A's utility function. This additional complication is still tractable in a general framework.

Proposition 11 *Suppose that there are no externalities, but decisions interact in $u_A(\cdot)$. The principal then optimally negotiates with agents in decreasing order of their bargaining power.*

The result shows that the insight derived in the case with two agents (Proposition 3) carries over to the case with N agents. As in the case with two agents, the intuition is based on the partial-surplus effect, which favors bargaining with weaker agents at later stages.

7.3 Exclusive Contracts

In the baseline model, we assumed that contracts between the principal and an agent cannot condition on the outcome that the principal reaches in the negotiation with the other agent. This assumption is reasonable in many real-world situations—e.g., such conditional contracts between firms are often illegal by antitrust law or difficult and costly to write, as they require the specification of several different contingencies (Dequiedt and Martimort, 2015). An exception are, however, exclusive dealing contracts, which, in some settings, are both legal and feasible. In this section, we therefore consider the robustness of our results to such exclusive contracts.

There are two ways to study exclusive contracts. The first is the single-contract scenario, which is considered by e.g. Rasmusen et al. (1991), Fumagalli and Motta (2006), and Marx and Shaffer (2007) in different environments. In this case, the two bargainers can either sign a contract that leaves the principal free to trade with the other agent (as in our main model), or an exclusive-dealing contract that commits the principal not to trade with the other agent. The second scenario is a menu of contracts, in which the proposer in the negotiation offers two contracts, one for the case in which the principal also trades with the other agent, and

one for the case of exclusive dealing. This scenario is considered by e.g. Segal and Whinston (2000) and Rey and Whinston (2013).[41] We consider the two scenarios in turn.

Single contract. In the single-contract scenario, we extend the contract space of the main model in the following way: in the negotiation in the first stage, the two bargainers can either sign an exclusive contract or a non-exclusive contract. The latter contract is the same as in our main model above and specifies a decision and a transfer, without restricting the contract that the principal may sign with the other agent. The exclusive contract also specifies a decision and a transfer, but restricts the principal to not negotiate with the second agent. Analyzing this scenario, we obtain the following result:

Proposition 12 *If the contract space is extended to the scenario in which the bargainers can either sign an exclusive or a non-exclusive contract, the sequences given in Propositions 1 - 4 are still optimal.*

The proposition shows that the possibility to negotiate an exclusive dealing contract does not affect our main insights. In what follows, we explain the intuition behind this result. Without an exclusive dealing contract, the principal is free to negotiate with the second agent. Therefore, if the principal prefers no exclusive dealing contract, her expected payoff must be (weakly) higher than the payoff without reaching an agreement in the second stage because she will reject any offer that gives her a lower payoff. Consider now the case without externalities. In this case, the payoff of the first agent is not affected by the second-stage decision. Therefore, the joint surplus of the principal and the first agent is (weakly) higher without an exclusive contract. With positive externalities between agents, excluding the second agent has the additional disadvantage that this externality on the first agent is not realized. As a consequence, with positive and no externalities, exclusion does not occur in equilibrium.

By contrast, with negative externalities, exclusive contracts may increase the joint surplus of the principal and the first agent, as the principal does not consider the negative externality on the first agent when bargaining with the second agent. Without exclusion, however, the principal benefits from bargaining with the second agent. These benefits are the larger the lower the bargaining power of the second agent. Therefore, if exclusion is used in sequence BC (where the second agent is weak), it will also be used in CB (where the second agent is strong), but not vice versa.

[41] The second scenario can also be interpreted as one in which the contract specifies a penalty for contract breach from exclusivity; see Fumagalli et al. (2018).

The principal's expected payoff from an exclusive dealing contract is, however, independent of the sequence of negotiations. The reason is that, by a similar argument as in Section 3, if an exclusive contract maximizes the joint surplus of those who bargain in stage 1, each player proposes the exclusive contract in each negotiation, which implies that the principal obtains the respective payoff with probability $1-\beta\gamma$.[42] Therefore, if an exclusive contract is optimal in both sequences, A is indifferent between the two sequences. By the argument above, exclusion is, however, more profitable in sequence CB than in sequence BC. It follows that if exclusion maximizes the joint surplus of the first-stage negotiation in sequence CB but not in sequence BC, the joint surplus of A and the first agent must be higher in the latter sequence. Therefore, A prefers sequence BC and no exclusive contract.

These considerations show that our key results on A's preferences over the negotiation sequences also hold if exclusive contracts are feasible. A minor caveat is that, with negative externalities, the principal's preference for sequence BC is not strict if there is exclusion in both sequences, while she typically has a strict preference for BC if exclusive contracts are not feasible (see Proposition 3).

Menu of contracts. We next turn briefly to the scenario in which the proposer in each negotiation can offer a menu of contracts. This menu consists of two contracts, (d_i^e, t_i^e) and (d_i, t_i), where the exclusive contract (d_i^e, t_i^e) is executed if the decision reached with the other agent $j \neq i$ is zero ($d_j = 0$), and (d_i, t_i) becomes relevant otherwise.

In this scenario, the principal prefers the sequence CB, independent of the externalities. With a menu of contracts, the exclusive dealing contract negotiated with the first agent matters in the second-stage bargaining, despite the fact that the respective transfer and decision may not be implemented on the equilibrium path. The reason is that this contract determines the principal's outside option against the second bargainer. By choosing the contract in an appropriate way, the surplus of the second agent can be fully extracted, independent of his bargaining power.[43] This implies that the principal and the first agent fully internalize the effect of their decision on u_A and on the utility of the second agent. As a consequence, both sequences lead to the same joint surplus of all three players. Because the second agent obtains no rent regardless of the sequence and the principal receives a higher share of the joint surplus if she bargains with the weaker agent first, her optimal sequence

[42] Formally, in timing BC, A's payoff is $\max_b \{u_A(b,0) + u_B(b,0)\}$ if she offers an exlusive deal in stage 1. If B proposes an exclusive deal, A's payoff is her outside option $(1-\gamma)\max_c \{u_A(0,c) + u_C(0,c)\}$. In expectation, A therefore obtains $(1-\beta\gamma)\max_b \{u_A(b,0) + u_B(b,0)\}$, which depends only on the product $\beta\gamma$, and is equal to A's expected payoff in timing CB.

[43] This type of rent extraction via a menu of contracts is similar to the one shown by Aghion and Bolton (1987) and Segal and Whinston (2000).

is CB.

8 Conclusion

The situation in which a principal bargains with multiple agents bilaterally, and the decision in each negotiation has an effect on the payoff of other agents, is prevalent in many economic environments. The principal then has the choice how to sequence the negotiations with the agents. This paper has shown that the difference in the agents' bargaining power is crucial for the optimal sequence, and therefore the efficiency of decisions.

To understand the driving forces behind the sequencing decision, we considered a general framework with one principal and two agents. We show that the optimal sequence is driven by the interplay of two effects: the *partial-surplus effect* and the *anticipated-externality effect*. Because of the partial-surplus effect, the sequence that generates the highest joint surplus is the one on which the principal bargains with the strong agent first, independent of whether externalities are positive or negative. By contrast, the sequence chosen by the principal depends on the externalities.

If externalities are negative, both effects point in the same direction and favor the efficient sequence. Instead, with positive externalities, we identify conditions under which the equilibrium timing is to bargain with the weak agent first. This is the case if, for instance, externalities are small and the principal's utility function is additively separable. The anticipated-externality effect then dominates the partial-surplus effect, which induces the principal to choose the sequence that does not maximize the joint surplus. We show that the resulting surplus loss can be substantial. In addition, we also compare sequential with simultaneous bargaining, and show that simultaneous negotiations can be optimal if externalities are positive.

In our study, we have focused on the role of bargaining power, as we think it is an important, yet understudied, topic in the context of contracting with externalities. To bring out the effects of the bargaining power in a clear way, we have derived our main results for the case in which agents are symmetric except for bargaining power.[44] As our framework is relatively general, it lends itself naturally to explore the effects of other dimensions on optimal negotiations. For example, agents may differ in their contribution to the total surplus instead of the bargaining power. Also, agents may be asymmetric in the externalities they exert on each other. These differences can affect the surplus-maximizing sequence and the one chosen

[44] Without the assumption of symmetry, we could show that if one agent has all bargaining power, the principal will always negotiate with this agent first if externalities are negative, but with the weaker agent first if externalities are positive.

by the principal. In particular, asymmetries in those other dimensions may bring in new effects that could strengthen or qualify the effects shown in this paper. We think this is an interesting avenue for future research.

Appendix

Proof of Proposition 1

The proof of the proposition uses the following lemma:

Lemma 1 *Suppose that* $w : \mathcal{B} \to \mathbb{R}$ *and* $v : \mathcal{B} \to \mathbb{R}$ *are functions,* $0 \leq \gamma_0 < \gamma_1 \leq 1$ *and*

$$b_i = \arg\max_{b \in \mathcal{B}} \{(1 - \gamma_i) w(b) + \gamma_i v(b)\} \text{ for } i = 0, 1.$$

Then, $w(b_1) \leq w(b_0)$.

Proof. If $\gamma_0 = 0$, $b_0 = \arg\max_{b \in \mathcal{B}} w(b)$, hence $w(b_0) \geq w(b_1)$. The rest of the proof considers the case $\gamma_0 > 0$. Towards a contradiction, suppose that $w(b_1) > w(b_0)$. By assumption, $(1 - \gamma_0) w(b_0) + \gamma_0 v(b_0) \geq (1 - \gamma_0) w(b_1) + \gamma_0 v(b_1)$, or equivalently,

$$(1 - \gamma_0)(w(b_0) - w(b_1)) \geq \gamma_0 (v(b_1) - v(b_0)). \tag{11}$$

Since $w(b_1) > w(b_0)$ and $0 < \gamma_0 < \gamma_1 \leq 1$, the left side of inequality (11) is strictly negative; hence, $v(b_1) < v(b_0)$.

