A Theory of Digital Ecosystems

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A Theory of Digital Ecosystems*

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Abstract

We develop a theory of digital ecosystems built on the premise that a multi-market firm can steer users it has in one market toward its products in other markets. Due to this "cross-market leverage," a leader in an "access-point" market (where users begin their online journeys) derives a high value from offering services in connected markets (where users continue their journeys), and can thus make profitable takeovers. Indeed, because the firm has the outside option of acquiring, and steering users toward, its target's competitor, it can take over the target at a discount. In contrast, other firms have no or smaller incentives for takeovers, explaining why ecosystems grow out of market leaders at access points. Conversely, cross-market leverage also implies that once an ecosystem has grown, it has an increased value of controlling access points, explaining why some firms go to great lengths to dominate these markets.

Our theory informs recent discussions and proposals on the role and regulation of ecosystems. Under an arguably weak additional assumption, an ecosystem consolidates and steers consumers toward firms that are appealing in the first place, and this may benefit consumers in the short run. But an ecosystem's takeovers and dominance of access points lower the contestability of access-point as well as follow-on markets. Hence, policies that weaken cross-market leverage or restrict payments for an access-point default position raise contestability.

Keywords: digital ecosystems, takeover, contestability, entry, envelopment, default effects, steering.

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1 Introduction

Digital ecosystems — firms that offer a wide range of online services — have become so dominant that they are familiar to any consumer with internet access. In this paper, we formulate a novel theory of digital ecosystems by connecting two observations that are central in policy reports as well as academic research on the topic. First, ecosystems aim to keep consumers through setting defaults, manipulating attention, or employing other means of steering. Second, ecosystems tend to develop in a major part through takeovers.

Based on the first fact, we make the core assumption that an ecosystem can use a sale in one service to steer consumers toward its product for a follow-on service. For instance, Google Search preferentially lists videos from Google-owned Youtube and connections from Google Flights, surely to increase traffic to the latter services (Schechner et al., 2020).[1] Due to this "cross-market leverage," a leader in an "access-point" market (where users begin their online journeys) has a high value from taking over firms that offer services in connected markets. Indeed, because the acquirer has the threatening outside option of taking over and then steering users toward a target's competitor, it can obtain the target at a discount. We show that other firms have lower incentives for takeovers, explaining the second observation, and adding the prediction that ecosystems grow out of firms that control access points.

Conversely, cross-market leverage also means that once an ecosystem has grown, it can have an extremely high value for attracting consumers at access points. It is thus willing to spend tremendous resources to steer consumers toward itself in these markets, for instance by buying the default position there or developing its own product to be the first mover.

These predictions distinguish our theory from existing explanations based on complementarities between markets. Such complementarities provide a motive for integration, but existing models do not explain why ecosystems grow out of access points in particular. In addition, these theories do not explain why established ecosystems devote so many resources to securing access-point defaults.

Our theory informs recent discussions and proposals on the role and regulation of ecosystems.

[1] Or, as Mark Zuckerberg put it in a leaked email, "the more cross-linking [between services], the better" (`https://t1p.de/x3w8o`, accessed July 19, 2024).

Under an arguably weak additional assumption, an ecosystem consolidates and steers consumers toward firms that are most "appealing" in their markets (in a sense we make precise), and if appeal reflects quality, this benefits consumers in the short run. But an ecosystem's takeovers and dominance of access points lower the contestability of — i.e., incentives for entry into — all markets. Hence, policies that weaken cross-market leverage or restrict payments for an access-point default position raise contestability.

We introduce our model in Section 2. Consumers use services a and b in sequence, with single-market firms operating in one of these markets and ecosystems, if present, operating in both. In each market, consumers are steered toward one "default" firm, which obtains an increase in its market share at the expense of other firms.[2] In our basic model, the default at the access point a is allocated exogenously. Default determination in market b, in turn, captures cross-market leverage: if the consumer bought from an ecosystem in market a, then the ecosystem becomes the consumer's default in market b. Otherwise, the consumer's default in market b is selected randomly.

In Section 3, we study the emergence of ecosystems from an economy with only single-market firms. Suppose that a single firm G in market a can make sequential take-it-or-leave-it acquisition offers to firms in market b until one accepts or all reject. Because of cross-market leverage, G can direct its users in market a toward the service of an acquired target, so it can make more profits than the target can by itself. But in addition to this valuation incentive for growing, G also has a strategic advantage in the process. Namely, if a target t rejected G's offer, then G would take over a competitor t' of t, and direct its users to t'. Because this would divert users away from t, it diminishes the position of t. As a result, G can take over any preferred target t at a discount.