Similarly, $(1 - \gamma_1) w(b_1) + \gamma_1 v(b_1) \geq (1 - \gamma_1) w(b_0) + \gamma_1 v(b_0)$, or equivalently

$$-(1 - \gamma_1)(w(b_0) - w(b_1)) \geq -\gamma_1 (v(b_1) - v(b_0)). \tag{12}$$

Adding (12) to (11) shows that

$$(\gamma_1 - \gamma_0)(w(b_0) - w(b_1)) \geq (\gamma_0 - \gamma_1)(v(b_1) - v(b_0)).$$

This is a contradiction since the left-hand side is strictly negative and the right-hand side strictly positive. ■

Using this lemma, we first show that S^{BC} is decreasing in γ and constant in β. It is evident from (2) that the equilibrium decisions $\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ do not depend on β. Therefore, $S^{BC} =$

$S\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is constant in β. Moreover, $b^{BC} = \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$, where

$$\begin{aligned} S_{AB}^{BC}(b) &= u_B(b, c^*(b)) + (1-\gamma)\left(u_A(b, c^*(b)) + u_C(b, c^*(b))\right) + \gamma u_A(b, 0) \\ &= (1-\gamma) S(b, c^*(b)) + \gamma\left[u_A(b, 0) + u_B(b, c^*(b))\right]. \end{aligned}$$

Applying Lemma 1 with $w(b) = S(b, c^*(b))$ and $v(b) = u_A(b, 0) + u_B(b, c^*(b))$ shows that $S\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is decreasing in γ. In a similar way, we can establish that S^{CB} is decreasing in β and constant in γ.

If $\beta = \gamma$, sequences BC and CB differ only in the names of the agents and $S^{BC} = S^{CB}$. Because S^{BC} is constant in β and decreasing in γ, whereas S^{CB} is decreasing in β and constant in γ, it follows that, for $\beta > \gamma$, $S^{BC} \geq S^{CB}$. ■

Proof of Proposition 3

The symmetry of the agents has two implications that will be used in the proof. First,

$$\arg\max_{c\in\mathcal{C}}\left\{u_A(x, c) + u_C(x, c)\right\} = \arg\max_{b\in\mathcal{B}}\left\{u_A(b, x) + u_C(b, x)\right\} =: f(x) \tag{13}$$

for all $x \in \mathcal{B} = \mathcal{C}$. The function $f(x)$ defined in (13) gives the second-stage decision that ensues after a first-stage decision x; under symmetry, it is the same function in both sequences. Second, symmetry implies that $\max_{c\in\mathcal{C}}\left\{u_A(0, c) + u_C(0, c)\right\} = \max_{b\in\mathcal{B}}\left\{u_A(b, 0) + u_B(b, 0)\right\}$. Since the outside options of A in stage 1 are, respectively, $O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0, c) + u_C(0, c)\right\}$ and $O_A^{CB} = (1-\beta)\max_{b\in\mathcal{B}}\left\{u_A(b, 0) + u_B(b, 0)\right\}$, it follows that symmetry implies that

$$\beta O_A^{BC} - \gamma O_A^{CB} = (\beta - \gamma)\max_{c\in\mathcal{C}}\left\{u_A(0, c) + u_C(0, c)\right\}. \tag{14}$$

The surplus of A and B in sequence BC as a function of b is

$$S_{AB}^{BC}(b) = (1-\gamma)\left(u_A(b, f(b)) + u_C(b, f(b))\right) + \gamma u_A(b, 0) + u_B(b, f(b)).$$

In equilibrium of sequence BC, $b = b^{BC} = \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$. Similarly, the surplus of A and C in sequence CB as a function of c is

$$S_{AC}^{CB}(c) = (1-\beta)\left(u_A(f(c), c) + u_B(f(c), c)\right) + \beta u_A(0, c) + u_C(f(c), c).$$

In equilibrium of sequence CB, $c = c^{CB} = \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$. The expected payoffs of A in sequences BC and CB are, respectively, $U_A^{BC} = (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}$ and $U_A^{CB} = (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) + \gamma O_A^{CB}$.

Since $b^{BC} = \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$,

$$S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right). \tag{15}$$

Moreover, by symmetry,

$$S_{AB}^{BC}\left(c^{CB}\right) = (1-\gamma)\left(u_A\left(f\left(c^{CB}\right), c^{CB}\right) + u_B\left(f\left(c^{CB}\right), c^{CB}\right)\right) + \gamma u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)$$

and therefore

$$(1-\beta)\, S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma)\, S_{AC}^{CB}\left(c^{CB}\right) = (\gamma-\beta)\left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right). \tag{16}$$

From (14), (15), and (16),

$$\begin{aligned} U_A^{BC} - U_A^{CB} &= (1-\beta)\, S_{AB}^{BC}\left(b^{BC}\right) - (1-\gamma)\, S_{AC}^{CB}\left(c^{CB}\right) + \beta O_A^{BC} - \gamma O_A^{CB} \\ &\geq (1-\beta)\, S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma)\, S_{AC}^{CB}\left(c^{CB}\right) + \beta O_A^{BC} - \gamma O_A^{CB} \\ &= (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\left\{u_A(0,c) + u_C(0,c)\right\} - \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)\right) \\ &\geq (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\left\{u_A(0,c) + u_C(0,c)\right\} - \max_{c\in\mathcal{C}}\left\{\left(u_A(0,c) + u_C\left(f(c), c\right)\right)\right\}\right) \\ &\geq 0. \end{aligned} \tag{17}$$

The final inequality follows from negative externalities, which imply $u_C(0,c) \geq u_C(b,c)$ for all $b \geq 0$. Moreover, whenever externalities are strictly negative and $b > 0$, $u_C(0,c) > u_C(b,c)$ for all $c > 0$; hence, $U_A^{BC} > U_A^{CB}$.

The chain of inequalities in (17) establishes that $U_A^{BC} \geq U_A^{CB}$ also in case of no externalities. Moreover, if $b^{BC} \neq c^{CB}$, then inequality (15) is strict, which implies that $U_A^{BC} > U_A^{CB}$.

We now turn to the sufficient conditions (i)-(iii) for $b^{BC} \neq c^{CB}$ mentioned in the main text. Suppose there are no externalities. By (iii), $S_{AB}^{BC}(b)$ and $S_{AC}^{CB}(c)$ are differentiable. Since $b^{BC} = \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ is interior by (ii), it satisfies the first-order condition

$$\begin{aligned} \frac{\partial S_{AB}^{BC}\left(b^{BC}\right)}{\partial b} &= \frac{\partial u_B\left(b^{BC}\right)}{\partial b} + (1-\gamma)\frac{\partial}{\partial b} u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b} u_A\left(b^{BC}, 0\right) \\ &\quad + (1-\gamma)\left(\frac{\partial}{\partial c}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right)\frac{df\left(b^{BC}\right)}{db}\right) = 0. \end{aligned}$$

Since $f\left(b^{BC}\right)$ is interior by (ii), and $u_A(b,c) + u_C(c)$ is differentiable by (iii), the first-order

condition

$$\frac{\partial}{\partial c}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right) = 0$$

holds. Therefore,

$$\frac{\partial u_B\left(b^{BC}\right)}{\partial b} + (1-\gamma)\frac{\partial}{\partial b}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b}u_A\left(b^{BC}, 0\right) = 0.$$

In addition, $f\left(b^{BC}\right) > 0$ since it is interior by (ii). Condition (i) then implies

$$\frac{\partial}{\partial b}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) \neq \frac{\partial}{\partial b}u_A\left(b^{BC}, 0\right).$$

Since $\beta > \gamma$,

$$\begin{aligned}
&\frac{\partial u_B\left(b^{BC}\right)}{\partial b} + (1-\gamma)\frac{\partial}{\partial b}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b}u_A\left(b^{BC}, 0\right) \\
&\neq \frac{\partial u_B\left(b^{BC}\right)}{\partial b} + (1-\beta)\frac{\partial}{\partial b}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \beta\frac{\partial}{\partial b}u_A\left(b^{BC}, 0\right) \\
&= \frac{\partial u_C\left(b^{BC}\right)}{\partial c} + (1-\beta)\frac{\partial}{\partial c}u_A\left(f\left(b^{BC}\right), b^{BC}\right) + \beta\frac{\partial}{\partial c}u_A\left(0, b^{BC}\right) \\
&= \frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c},
\end{aligned}$$

where the first equality is due to symmetry and the last equality can be established by using the first-order condition of the second stage, as above. We have thus shown that $\partial S_{AC}^{CB}\left(b^{BC}\right)/\partial c \neq 0$.

Because any $c^{CB} = \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$ is interior by (ii), it satisfies the first-order condition $\partial S_{AC}^{CB}\left(c^{CB}\right)/\partial c = 0$. Therefore, $b^{BC} \neq c^{CB}$. ■

Proof of Proposition 4

In what follows, we denote the first-stage decision in sequence BC, which depends on k, by $b^{BC}(k)$. We also denote the second-stage decision in sequence BC by $c^*(b,k) := \arg\max_{c\in\mathcal{C}}\{u_A(b,c) + u_C(b,c,k)\}$, and define $b^*(c,k)$ similarly as the optimal second-stage decision in sequence CB. By symmetry, for any given first-stage decision x,

$$c^*(x,k) = b^*(x,k) =: f(x,k).$$

The proof of statement (i) rests on the Envelope Theorem, which we can use although the choices in the second stage do, in general, not maximize the joint surplus of the principal and the agent who bargain in the first stage. With parametric externalities, however, at

$k = 0$, the second-stage decision also maximizes the surplus of the negotiation in the first stage, therefore the corresponding terms disappear.

Lemma 2 *In case of parametric externalities,* $S_{AB}^{BC}(k) = \max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ *and* $S_{AC}^{CB}(k) = \max_{c\in\mathcal{C}} S_{AC}^{CB}(b^*(c,k), c, k)$ *are differentiable in* k*, and*

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k} u_B(b,c,k)\Big|_{\substack{k=0\\ b=c=f(0,0)}} > 0.$$

Proof. At $k = 0$, the bargaining problems are completely independent, and (by symmetry) all equilibrium decisions are identical. That is, for any b and c,

$$b^{BC}(0) = c^{CB}(0) = b^*(c,0) = c^*(b,0) = f(0,0).$$

By our smoothness assumptions, $S_{AB}^{BC}(k)$ is differentiable in k. By the envelope theorem,

$$\begin{aligned}\frac{d}{dk}\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)|_{k=0} = &\frac{\partial}{\partial k}\left(u_B(b,c,k) + (1-\gamma) u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}\\ &+\left(\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c}\frac{\partial c^*(b,k)}{\partial k}\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}.\end{aligned}$$

The first term of the right-hand side is the direct effect of k, keeping b and c constant, whereas the second term captures that the second-stage equilibrium decision depends on k.

We next show that

$$\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c}\Big|_{\substack{k=0\\ b=c=f(0,0)}} = 0.$$

We have

$$\frac{\partial S_{AB}^{BC}(b,k)}{\partial c} = \frac{\partial}{\partial c} u_B(b, c^*(b), k) + (1-\gamma)\frac{\partial}{\partial c}\left(u_A(b, c^*(b,k)) + u_C(b, c^*(b,k), k)\right).$$

Evaluated at $k = 0$, u_B does not depend on c; hence, the first term on the right-hand side is zero. Moreover, $c^*(b,k)$ maximizes $u_A(b,c) + u_C(b,c,k)$, and by the first-order condition of this maximization problem, the second term on the right hand side is also zero.

It follows that

$$\frac{d}{dk} S_{AB}^{BC}(k)|_{k=0} = \frac{\partial}{\partial k}\left(u_B(b,c,k) + (1-\gamma) u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}.$$

Similarly,

$$\begin{aligned}\frac{d}{dk}S_{AC}^{CB}(k)|_{k=0} &= \frac{\partial}{\partial k}\left(u_C(b,c,k)+(1-\beta)u_B(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}\\ &= \frac{\partial}{\partial k}\left(u_B(b,c,k)+(1-\beta)u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}},\end{aligned}$$

where the second line uses symmetry. Therefore,

$$\frac{d}{dk}\left((1-\gamma)S_{AC}^{CB}(k)-(1-\beta)S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k}u_B(b,c,k)\Big|_{\substack{k=0\\ b=c=f(0,0)}},$$

which is strictly positive since $b=c=f(0,0)>0$. ■

We are now in a position to prove the first statement of Proposition 4.

Since u_A and $u_C(0,c,k)$ do not depend on k,

$$O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c)+u_C(0,c,k)\}$$

does not depend on k. Similarly, O_A^{CB} is independent of k. The payoff of A in sequences BC and CB is $U_A^{BC}(k) := (1-\beta)S_{AB}^{BC}(k)+\beta O_A^{BC}$ and $U_A^{CB}(k) := (1-\gamma)S_{AC}^{CB}(k)+\gamma O_A^{CB}$, respectively. Therefore, Lemma 2 implies that

$$\frac{\partial}{\partial k}\left(U_A^{CB}(k)-U_A^{BC}(k)\right)|_{k=0}>0.$$

If $k=0$, the bargaining problems do not interact, and $U_A^{BC}(0)=U_A^{CB}(0)$. It follows that for sufficiently small $k>0$, $U_A^{CB}(k)>U_A^{BC}(k)$.

We next show statement (iii). To do so, we start with a similar argument as in the proof of Proposition 3. Since $c^{CB}=\arg\max_{c\in\mathcal{C}}S_{AC}^{CB}(c)$, $S_{AB}^{CB}\left(c^{CB}\right)\geq S_{AB}^{CB}\left(b^{BC}\right)$, and, hence,

$$\begin{aligned}&U_A^{CB}-U_A^{BC}\\ &=(1-\gamma)S_{AC}^{CB}\left(c^{CB}\right)-(1-\beta)S_{AB}^{BC}\left(b^{BC}\right)+\gamma O_A^{CB}-\beta O_A^{BC}\\ &\geq(1-\gamma)S_{AC}^{CB}\left(b^{BC}\right)-(1-\beta)S_{AB}^{BC}\left(b^{BC}\right)+\gamma O_A^{CB}-\beta O_A^{BC}\\ &=(\beta-\gamma)\left(\left(u_A\left(b^{BC},0\right)+u_B\left(b^{BC},f\left(b^{BC}\right)\right)\right)-\max_{b\in\mathcal{B}}\{u_A(b,0)+u_B(b,0)\}\right).\end{aligned}$$

Therefore it is sufficient to establish that, for sufficiently large k,

$$u_A\left(b^{BC},0\right)+u_B\left(b^{BC},f\left(b^{BC}\right)\right)>\max_{b\in\mathcal{B}}\{u_A(b,0)+u_B(b,0)\}. \tag{18}$$

By assumption, u_B is constant in k if $c = 0$, and u_A does not depend on k. Therefore, the right-hand side of (18) is constant in k. In contrast, the left-hand side grows beyond bounds as $k \to \infty$, as we show next.

By assumption, equilibrium decisions b^{BC} and $f\left(b^{BC}\right)$ converge to $\lim_{k\to\infty} b^{BC} = \bar{b} > 0$ and $\lim_{k\to\infty} f\left(b^{BC}, k\right) = \bar{c} > 0$. Since the sets $\mathcal{B}$ and $\mathcal{C}$ are compact, $\bar{b}$ and $\bar{c}$ are finite. Making the dependence of u_B and f on k explicit, and using continuity, we have $\lim_{k\to\infty} u_B\left(b^{BC}, f\left(b^{BC}, k\right), k\right) = \lim_{k\to\infty} u_B\left(\bar{b}, \bar{c}, k\right) = \infty$. By contrast, $\lim_{k\to\infty} u_A\left(b^{BC}, 0\right) = u_A\left(\bar{b}, 0\right)$ is finite. Therefore, (18) holds for sufficiently large k. This proves statement (iii).

Finally, for intermediate k, the sequence CB no longer dominates BC as the effects that work in favor of either sequence may dominate. We show in the supplier-retailers example that either sequence can indeed be optimal (statement (ii) of the proposition). ■

Proof of Proposition 5

We start with the first-best decisions in the club good example described in the main text. Since participation and choosing the low investment is costless, both agents optimally participate. Denoting the total surplus as a function of the decisions (b, c) by $S(b, c)$, we also have $S(1, 2) = S(2, 1) = 2\phi(3) - z > 2\phi(2) = S(1, 1)$ by (4). Furthermore, $S(2, 2) = 2(\phi(4) - z) > 2\phi(3) - z = S(1, 2)$ iff $\phi(4) \geq \phi(3) + z/2$. Therefore, the first best decisions are $(b, c) \in \{(2, 1), (1, 2)\}$ if $\phi(4) < \phi(3) + z/2$, and $(b, c) = (2, 2)$ if $\phi(4) \geq \phi(3) + z/2$.[45]

We next turn to the equilibrium decisions. The second stage decision is 1, no matter which sequence is chosen or what the first-stage decision d_1 is, that is,

$$\arg\max_{d_2} u_C(d_1, d_2) = \arg\max_{d_2} u_B(d_2, d_1) = 1$$

for all $d_1 \in \{0, 1, 2\}$.

Consider the sequence BC. The first-stage decision maximizes the joint surplus of A and B

$$S_{AB}^{BC}(b) = u_B(b, 1) + (1 - \gamma) u_C(b, 1),$$

where $S_{AB}^{BC}(1) = \phi(2) + (1 - \gamma)\phi(2)$ and $S_{AB}^{BC}(2) = \phi(3) - z + (1 - \gamma)\phi(3)$. We have $S_{AB}^{BC}(2) - S_{AB}^{BC}(1) = 2\phi(3) - z - 2\phi(2) - \gamma(\phi(3) - \phi(2))$. This difference is strictly decreasing in γ, and equals zero if $\gamma = \kappa$. Since $\gamma < \kappa$, in sequence BC the first-stage decision is $b^{BC} = 2$. By symmetry, since $\beta > \kappa$ the first-stage decision in sequence CB is $c^{CB} = 1$.

We now show that A strictly prefers sequence CB if β is sufficiently high. The payoffs of A in the sequences BC and CB are $U_A^{BC} = (1 - \beta)(\phi(3)(2 - \gamma) - z) + \beta(1 - \gamma)\phi(1)$ and

[45] In case $\phi(4) = \phi(3) + z/2$, any $(b, c) \in \{(2, 1), (1, 2), (2, 2)\}$ is first best and leads to the same total surplus.

$U_A^{CB} = (1-\gamma)(\phi(2)(2-\beta)) + \gamma(1-\beta)\phi(1)$, respectively. Therefore,

$$U_A^{CB} - U_A^{BC} = (1-\gamma)(\phi(2)(2-\beta)) - (1-\beta)(\phi(3)(2-\gamma) - z) - (\beta-\gamma)\phi(1).$$

Note that, if $\beta = 1$, $U_A^{CB} - U_A^{BC} = (1-\gamma)(\phi(2) - \phi(1)) > 0$. Therefore, for β sufficiently close to 1, $U_A^{CB} > U_A^{BC}$.

It remains to show that A strictly prefers CB whenever $\gamma < \kappa < \beta$. If $\beta = \gamma = \kappa$ then $U_A^{CB} - U_A^{BC} = 0$. Moreover, given $\beta > \kappa > \gamma$, $U_A^{CB} - U_A^{BC}$ is (i) strictly increasing in β and (ii) strictly decreasing in γ, as we show next. First, since $\gamma < \kappa$ and $0 < \phi(1) < \phi(2) < \phi(3)$,

$$\begin{aligned}
\frac{\partial}{\partial\beta}\left(U_A^{CB} - U_A^{BC}\right) &= -(1-\gamma)\phi(2) + (\phi(3)(2-\gamma) - z) - \phi(1) \\
&= 2\phi(3) - \phi(2) - \phi(1) - z - \gamma(\phi(3) - \phi(2)) \\
&> 2\phi(3) - \phi(2) - \phi(1) - z - \kappa(\phi(3) - \phi(2)) \\
&= 2\phi(3) - \phi(2) - \phi(1) - z - \left(2 - \frac{z}{\phi(3) - \phi(2)}\right)(\phi(3) - \phi(2)) \\
&= \phi(2) - \phi(1) > 0.
\end{aligned}$$

Second, using $\beta > \kappa$,

$$\begin{aligned}
\frac{\partial}{\partial\gamma}\left(U_A^{CB} - U_A^{BC}\right) &= -(\phi(2)(2-\beta)) + (1-\beta)\phi(3) + \phi(1) \\
&= \phi(3) + \phi(1) - 2\phi(2) - \beta(\phi(3) - \phi(2)) \\
&< \phi(3) + \phi(1) - 2\phi(2) - \kappa(\phi(3) - \phi(2)) \\
&= \phi(3) + \phi(1) - 2\phi(2) - \left(2 - \frac{z}{\phi(3) - \phi(2)}\right)(\phi(3) - \phi(2)) \\
&= \phi(1) + z - \phi(3) < 0,
\end{aligned}$$

where the last inequality follows from assumption (4).