To sharpen our understanding of takeover incentives, we study several alternative games. First, taking the perspective that the most profitable takeovers are the most likely to occur, we compare the profits of all cross-market acquisitions. We establish that the takeover in which the leader (highest-demand firm) in market a acquires a firm in market b is the most profitable. In particular, this forward integration is more profitable than backward integration, where a firm in market b

[2] In the Appendix, we formally model several possible microfoundations for the default firm's benefit, such as the ability to introduce switching costs, manipulate a consumer's consideration set or reference point, or make a recommendation to her.

acquires the leader in market a. The reason is that a firm in market b cannot steer users away from a firm in market a, so it cannot negotiate a takeover discount. Second, we show in an example with multiple equally valuable targets in market b that the takeover discount is not an artifact of a monopsonistic acquirer. Intuitively, with multiple acquirers the equilibrium price for takeovers is determined by the last target's value, which is low because some consumers are steered toward already acquired firms. Third, we argue that if appropriate targets are unavailable, a dominant firm at the access point has an incentive to develop products for connected markets itself. Because of cross-market leverage, it benefits more from the new service than other firms. But the takeover discount means that it has a preference for takeovers over internal development.

Taken as a whole, the above logic implies that market leaders at access points grow into digital ecosystems, primarily by taking over other firms, and alternatively by developing new services. To mention a few of numerous examples, Google added both YouTube and Google Flights to its services through takeovers, and Microsoft (a leader in operating systems) spent tremendous resources to develop its own browser and search engine. Cross-market leverage also explains why it is increasingly tech firms rather than investors specializing in tech that make takeovers in digital markets (Eisfeld, 2025), and why ecosystems initially emerged through the takeover of "consumer-facing" rather than "data-intensive" firms (Jin et al., 2023).

In Section 4, we return to the beginning of the consumer's journey, the access-point market a. We endogenize the determination of the default firm by assuming that an outside seller auctions off this position to firms in the market. This corresponds to a situation where an original equipment manufacturer (OEM) like Apple or Samsung sets the default search engine for a fee.[3] We show that once an ecosystem has developed and benefits from cross-market leverage, it may outbid otherwise similar rivals at the access point. Its payment for the default position can be especially high when there is a second ecosystem, and both aim to prevent the other from drawing consumers into its web of services. A striking example consistent with this prediction is the $18-20 billion annual payment that Google makes for being the default search engine on Apple devices — which constitutes 36%

[3] Other negotiation mechanisms, for instance bilateral bargaining, yield similar qualitative insights. Alternatively, the race to offer a new access-point product first may often be seen as an all-pay auction whose winner obtains the default position through prominence, again yielding similar insights.

of its search advertising revenue from these devices.[4]

While the above results derive from a single non-standard assumption, cross-market leverage, in Section 5 we develop a further insight by putting more structure on the nature of steering. We say that a firm is more appealing than another if, in the symmetric situation in which the default is assigned randomly, it would have higher market share. Then, we say that steering complements appeal if more appealing firms benefit more from holding the default position, drawing more consumers away from all rivals.[5] Then, we establish that if complementarity holds and the default position in market a is assigned randomly or acquired by the most appealing firm, an ecosystem with system-wide dominance emerges: the most appealing firm in market a takes over the most appealing firm in market b and secures the default position in market a, enhancing its superior position in both markets.

In Section 6, we build on the above analysis to identify policy implications. Reflecting the core mechanism of our paper, we focus on interventions that disrupt the ability of ecosystems to steer consumers. These interventions include "leverage policies" aimed at weakening cross-market leverage, and "access-point policies" restricting the sale of defaults. For instance, Articles 6(5) and 6(6) of the European Union's Digital Market Act, which prohibit access-point firms from favoring their own products through rankings or impediments to switching, can be seen as leverage policies. At the same time, Articles 6(3) and 6(4), which mandate easy changeability of default settings on operating systems, virtual assistants, web browsers, and app stores, can be seen as access-point policies.

On the one hand, if greater appeal reflects higher consumer surplus, the equilibrium ecosystem steers consumers toward the best services in both markets. Both types of policy then weaken this endogenous benefit. On the other hand, the ecosystem's control of defaults reduces the profits of potential entrants. An access-point policy therefore makes market a more contestable; and either

[4] See, for example, `https://t1p.de/r9drd` (accessed on June 11, 2024). Google also pays $400-$450 million per year for the default position on Firefox (see `https://t1p.de/1gvpq` accessed on June 11, 2024). That capturing the consumer's attention at access points is a key part of Google's strategy is clear from Google's internal communications as well. For instance: "There's tremendous power in the default OS access points but it's pay to play [...]. There is no substitute for the default access points: we should continue to explore broad default access across all OS (including newer and emerging access points)" (see, e.g., `https://www.justice.gov/d9/2023-09/416682.pdf`).