Finally, we show that if β is sufficiently close to 1, S^{CB} can be arbitrarily close to zero while S^{BC} can be arbitrarily high. Let $\varepsilon > 0$ be arbitrarily small and set $\phi(2) = \varepsilon/3$. Note that as $\varepsilon \to 0$ and therefore $\phi(2) \to 0$, we also have $\phi(1) \to 0$; hence, $z \to \phi(3)$ by (3) and (4). This implies that $\kappa = 2 - z/(\phi(3) - \phi(2)) \to 1$. Therefore, β must be sufficiently close to 1 to fulfill the assumption $\beta > \kappa$ for small $\varepsilon > 0$. Continuing to assume that $\beta > \kappa > \gamma$, total surplus in sequence CB is $S^{CB} = 2\phi(2) = 2\varepsilon/3 < \varepsilon$. This shows that S^{CB} can be arbitrarily small. Moreover, as $\varepsilon \to 0$, $S^{BC} = 2\phi(3) - z \to \phi(3)$, which can be arbitrarily high by choosing $\phi(4)$ even higher and letting $\phi(1)$ converge to zero sufficiently fast so that

(4) holds.[46]

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Sequencing Bilateral Negotiations with Externalities

Online Appendix (Not for Publication)

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This Online Appendix consists of 4 sections. In Section A, we provide the proofs of the propositions of the section on simultaneous negotiations—i.e., Section 6 of the paper (page 1ff.). In Section B, we provide the proofs of the propositions of the section with agent asymmetry beyond bargaining power—i.e., Section 7.1 of the paper (page 4ff.). In Section C, we provide the proofs of the propositions of the section with a general number of agents—i.e., Section 7.2 of the paper (page 5ff.). In Section D, we provide the proofs of the propositions of the section on exclusive contracts—i.e., Section 7.3 of the paper (page 17ff.).

A Proof of the propositions of Section 6

A.1 Proof of Proposition 6

Proof. We first prove Statement (i) of the proposition. The principal's payoff in sequence BC is

$$U_A^{BC} = (1-\beta)\, S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}, \tag{19}$$

with

$$S_{AB}^{BC}\left(b^{BC}\right) = u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + (1-\gamma)\left(u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right) + \gamma u_A\left(b^{BC}, 0\right)$$

and

$$O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A\left(0,c\right) + u_C\left(0,c\right)\right\}.$$

Inserting the last two expressions into (19) yields

$$(1-\beta)\left\{u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + (1-\gamma)\left(u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right) + \gamma u_A\left(b^{BC}, 0\right)\right\}$$

$$+\beta(1-\gamma)\left\{u_A\left(0, c^*(0)\right) + u_C\left(0, c^*(0)\right)\right\}.$$

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This can be written as

$$U_A^{BC} = (1-\beta)(1-\gamma)\left\{u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right\} \quad (20)$$

$$+(1-\beta)\gamma\left\{u_A(b^{BC}, 0) + u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right\} + \beta(1-\gamma)\left\{u_A\left(0, c^*(0)\right) + u_C\left(0, c^*(0)\right)\right\}.$$

We now compare (20) with (7). We start with the last term of (7). If u_A is additive-separable, then $u_A(b,c) = u_A(b,0) + u_A(0,c)$ for all b and c. Therefore, the last term of (7) is zero.

Looking at the first and the second term of (20), it is easy to see that the structure is the same as the one of the first two terms of (7). However, the arguments are different. In (7), they are $b^\star$ and $c^\star$ or $b^\star$ and 0, whereas in (20) they are b^{BC} and $c^*\left(b^{BC}\right)$ or b^{BC} and 0. If b^{BC} were equal to $b^\star$, then $c^*\left(b^{BC}\right)$ would also be equal to $c^\star$ because the maximization problem with respect to c is then the same in the simultaneous and the sequential timing. However, b^{BC} is chosen to maximize the first two terms of (20) (i.e., taken into account the reaction of c in the second stage). Therefore, by a revealed preference argument, if b^{BC} differs from $b^\star$, the first two terms of (20) must be larger than the corresponding ones of (7).

Finally, we need to compare the last term of (20) (i.e., $\beta(1-\gamma)\left\{u_A\left(0, c^*(0)\right) + u_C\left(0, c^*(0)\right)\right\}$), with the third term of (7) (i.e., $\beta(1-\gamma)\left\{u_A(0,c^\star) + u_C(b^\star, c^\star)\right\}$). Since $b^\star \geq 0$ and $c^*(0)$ maximizes $u_A(0,c) + u_C(0,c)$, it is evident that the latter term is lower then the former if externalities are negative. It follows that all terms in (7) are weakly lower than those in (20) if externalities are negative and u_A is additively separable. It follows that $U_A^{BC} \geq U_A^{sim}$ in that case. In addition, unless equilibrium decisions are not the same in both cases, the inequality is strict if externalities are strictly negative.

We next turn to Statement (ii) and again compare (20) with (7). Starting with the first two terms of each expressions, the argument made in the first part of the proof does not depend on externalities: as b^{BC} is chosen to maximize these terms whereas $b^\star$ is not, these terms must be weakly larger in (20) than in (7). Comparing the third term of (7) with the last term of (20), as there are no externalities, the difference in the first argument of u_C in both terms is irrelevant. Therefore, the driving force in the difference between these two terms is that $c^*(0)$ maximizes $u_A(0,c) + u_C(0,c)$ but $c^\star$ does not necessarily do so. As a consequence, this term is also weakly larger in (20) than in (7). Finally, if u_A is super-modular, than $u_A(b,c) \geq u_A(b,0) + u_C(0,c)$, which implies that the last term of (7) is negative. It follows that sequence BC is preferred by the principal if u_A is super-modular. Moreover, the principal strictly prefers sequence BC if u_A is strictly super-modular and equilibrium decisions are not zero.

We now turn to the case in which u_A is sub-modular. Then the difference in the first

three terms between (20) and (7) is non-negative, as established above. If, in addition, $\gamma = 0$, the last term in (7) drops out; hence, $U_A^{BC} \geq U_A^{sim}$. Moreover, at $\gamma = 0$ we have $U_A^{BC} > U_A^{sim}$ if equilibrium decisions differ in sequential versus simultaneous negotiations. By continuity, the result then also holds for γ close to 0.

Finally, suppose $\gamma \to 1$, which implies that $\beta = 1$ (since $\gamma < \beta$). By (20), $U_A^{BC} = 0$. By (7), $U_A^{sim} = u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star) \geq 0$ because u_A is sub-modular. Hence, $U_A^{sim} \geq U_A^{BC}$. Moreover, for $\gamma = 1$ we have $U_A^{sim} > U_A^{BC}$ if u_A is strictly sub-modular and equilibrium decisions are not zero. By continuity, the result then also holds for γ close to 1. ■

A.2 Proof of Proposition 7

Proof. From the proof of Proposition 4 we know that, when evaluated at $k = 0$,

$$\frac{d}{dk} U_A^{CB}(k)\,|_{k=0} = \frac{\partial}{\partial k}\left((1-\gamma)u_C(b,c,k) + (1-\gamma)(1-\beta)\,u_B(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}.$$

Applying the same logic to (7) , we obtain

$$\begin{aligned}\frac{d}{dk} U_A^{sim}(k)\,|_{k=0} = \frac{\partial}{\partial k}\Big((1-\beta)(1-\gamma)\left(u_B(b,c,k) + u_C(b,c,k)\right)\\ + (1-\beta)\,\gamma u_B(b,c,k) + \beta(1-\gamma)u_C(b,c,k)\Big)\Big|_{\substack{k=0\\ b=b^\star\\ c=c^\star}}\\ = \frac{\partial}{\partial k}\left((1-\gamma)u_C(b,c,k) + (1-\beta)\,u_B(b,c,k)\right)\Big|_{\substack{k=0\\ b=b^\star\\ c=c^\star}}\end{aligned}$$

Symmetry of agents, no externalities ($k = 0$), and u_A being additively separable imply that $b^\star = c^\star = f(0,0)$. As a consequence,

$$\frac{d}{dk}\left\{U_A^{sim}(k) - U_A^{CB}(k)\right\}|_{k=0} = \frac{\partial}{\partial k}\left(\gamma(1-\beta)u_B(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}} > 0\,.$$

If $k = 0$, the bargaining problems do not interact, and $U_A^{sim}(0) = U_A^{CB}(0)$. It follows that for sufficiently small $k > 0$, $U_A^{sim}(k) > U_A^{CB}(k)$. ■

B Proof of the propositions of Section 7.1

B.1 Proof of Proposition 8

Proof. In sequence BC, the expected payoff of A is

$$U_A^{BC} = (1-\beta) S_{AB}^{BC} + \beta O_A^{BC}$$

where

$$S_{AB}^{BC} = u_B(1,1) + (1-\gamma)(u_A(1,1) + u_C(1,1)) + \gamma u_A(1,0)$$

is the joint surplus of A and B and

$$O_A^{BC} = (1-\gamma)(u_A(0,1) + u_C(0,1))$$

is the outside option of A in stage 1. Thus,

$$\begin{aligned} U_A^{BC} &= (1-\beta)(u_B(1,1) + (1-\gamma)(u_A(1,1) + u_C(1,1)) + \gamma u_A(1,0)) \\ &\quad +\beta(1-\gamma)(u_A(0,1) + u_C(0,1)). \end{aligned}$$

Similarly, the expected payoff of A in sequence CB is

$$\begin{aligned} U_A^{CB} &= (1-\gamma)(u_C(1,1) + (1-\beta)(u_A(1,1) + u_B(1,1)) + \beta u_A(0,1)) \\ &\quad +\gamma(1-\beta)(u_A(1,0) + u_B(1,0)). \end{aligned}$$

Therefore,

$$\begin{aligned} U_A^{BC} - U_A^{CB} &= (1-\beta)\gamma(u_B(1,1) - u_B(1,0)) \\ &\quad -\beta(1-\gamma)(u_C(1,1) - u_C(0,1)) \\ &= (\beta-\gamma)(u_B(1,0) - u_B(1,1)) \\ &\quad +\beta(1-\gamma)(u_B(1,1) - u_B(1,0) - (u_C(1,1) - u_C(0,1))). \end{aligned}$$

■

B.2 Proof of Proposition 9

Proof. Since $\beta = 1$, $U_A^{BC} = O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\}$. In contrast, in sequence CB, $U_A^{CB} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}$, where

$$b^*(c) = \arg\max_{b\in\mathcal{B}}\{u_A(b,c) + u_B(b,c)\}.$$

Therefore,

$$U_A^{BC} - U_A^{CB} = (1-\gamma)\left(\max_{c\in\mathcal{C}}\{(u_A(0,c) + u_C(0,c))\} - \max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}\right).$$

When there are negative externalities of b on C, then $u_C(0,c) \geq u_C(b,c)$ for all b, c. Therefore, $U_A^{BC} \geq U_A^{CB}$.[1] The results on positive and no externalities can be established similarly. ■

C Proofs of the propositions of Section 7.2

We call the agent with whom A negotiates in stage t agent t. The bargaining power of agent t in sequence T is denoted by $\beta_{T,t}$. The decision in stage t is denoted by $d_{T,t}$. We omit the reference to the sequence T when it is clear from the context. As mentioned in the main text, we assume that each negotiation subgame has a unique subgame perfect Nash equilibrium (SPNE), after breaking ties that a player accepts when indifferent between accepting or rejecting an offer. Moreover, A commits to a sequence of negotiations ex ante. Prior to the negotiations, the sequence chosen by A becomes common knowledge.

The section proceeds in several steps. We first derive a tractable expression for the principal's expected payoff in a given sequence (Section C.1). To do so, we show that this payoff has a recursive structure, which can be used to find an algorithm that determines A's payoff. We next compare two different sequences with each other, in which we exchange the positions of two adjacent agents (Section C.2). We finally use the insights from this comparison to prove Propositions 10 and 11 (Section C.3).

C.1 The principal's expected payoff

This subsection investigates the structure of A's equilibrium payoff in a given sequence. We allow for externalities between agents, interactions of the decisions in u_A, and do not assume

[1] Moreover, when externalities are strictly negative and $c^{CB} \neq 0 \neq b^*(c)$, then $U_A^{BC} > U_A^{CB}$.

symmetry.[2] We will only use symmetry later in the proofs of Propositions 10 and 11.

Recursion relation for A's payoff To obtain a recursive structure for A's payoff, we determine A's expected payoff in the subgame starting at the beginning of stage t — i.e., before nature draws the proposer in stage t. We denote by h_{t-1} the history of decisions until this stage, that is, $h_{t-1} = (d_1, ..., d_{t-1})$. Let then $U_A^t\left|_{h_{t-1}}\right.(T)$ be A's expected SPNE payoff in the subgame starting at stage t, given sequence T. This payoff is net of the transfers from the agents who have already bargained with A, but includes expected future transfers.[3] Let h_0 denote the beginning of the game. For example, $U_A^1\left|_{h_0}\right.(T)$ is A's expected payoff at the beginning of the game. Let $f_T(h_t)$ denote the path of SPNE decisions in a subgame starting after history h_t in sequence T.

Consider stage t, and a history of decisions h_{t-1}. With probability β_t, the agent proposes. In this case, the principal's payoff is given by her outside option, which is to continue the game in $t+1$ with a decision $d_t = 0$ and a transfer $t_t = 0$. With probability $(1-\beta_t)$, the principal proposes. The principal will then set the transfer equal to the largest transfer the agent is willing to accept, $t_t = u_t(h_{t-1}, d_t, f_T(h_{t-1}, d_t))$. She will choose the decision d_t to maximize her payoff, which is (again net of transfers prior to stage t, which are sunk in stage t) :

$$u_t(h_{t-1}, d_t, f_T(h_{t-1}, d_t)) + U_A^{t+1}\left|_{(h_{t-1},d_t)}\right.(T).$$

These considerations show that the following recursion relation for A's payoffs holds for $t = 1, ..., N-1$:

$$\begin{aligned} &U_A^t\left|_{h_{t-1}}\right.(T) \\ &= \beta_{T,t} U_A^{t+1}\left|_{(h_{t-1},0)}\right.(T) + (1-\beta_{T,t}) \max_{d_t}\left\{u_t(h_{t-1}, d_t, f_T(h_{t-1}, d_t)) + U_A^{t+1}\left|_{(h_{t-1},d_t)}\right.(T)\right\} \end{aligned} \tag{21}$$

A similar reasoning applied to stage N shows that the following initial condition holds:

$$\begin{aligned} &U_A^N\left|_{h_{N-1}}\right.(T) \\ &= \beta_{T,N} u_A(h_{N-1}, 0) + (1-\beta_{T,N}) \max_{d_N}\{u_N(h_{N-1}, d_N) + u_A(h_{N-1}, d_N)\}. \end{aligned} \tag{22}$$

[2]The precise definition of symmetry in case of N agents is that (i) u_A is symmetric in that any permutation of $(d_1, ..., d_n)$ leaves u_A constant, (ii) for any two agents i and $j \neq i$, $u_i(d') = u_j(d)$ where d' denotes the vector of decisions obtained from $d = (d_1, ..., d_n)$ by exchanging d_i with d_j, and (iii) any permutation of the decisions of agents $k \neq i$ leaves u_i constant.

[3]Note that the history of decisions h_{t-1} is comprised only of the parts of the history up to stage t, which are relevant for the equilibrium in the subgame. In contrast, the past transfers, who proposed in which stage, and the acceptance or rejection decisions do not influence the equilibrium of the subgame after a history of decisions h_{t-1}.

Let $f_{T,t}(h_{t-1})$ denote the SPNE decision taken in stage t of sequence T, after a history of decisions h_{t-1}. Note that $f_{T,t}(h_{t-1})$ is the first component of the vector $f_T(h_{t-1})$.[4] In a SPNE, the decision taken in stage t maximizes the joint surplus of the principal and agent t, regardless who proposes. Therefore,

$$f_{T,N}(h_{N-1}) = \arg\max_{d_N}\{u_N(h_{N-1}, d_N) + u_A(h_{N-1}, d_N)\} \tag{23}$$

and

$$f_{T,t}(h_{t-1}) = \arg\max_{d_t}\left\{u_t(h_{t-1}, d_t, f_T(h_{t-1}, d_t)) + U_A^{t+1}\big|_{(h_{t-1},d_t)}(T)\right\}. \tag{24}$$

Expected payoff in different subgames We now define the *fictitious history*, a concept we will use to find a succinct expression for the principal's equilibrium payoff. Let $r_t = 1$ if A responds in stage t, and $r_t = 0$ if A proposes in stage t. An event r is a vector $r = (r_1, ..., r_N)$ describing who proposes in which stage. Using that $f_{T,1}(h_0)$ is the SPNE decision in $t = 1$ in sequence T, we now recursively define a *fictitious history* as follows. Let

$$\eta_{T,1} = \begin{cases} f_{T,1}(h_0), & \text{if } r_1 = 0, \\ 0, & \text{if } r_1 = 1; \end{cases}$$

moreover, for $t = 2, ..., N$:

$$\eta_{T,t} = \begin{cases} (\eta_{T,t-1}, f_{T,t}(\eta_{T,t-1})), & \text{if } r_t = 0, \\ (\eta_{T,t-1}, 0), & \text{if } r_t = 1, \end{cases}$$

where $\eta_{T,t}$ is a vector of t decisions. In general, it differs across sequences and events. The relevant sequence is indicated in the first subscript. We sometimes write $\eta_{T,t}(r)$ to indicate the relevant event. Note that $\eta_{T,t}(r)$ depends on $(r_1, ..., r_t)$, but not on $(r_{t+1}, ..., r_N)$.

In words, the fictitious history is built up as follows. If A proposes in a stage t ($r_t = 0$), the decision in t is the SPNE decision $f_{T,t}(\eta_{t-1})$. If A responds in a stage t ($r_t = 1$), the decision in t is 0.

We can similarly define a fictitious history after a given history of decisions h_{t-1}: let

$$\eta_{T,t}(h_{t-1}, r) = \begin{cases} (h_{t-1}, f_{T,t}(h_{t-1})), & \text{if } r_t = 0, \\ (h_{t-1}, 0), & \text{if } r_t = 1, \end{cases}$$

[4]Recall that $f_T(h_{t-1})$ is a path of SPNE decisions. Since h_{t-1} is a history of $t-1$ decisions, $N-(t-1)$ agents remain, thus $f_T(h_{t-1})$ is vector of $N-(t-1)$ decisions.

and for $\tau = t+1, ..., N$

$$\eta_{T,\tau}(h_{t-1}, r) = \begin{cases} (\eta_{T,\tau-1}(h_{t-1}, r), f_{T,\tau}(\eta_{T,\tau-1}(h_{t-1}, r))), & \text{if } r_\tau = 0, \\ (\eta_{T,\tau-1}(h_{t-1}, r), 0), & \text{if } r_\tau = 1. \end{cases}$$

For example, $\eta_{T,\tau}(h_{t-1}, r)$ is a vector of length τ consisting of the decisions h_{t-1} in the first $t-1$ stages, and the fictitious history for decisions in stage t up to stage τ. Note that for $\tau = t+1, ..., N$, $\eta_{T,\tau}(h_{t-1}, r)$ depends on $(r_t, ..., r_\tau)$, but is independent of $(r_1, ..., r_{t-1})$ and $(r_{\tau+1}, ..., r_N)$, as the former is included in h_{t-1} and the latter is not decided yet. This implies that $\eta_{T,N}(h_{t-1}, r)$ denotes a history of N decisions where the first $t-1$ decisions are given by h_{t-1}, and the decisions in the remaining stages are given by the fictitious history, started at t.

Iteratively applying the recursion relation (21) to the initial condition (22) shows that the expected payoff of A at the start of a subgame in stage t (net of transfers already received from agents $1, ..., t-1$) is

$$U_A^t|_{h_{t-1}}(T) = \sum_{r \in \{0,1\}^{N-t+1}} \prod_{i=t}^{N} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} [u_A(\eta_{T,N}(h_{t-1}, r)) + \sum_{j=t}^{N} u_j(\eta_{T,j}(h_{t-1}, r), f_T(\eta_{T,j}(h_{t-1}, r)))]. \tag{25}$$

In particular, using the definitions of the fictitious history, whenever $r_t = 1$ (i.e., whenever the agent proposes), the decision that enters the fictitious history $\eta_{T,t}$ as the second argument equals zero, which implies $\eta_{T,t}(h_{t-1}, 1) = (h_{t-1}, 0)$. This occurs with probability $\beta_{T,t}$. Instead, with probability $1-\beta_{T,t}$, the principal proposes. This implies $r_t = 0$ and $\eta_{T,t}(h_{t-1}, 0) = (h_{t-1}, f_{T,t}(h_{t-1}))$.

Since equations (21) and (22) uniquely pin down the expected payoff in any subgame, net of previous transfers, A's expected payoff is as given in equation (25).[5] Evaluating (25) at $t = 1$ then yields A's ex ante expected payoff:

$$U_A(T) = U_A^1|_{h_0}(T) = \tag{26}$$
$$= \sum_{r \in \{0,1\}^N} \prod_{i=1}^{N} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} \left(u_A(\eta_{T,N}(r)) + \sum_{j=1}^{N} u_j(\eta_{T,j}(r), f_T(\eta_{T,j}(r))) \right).$$

Following this procedure, which automatically sets the decision d_i to zero, if the agent i

[5] Indeed, evaluating (25) at $t = N$ yields (22).

proposes in stage i, and sets $d_i = f_{T,i}\left(\eta_{T,i-1}\left(r\right)\right)$ if A proposes in stage i, is not a game, but rather an algorithm that helps to tractably analyze A's payoff for different sequences.

C.2 Comparison between different sequences

In this section, we use (26) to compare two sequences, denoted by S and T. Recall from above that the SPNE decisions in the negotiation with agent i in case of sequence T are denoted by $f_T\left(\cdot\right)$ whereas the ones in case of sequence S are $f_S\left(\cdot\right)$, and the two may differ.

To compare the two sequences, suppose that the SPNE decisions of sequence T are used in sequence S. We denote A's expected payoff in this case by $V_A\left(S,T\right)$, with

$$V_A\left(S,T\right) := \sum_{r\in\{0,1\}^N}\prod_{i=1}^{N}\beta_{S,i}^{r_i}\left(1-\beta_{S,i}\right)^{1-r_i}\left(u_A\left(\eta_{T,N}\left(r\right)\right)+\sum_{i=1}^{N}u_i\left(\eta_{T,i}\left(r\right),f_T\left(\eta_{T,i}\left(r\right)\right)\right)\right). \tag{27}$$

By construction, if S and T are the same sequence, then the expected payoff equals A's expected payoff from this sequence of the bargaining game. Thus $V_A\left(T,T\right) = U_A\left(T\right)$. Moreover, as we will show in Subsection C.3 below, under the assumptions of Propositions 10 and 11, $V_A\left(S,T\right)$ is a lower bound on A's equilibrium payoff in the sequence S: $U_A\left(S\right) \geq V_A\left(S,T\right)$.[6] Therefore, to show that A's expected payoff is higher in sequence S, it is enough to show that $V_A\left(S,T\right) \geq V_A\left(T,T\right)$.

We next provide two preliminary steps (shown in Lemmas 3 and 4) that lead to a tractable formula, which will help us to determine the conditions under which the principal bargains with the agents in descending order of their bargaining power. Suppose that in sequence T, the principal does not bargain in this order. This implies that there exist stages $\hat{\imath}$ and $\hat{\imath}+1$ such that $\beta_{T,\hat{\imath}} < \beta_{T,\hat{\imath}+1}$. We will show that exchanging agents $\hat{\imath}$ and $\hat{\imath}+1$, keeping the sequence of agents constant otherwise, increases A's payoff. The new sequence after this adjacent pairwise interchange will be denoted by S. Our aim is to show that $U_A\left(S\right) \geq U_A\left(T\right)$.

To do so, we first use the relation between the sequences S and T to derive a result on

[6] This is trivial under the assumptions of Proposition 10, where equilibrium decisions do not depend on the negotiation sequence and thus $U_A\left(S\right) = V_A\left(S,S\right) = V_A\left(S,T\right)$. Under the assumptions of Proposition 11, we prove that $V_A\left(S,S\right) \geq V_A\left(S,T\right)$ below.

the difference $V_A(S,T) - V_A(T,T)$. Let

$$V_A^t(S,T)\big|_{(r_1,\ldots,r_{t-1})} := \sum_{(r_t,\ldots,r_N)\in\{0,1\}^{N-t+1}} \prod_{i=t}^{N} \beta_{S,i}^{r_i}\left(1-\beta_{S,i}\right)^{1-r_i} \tag{28}$$
$$* \left(u_A\left(\eta_{T,N}(r)\right) + \sum_{i=1}^{N} u_i\left(\eta_{T,i}(r), f_T\left(\eta_{T,i}(r)\right)\right)\right),$$

that is, $V_A^t(S,T)$ is A's interim expected payoff starting from stage t when using sequence S, but the decisions are the SPNE decisions from sequence T. Note that

$$V_A(S,T) = \sum_{(r_1,\ldots,r_{t-1})\in\{0,1\}^{t-1}} \prod_{i=1}^{t-1} \beta_{S,i}^{r_i}\left(1-\beta_{S,i}\right)^{1-r_i} V_A^t(S,T)\big|_{(r_1,\ldots,r_{t-1})}. \tag{29}$$

Lemma 3 *Consider a sequence T such that $\beta_{T,\hat{\imath}} < \beta_{T,\hat{\imath}+1}$, and a sequence S which is identical to sequence T except that agents $\hat{\imath}$ and $\hat{\imath}+1$ change places. Then*

$$V_A(S,T) - V_A(T,T)$$
$$= \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i}\left(\beta_{T,\hat{\imath}+1}-\beta_{T,\hat{\imath}}\right)\left(V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},0,1)}\right).$$

Proof. From (29),

$$V_A(T,T) = \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} V_A^{\hat{\imath}}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})}.$$

Similarly,

$$V_A(S,T) = \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{S,i}^{r_i}\left(1-\beta_{S,i}\right)^{1-r_i} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})}$$
$$= \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})},$$

where the second equality follows from $\beta_{S,i} = \beta_{T,i}$ for $i < \hat{\imath}$. Thus,

$$V_A(S,T) - V_A(T,T) \tag{30}$$
$$= \sum_{(r_1,...,r_{\hat{\imath}-1}) \in \{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} \left(V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} - V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} \right).$$

By the construction of (28), we can write

$$V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} = \beta_{T,\hat{\imath}} V_A^{\hat{\imath}+1}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1)} + (1-\beta_{T,\hat{\imath}}) V_A^{\hat{\imath}+1}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0)}.$$

Iterating one more time gives

$$\begin{aligned} V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} &= \beta_{T,\hat{\imath}}\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ &+ \beta_{T,\hat{\imath}}(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ &+ (1-\beta_{T,\hat{\imath}})\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ &+ (1-\beta_{T,\hat{\imath}})(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

Similarly,

$$\begin{aligned} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{t-1})} &= \beta_{S,\hat{\imath}}\beta_{S,\hat{\imath}+1} V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ &+ \beta_{S,\hat{\imath}}(1-\beta_{S,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ &+ (1-\beta_{S,\hat{\imath}})\beta_{S,\hat{\imath}+1} V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ &+ (1-\beta_{S,\hat{\imath}})(1-\beta_{S,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

For agents $i = \hat{\imath}+1, ..., N$, we have $\beta_{S,i} = \beta_{T,i}$. Thus for for any given $(r_1, ..., r_{\hat{\imath}+1})$,

$$V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}+1})} = V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}+1})}.$$

By assumption, $\beta_{S,\hat{\imath}} = \beta_{T,\hat{\imath}+1}$ and $\beta_{S,\hat{\imath}+1} = \beta_{T,\hat{\imath}}$. Thus

$$\begin{aligned} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{t-1})} &= \beta_{T,\hat{\imath}}\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ &+ \beta_{T,\hat{\imath}+1}(1-\beta_{T,\hat{\imath}}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ &+ (1-\beta_{T,\hat{\imath}+1})\beta_{T,\hat{\imath}} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ &+ (1-\beta_{T,\hat{\imath}})(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

Therefore,

$$V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} - V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})}$$
$$= (\beta_{T,\hat{\imath}+1} - \beta_{T,\hat{\imath}})\left(V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}\right).$$

Together with (30), this concludes the proof of the lemma. ■

Next, we will investigate the difference $V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}$. Consider the events $(r_1,...,r_{\hat{\imath}-1},1,0)$ and $(r_1,...,r_{\hat{\imath}-1},0,1)$. In stages $1,...,\hat{\imath}-1$, A accumulates the same transfers from the agents in the two events under consideration, since these two events differ only in later stages. Moreover, in the fictitious history, A proposes and collects a transfer in $\hat{\imath}+1$ in event $(r_1,...,r_{\hat{\imath}-1},1,0)$, but does not receive a transfer in $\hat{\imath}$, where she responds. In contrast, in event $(r_1,...,r_{\hat{\imath}-1},0,1)$, A receives a transfer in $\hat{\imath}$ but not in $\hat{\imath}+1$. The next lemma uses this difference to determine an expression for $V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}$.

Let

$$r_{(0,1)} := (r_1,...,r_{\hat{\imath}-1},0,1,r_{\hat{\imath}+2},...,r_n)$$
$$r_{(1,0)} := (r_1,...,r_{\hat{\imath}-1},1,0,r_{\hat{\imath}+2},...,r_n)$$

Note that $\eta_{T,\hat{\imath}-1}\left(r_{(0,1)}\right) = \eta_{T,\hat{\imath}-1}\left(r_{(1,0)}\right)$. To shorten notation, let

$$\hat{\eta}_{\hat{\imath}-1} := \eta_{T,\hat{\imath}-1}\left(r_{(0,1)}\right) = \eta_{T,\hat{\imath}-1}\left(r_{(1,0)}\right).$$

Lemma 4 *For all* $(r_1,...,r_{\hat{\imath}-1})$,

$$V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}$$
$$= u_{\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0,f_T\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T)$$
$$- \left(u_{\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1},f_T\left(\hat{\eta}_{\hat{\imath}-1}\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)}(T)\right).$$

Proof. By definition,

$$V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} = \sum_{(r_{\hat{\imath}+2},...,r_N)}\prod_{i=\hat{\imath}+2}^{N}\beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i}$$
$$* \left(u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) + \sum_{i=1}^{N}u_i\left(\eta_{T,i}\left(r_{(1,0)}\right),f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right)\right)$$

and

$$V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} = \sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} * \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=1}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right)$$

The vectors $r_{(0,1)}$ and $r_{(1,0)}$ do not differ in their components 1 to $\hat{\imath}-1$. Therefore,

$$\sum_{i=1}^{\hat{\imath}-1} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right) = \sum_{i=1}^{\hat{\imath}-1} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right).$$

Moreover, in the event $r_{(1,0)}$, A responds in $\hat{\imath}$; hence, in the fictitious history, she does not obtain a transfer in this stage. Similarly, in event $r_{(0,1)}$, A does not obtain a transfer in the fictitious history in stage $\hat{\imath}+1$. Therefore,

$$\begin{aligned}
&V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\
&= \sum_{(r_{\hat{\imath}+2},...,r_N)\in\{0,1\}^{N-t}} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} \\
&* [u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) \\
&+ u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right) \\
&\quad - u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) \\
&\quad - u_{\hat{\imath}}\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right), f_T\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right)\right)\right) - \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)].
\end{aligned}$$

The fictitious history $\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right)$ does not depend on $(r_{\hat{\imath}+2},...,r_N)$; hence, the transfers paid in stage $\hat{\imath}+1$ in event $r_{(1,0)}$, which is $u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right)$, can be taken out of the expectation. Similarly, the transfer paid in stage $\hat{\imath}$ in event $r_{(0,1)}$ can

be taken out of the expectation. Thus

$$
\begin{aligned}
&V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\
&= u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right) - u_{\hat{\imath}}\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right), f_T\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right)\right)\right) \\
&+ \sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \left(u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right)\right) \\
&- \sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right).
\end{aligned}
$$

The result then follows from (25). To see this, evaluate (25) at $t = \hat{\imath}+2$ and history $(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0))$. Note that by definition

$$
\eta_{T,i}\left(\left(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1}, 0\right)\right), r\right) = \eta_{T,i}\left(r_{(1,0)}\right)
$$

for $i = \hat{\imath}+2, ..., N$. This shows

$$
\begin{aligned}
U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0)\right)}(T) = &\sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \\
&* \left(u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right)\right).
\end{aligned}
$$

Similarly, evaluating (25) at $t = \hat{\imath}+2$ and history $(\hat{\eta}_{\hat{\imath}-1}, f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}), 0)$ shows

$$
\begin{aligned}
U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0\right)}(T) = &\sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \\
&* \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right).
\end{aligned}
$$

■

Recall that $\beta_{T,\hat{\imath}+1} > \beta_{T,\hat{\imath}}$ by assumption. By the last two lemmas, $V_A(S,T) \geq V_A(T,T)$ if

$$
\begin{aligned}
&u_{\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1}, 0, f_T\left(\hat{\eta}_{\hat{\imath}-1}, 0\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0)\right)}(T) \\
&\geq u_{\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}, f_T\left(\hat{\eta}_{\hat{\imath}-1}\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0\right)}(T).
\end{aligned} \tag{31}
$$

We will next show that this inequality is fulfilled under the assumptions of Propositions 11,

and 10 with negative externalities. As mentioned above, we will also show that $U_A(S) \geq V_A(S,T)$ under these assumptions.

C.3 Proof of Propositions 10 and 11

Proof of Proposition 10 We prove inequality (31) under the assumptions of Proposition 10. Consider the case of binary decisions, $d_i \in \{0,1\}$, and assume that participation is always optimal. That is, $f_{T,t}(h_{t-1}) = f_{S,t}(h_{t-1}) = 1$ for all t and h_{t-1}. By symmetry,

$$U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0))}(T) = U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0)}(T),$$

since the same number of agents has participated before stage $\hat{\imath}+2$ in history $(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0))$ as in history $(\hat{\eta}_{\hat{\imath}-1}, f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}), 0)$. Therefore, inequality (31) simplifies to

$$u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0)) \geq u_{\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1})). \tag{32}$$

We focus on the case of negative externalities. (The proof for positive externalities is similar.) In the history $(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0))$, agent $\hat{\imath}$ does not participate, whereas in $(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1}))$, agent $\hat{\imath}$ does participate; in all other respects these histories are identical. By negative externalities and symmetry, it follows that

$$u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0)) \geq u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 1, f_T(\hat{\eta}_{\hat{\imath}-1}), 1) = u_{\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1})).$$

Finally, with binary decisions and participation of all agents, $f_{T,t}(h_{t-1}) = f_{S,t}(h_{t-1}) = 1$, which implies $V_A(S,S) = V_A(S,T)$. This concludes the proof of Proposition 10. ■

It is evident that the proof of inequality (32) follows the logic of the expected-externality effect.

Proof of Proposition 11 Suppose there are no externalities between agents. Then (31) simplifies to

$$\begin{aligned} &u_{\hat{\imath}+1}(f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0)) + U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0))}(T) \\ &\geq u_{\hat{\imath}}(f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1})) + U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0)}(T). \end{aligned} \tag{33}$$

We now prove inequality (33).

By (24), evaluated at $t = \hat{\imath}+1$ and $h_{t-1} = (\hat{\eta}_{\hat{\imath}-1}, 0)$, and the fact that there are no

externalities between agents,

$$\begin{aligned}
&u_{\hat{\imath}+1}\left(f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T)\\
&=\max_{d_{\hat{\imath}+1}}\left\{u_{\hat{\imath}+1}\left(d_{\hat{\imath}+1}\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,d_{\hat{\imath}+1}\right)}(T)\right\}\\
&\geq u_{\hat{\imath}+1}\left(f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)}(T)\\
&=u_{\hat{\imath}}\left(f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)},
\end{aligned}$$

where the last line uses symmetry.

It remains to show that A's utility in the sequence S (i.e., $U_A(S)=V_A(S,S)$) is indeed (weakly) larger than $V_A(S,T)$. To do so, recall first that $V_A(S,S)$ was obtained by automatically setting the decision to $d_t=f_{S,t}(h_{t-1})$ and the transfer to $t_t=u_t(d_t)$,[7] in case the principal proposes in stage t after a history of decisions h_{t-1}, whereas both the decision and the transfer are zero in case the agent proposes.

Suppose that instead of automatically assigning these decisions and transfers, if $r_t=0$, the principal *chooses* a decision d_t which automatically creates a transfer $t_t=u_t(d_t)$. However, if $r_t=1$, the decision d_t and the transfer t_t are automatically set to zero.

This defines a dynamic decision problem for the principal.[8] It is not a game since the agents are replaced by automata. It has two features that are crucial here. First, by construction, the optimal decision of the principal in stage t after a history of decisions h_{t-1} is

[7] The transfer depends only on d_t because there are no externalities between agents.

[8] It might be useful to describe the dynamic decision problem of A in a way that makes it clear it is a standard finite horizon stochastic dynamic programming problem. For this, we have to define a state variable, a set of admissible controls (decisions) for each stage, and the payoff of A in a given stage.

In a standard dynamic programming problem, the payoff of the decision maker in any stage t depends only on the value of the state variable in t, on controls taken in t, and possibly random terms. In our setting, A's payoff in the final stage depends on the whole history of all decisions, because u_A depends on all the decisions. We use state augmentation to deal with this problem of time delay (Bertsekas 2017, Chapter 1.4): in stage t, define the state as $s_t=(h_{t-1},r_t)$. As above, h_0 denotes the beginning of the game. For $t>1$, h_{t-1} is a vector of $t-1$ decisions. At the beginning of stage t, the binary random variable r_t realizes. The distribution of r_t is given by $\Pr(r_t=1)=\beta_{S,t}$ (which does not depend on any decisions and can thus be treated as an uncontrollable state component).

The set of decisions available in stage t is

$$D_t=\begin{cases}\{0\}, & \text{if } r_t=1,\\ \mathcal{D}, & \text{if } r_t=0,\end{cases}$$

where $\mathcal{D}$ is the set of decisions in the bargaining game.

In stage t, the random variable r_t realizes, A observes the realization, and chooses a decision $d_t\in D_t$. Note that if $r_t=1$, then A has no other choice than $d_t=0$. In stage t, r_{t+1} realizes, and the state updates according to $s_{t+1}=(h_{t-1},d_t,r_{t+1})$.

In $t=1,...,N-1$, the instantaneous payoff of A in t is $u_t(d_t)$. In $t=N$, the instantaneous payoff of A is $u_N(d_N)+u_A(h_N)$. The payoff of A is the sum of the instantaneous payoffs $u_A(h_N)+\sum_{t=1}^{N}u_t(d_t)$.

just $f_{S,t}(h_{t-1})$. Thus, the principal's expected payoff is equal to $V_A(S,S)$ in the solution to the dynamic decision problem, as in the bargaining game in sequence S. Second, by Bellmann's principal of optimality, the optimal decisions of the principal are time consistent. If the principal could commit to any other decision rule ex ante, she would not gain from this.[9] Therefore, replacing the decision rule f_S by f_T can only lead to a lower payoff. This shows that $V_A(S,S) \geq V_A(S,T)$. ■

Note that the time consistency argument for $V_A(S,S) \geq V_A(S,T)$ depends on there being no externalities between agents. In contrast, if there are externalities between agents, the transfer from an agent depends also on the future equilibrium decisions, and a reduction to a decision problem as in the last paragraphs of the proof is no longer possible.

D Proofs of the propositions of Section 7.3

D.1 Proof of Proposition 12

Call the agent who negotiates in stage $i = 1, 2$ agent i, with bargaining power β_i. Let d_i denote the decision with agent i, and t_i the transfer of agent i. A contract in stage 1 is either non-exclusive and fixes (d_1, t_1) independent of what happens in stage 2, or exclusive and fixes (d_1, t_1) together with the exclusion clause that $d_2 = 0$, which fully binds A.

Consider stage 2. If in stage 1 an exclusive contract was signed, then $d_2 = t_2 = 0$. If in stage 1 a non-exclusive contract was signed, stage 2 is as in our main model.

Consider next stage 1. The principal and agent 1 will agree on an exclusive dealing contract if

$$\begin{aligned} &\max_{d_1} \{u_1(d_1, 0) + u_A(d_1, 0)\} \\ &> \max_{d_1} \{u_1(d_1, f(d_1)) + \beta_2 u_A(d_1, 0) + (1-\beta_2)(u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)))\} \end{aligned} \tag{34}$$

The left-hand side is their joint surplus from an exclusive dealing contract, whereas the right-hand side is their joint surplus without exclusion (as in our main model). Without loss of generality, we break the tie in favor of non-exclusion.

Note that

$$\begin{aligned} u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) &= \max_{d_2} \{u_A(d_1, d_2) + u_2(d_1, d_2)\} \\ &\geq u_A(d_1, 0) + u_2(d_1, 0) = u_A(d_1, 0). \end{aligned}$$

[9] See, for example, Proposition 1.3.1 in Bertsekas (2017).

Therefore, the joint surplus of A and agent 1 without exclusion (given by the right-hand-side of (34)) is decreasing in β_2.

An exclusive dealing contract commits A not to deal with agent 2. Thus the joint surplus of A and agent 1 from an exclusive contract does not depend on β_2. By contrast, their joint payoff from a non-exclusive contract is decreasing in β_2, as shown above. It follows that, if exclusive contracts are used in sequence BC (where $\beta_2 = \gamma$) they are also used in sequence CB (where $\beta_2 = \beta > \gamma$). This leaves three possible cases: (i) there is exclusion both in BC and in CB, (ii) exclusion in CB but not in BC, and (iii) in both sequences there is no exclusion.

We begin with case (i): exclusion in BC and in CB.

Lemma 5 *Suppose that*

$$\begin{aligned} &\max_b \{u_B(b,0) + u_A(b,0)\} \\ &> \max_b \{u_B(b, f(b)) + \gamma u_A(b,0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\}. \end{aligned} \tag{35}$$

Then there is exclusion in sequence BC and in sequence CB; hence, $U_A^{BC} = U_A^{CB}$ and $S^{BC} = S^{CB}$.

Proof. By (34), there is exclusion in both sequences. Consider sequence BC. If the principal proposes, agent B is willing to pay up to $t_B = u_B(b,0)$, and A's payoff is $\max_b \{u_A(b,0) + u_B(b,0)\}$. If agent B proposes, the principal's payoff is given by her outside option

$$(1-\gamma) \max_c \{u_A(0,c) + u_C(0,c)\}.$$

In expectation, therefore, the principal's payoff in sequence BC is

$$\begin{aligned} U_A^{BC} &= (1-\beta) \max_b \{u_A(b,0) + u_B(b,0)\} + \beta(1-\gamma) \max_c \{u_A(0,c) + u_C(0,c)\} \\ &= (1-\beta\gamma) \max_b \{u_A(b,0) + u_B(b,0)\}. \end{aligned}$$

Note this depends only on the product $\beta\gamma$. Therefore, A's expected payoff from exclusive contracts is the same in sequences BC and CB : $U_A^{BC} = U_A^{CB}$. Moreover, the joint surplus of A, B and C is

$$S^{BC} = S^{CB} = \hat{S} := \max_{d_1} \{u_A(d_1,0) + u_1(d_1,0)\}$$

in both sequences. ■

We now turn to case (ii): exclusion in CB but not in BC.

Lemma 6 *Suppose that*

$$\begin{aligned}
&\max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\
&\geq \max_b \{u_B(b, 0) + u_A(b, 0)\} \\
&> \max_b \{u_B(b, f(b)) + \beta u_A(b, 0) + (1-\beta)(u_A(b, f(b)) + u_C(b, f(b)))\}
\end{aligned} \tag{36}$$

Then, there is exclusion in sequence CB, but not in BC. Moreover, $U_A^{BC} \geq U_A^{CB}$ and $S^{BC} \geq S^{CB}$.

Proof. By (34), there is exclusion in sequence CB, but not in BC. Since there is exclusion in sequence CB,

$$U_A^{CB} = (1-\beta\gamma) \max_b \{u_A(b, 0) + u_B(b, 0)\}$$

and $S^{CB} = \hat{S}$ as in case (i).

In sequence BC, our main analysis applies:

$$\begin{aligned}
U_A^{BC} = &(1-\beta) \max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\
&+ \beta(1-\gamma) \max_b \{u_A(b, 0) + u_B(b, 0)\}.
\end{aligned}$$

Thus,

$$\begin{aligned}
&U_A^{BC} - U_A^{CB} \\
&= (1-\beta)[\max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\
&- \max_b \{u_A(b, 0) + u_B(b, 0)\}].
\end{aligned}$$

By the first inequality in (36), it follows that $U_A^{BC} \geq U_A^{CB}$. Moreover, the joint surplus of A and B is

$$\begin{aligned}
&\max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\
&\geq \max_b \{u_B(b, 0) + u_A(b, 0)\} = \hat{S}.
\end{aligned}$$

The joint surplus of all three players, S^{BC}, is the joint surplus of A and B, plus the payoff of C, which is non-negative. Thus, $S^{BC} \geq \hat{S} = S^{CB}$. ■

Finally, if

$$\begin{aligned}&\max_b \{u_B(b, f(b)) + \beta u_A(b, 0) + (1-\beta)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ &\geq \max_b \{u_B(b, 0) + u_A(b, 0)\}, \end{aligned} \tag{37}$$

exclusive contracts are not used in equilibrium, irrespective of the sequence (case (iii)), and our main analysis applies.

In all three possible cases, the joint surplus is weakly higher in sequence BC than CB, as in Proposition 1. Which of these three cases applies depends on the externalities between agents. Suppose that externalities between agents are positive, or absent. Then,

$$\begin{aligned}&\max_{d_1} \{u_1(d_1, f(d_1)) + \beta_2 u_A(d_1, 0) + (1-\beta_2)(u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)))\} \\ &\geq \max_{d_1} \{u_1(d_1, f(d_1)) + u_A(d_1, 0)\} \\ &\geq \max_{d_1} \{u_1(d_1, 0) + u_A(d_1, 0)\}\end{aligned}$$

where the first inequality uses that the joint payoff is decreasing in β_2, and the second inequality uses positive or no externalities between agents. This chain of inequalities holds regardless of the sequence. It implies that (37) is fulfilled: the joint surplus of A and agent 1 is higher without exclusion, and they will agree on a non-exclusive contract in equilibrium. Therefore, with positive or no externalities between agents, there is no exclusion in equilibrium; hence, our analysis in the main text above applies. In particular, the part of Proposition 3 with no externalities and Proposition 4 hold.

Now consider negative externalities between agents. Then all three cases are possible. In cases (i) and (ii), A prefers sequence BC by the Lemmas above. In case (iii), there is no exclusion; hence, our main analysis applies. Therefore, with negative externalities between agents, A prefers sequence BC, as in Proposition 3 of the main text. Taking all results together implies that the statement of Proposition 2 on binary decisions holds as well.

D.2 Proof of the result that the principal prefers CB in case a menu of contracts is possible

A contract in stage 1 is a menu specifying (d_1^e, t_1^e, d_1, t_1), where the exclusive dealing contract (d_1^e, t_1^e) is executed when the decision reached with agent 2 is zero $(d_2 = 0)$, and (d_1, t_1) is relevant otherwise.

Consider stage 2. The joint surplus of A and agent 2 is $u_A(d_1^e, 0) + t_1^e$ if $d_2 = 0$, and

$u_A(d_1, d_2) + u_2(d_1, d_2) + t_1$ if $d_2 > 0$. The second stage decision will be

$$f(d_1) := \arg\max_{d_2} \{u_A(d_1, d_2) + u_2(d_1, d_2)\} > 0$$

if

$$\max_{d_2} \{u_A(d_1, d_2) + u_2(d_1, d_2)\} + t_1 \geq u_A(d_1^e, 0) + t_1^e, \tag{38}$$

and $d_2 = 0$ otherwise. (As above, we break ties in favor of non-exclusivity.)

Suppose A proposes in stage 2. If (38) holds, A will propose $d_2 = f(d_1)$ and $t_2 = u_2(d_1, d_2)$; A's payoff is $u_A(d_1, f(d_2)) + u_2(d_1, f(d_2)) + t_1$. If (38) does not hold, then $d_2 = 0$ and A's payoff is $u_A(d_1^e, 0) + t_1^e$.

Suppose agent 2 proposes in stage 2. If (38) holds, agent 2 will propose $d_2 = f(d_1)$ and t_2 such that A is indifferent between accepting and rejecting the offer. The payoff of A is $u_A(d_1^e, 0) + t_1^e$. This is also A's payoff if (38) is violated.

We now turn to stage 1. Consider first a contract menu such that (38) holds. With such contracts, it is possible to bring down agent 2 to his outside utility of zero, even if agent 2 proposes. To do so, choose (d_2^e, t_1^e) such that (38) holds with equality:

$$u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) + t_1 = u_A(d_1^e, 0) + t_1^e.$$

The principal's payoff is then $u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) + t_1$, no matter who proposes in stage 2. Moreover, the joint surplus of A and agent 1 is

$$S(d_1, f(d_1)) := u_A(d_1, f(d_1)) + u_1(d_1, f(d_1)) + u_2(d_1, f(d_1)).$$

If A proposes in stage 1, she will propose $d_1^* = \arg\max S(d_1, f(d_1))$ and $t_1 = u_1(d_1^*, f(d_1^*))$, giving her a payoff of

$$S^* := \max_{d_1} S(d_1, f(d_1)).$$

If agent 1 proposes, he will propose d_1^*, and t_1 such that A is brought down to her outside option,

$$(1 - \beta_2) \max_{d_2} \{u_A(0, d_2) + u_2(0, d_2)\}.$$

In either case, the joint payoff of A and agent 1 from a contract that satisfies (38) is S^*.

A contract menu that violates (38) leads to $d_2 = 0$. The joint surplus of A and agent 1 is then

$$\max_{d_1^e} \{u_A(d_1^e, 0) + u_1(d_1^e, 0)\} =: \hat{S}.$$

We now consider whether A and agent 1 will agree on a contract that satisfies or violates (38). There are three cases. First, if $\hat{S} < S^*$, the joint surplus of A and agent 1 from a contract that satisfies (38) is higher. Then the joint surplus of all three players is the same in both sequences, $S^{BC} = S^{CB} = S^*$. Moreover, $U_A^{BC} = (1-\beta)\, S^* + \beta\,(1-\gamma)\,\hat{S}$, $U_A^{CB} = (1-\gamma)\, S^* + \gamma\,(1-\beta)\,\hat{S}$, and therefore,

$$U_A^{CB} - U_A^{BC} = (\beta - \gamma)\left(S^* - \hat{S}\right) > 0;$$

hence, A prefers sequence CB.

Second, if $\hat{S} > S^*$, the joint surplus of A and agent 1 from a contract that violates (38) is higher. Then the joint surplus of all three players is $S^{BC} = S^{CB} = \hat{S}$. Moreover,

$$U_A^{BC} = (1-\beta)\,\hat{S} + \beta\,(1-\gamma)\,\hat{S} = (1-\beta\gamma)\,\hat{S} = U_A^{CB};$$

hence, A is indifferent between the sequences.

Third, if $\hat{S} = S^*$, satisfying or violating (38) gives A and agent 1 the same joint surplus, which implies that they are indifferent. No matter how they break the tie, the joint surplus of all three players is $S^{BC} = S^{CB} = \hat{S}$, and $U_A^{BC} = U_A^{CB} = (1-\beta\gamma)\,\hat{S}$; hence, A is indifferent between the sequences.

To summarize, in all three cases, $S^{BC} = S^{CB}$, and $U_A^{CB} \leq U_A^{BC}$ so A (weakly) prefers sequence CB.

References

[1] Bertsekas, D.P. (2017): *Dynamic Programming and Optimal Control*, Athena Scientific.