[5] In the Appendix, we show that our microfoundations often lead to consumer behavior satisfying complementarity.

type of policy makes market b more contestable, helps superior entrants compete for the access-point default, and encourages entry in further connected markets. In these senses, our theory validates recent regulatory concerns about the stifling effects of ecosystems on contestability (e.g., Crémer et al., 2019, Scott Morton et al., 2019, Furman et al., 2019) and supports policies restricting ecosystems' steering ability.

We conclude in Section 7 by speculating why ecosystems are less common in offline settings. While analogues do exist (e.g., tropical hotels with many services), in general offline firms appear to have less steering ability than their online counterparts.

Related Literature Our paper belongs to the quickly growing theoretical literature on ecosystems and the digital economy. No previous paper, however, analyzes our main question, the role of cross-market leverage and steering in the emergence of digital ecosystems. Indeed, although there are informal accounts (e.g., Eisenmann et al., 2011, Condorelli and Padilla, 2020), the only formal theories applicable to ecosystem growth are based on consumer-side (Chen and Rey, 2023, Rhodes and Zhou, 2019) or producer-side (Condorelli and Padilla, 2023) complementarities. Most closely related, Chen and Rey (2023) assume that consumers obtain a convenience or bundling benefit from buying multiple services from the same firm, which motivates conglomerate mergers. Our theory also features a potential consumer benefit from ecosystems, but this arises endogenously when mergers take place between appealing firms. More importantly, cross-market leverage yields two predictions, both consistent with observed ecosystem behavior, that existing complementarity-based theories do not make. First, our theory predicts that takeovers will be initiated by firms at (or closer to) consumers' access points, whereas previous theories are silent about the direction of takeovers. Indeed, without further assumptions, integration creates the same complementarity regardless of which side initiates the takeover. Second, our theory predicts that ecosystems will go to great lengths to secure access-point defaults, whereas previous models do not address this practice. Indeed, if multi-market firms have an inherent complementarity advantage, it is not obvious why they would need to spend a lot to steer consumers in their direction. Accordingly, our theory

informs access-point and leverage policies, which have no direct roles in previous theories.[6]

In addition, a small literature explores the implications of default effects or inertia in single-market models of the digital economy (Chen and Schwartz, 2023, Ostrovsky, 2023, Hovenkamp, 2023, Decarolis et al., 2025, Denicolò and Polo, 2024), without analyzing ramifications for ecosystems. Most closely related are Chen and Schwartz (2023) and Hovenkamp (2023), who model default effects as switching costs and show that better firms generally value the default position more. Our model yields a parallel result: under steering-appeal complementarity and with only single-market firms, the most appealing firm wins the default auction.[7]

Our assumption that firms can steer consumers builds on three observations about defaults. For starters, a vast literature in behavioral economics documents that an option's likelihood of being chosen is higher if it is the default (e.g., Madrian and Shea, 2001, Choi et al., 2004, Johnson and Goldstein, 2003, 2004, Jones, 2012, Chetty et al., 2014, Blumenstock et al., 2018, Jachimowicz et al., 2019, Brown and Previtero, 2020, Brot-Goldberg et al., 2023). In addition, academics, policymakers, as well as firms recognize that defaults are at least as important in digital settings as in traditional ones.[8] Finally, it is universally recognized that firms often have substantial control over consumers' defaults.

2 Basics: Firm Payoffs with Steering

We begin by specifying a model of demand when consumers are susceptible to default effects and steering. Because our main interest is studying firm behavior rather than the default effects per se, we specify firm payoffs in reduced form.

[6] While only distantly related to the access-point and leverage policies we discuss, Rhodes et al. (2025) analyze large digital firms' benefits of data-sharing across markets.

[7] Recent theoretical research also investigates aspects of the digital economy, including pricing (e.g., Jeon et al., 2023), within-market steering (e.g., Teh and Wright, 2022, Hidir and Vellodi, 2020, Heidhues et al., 2023b), entry for buyout (e.g., Bryan and Hovenkamp, 2020), and innovation (e.g., Madsen and Vellodi, forthcoming), that are completely different from our main focus.

[8] Among academics, see for instance Johnson et al. (2002), Altmann et al. (2019), Fletcher and Vasas (2023), and Fletcher et al. (2023). Among policy reports, see Scott Morton et al. (2019) and Furman et al. (2019). Among firms, extensive Google memos document that default settings "can be a powerful strategic weapon"; a view that is shared by its competitors, as is evident from the testimony by Neeva, DuckDuckGo, and Microsoft managers (see `https://t1p.de/sco1p`). Accessed July 8, 2024.

2.1 Setup

There are two markets, a and b, served by firms 1 through n. Market a is an access point, such as a browser, search engine, map, operating system, agentic AI, or in the future potentially a service on a wearable or self-driving car. Market b is a follow-on service, such as an online-video, travel-booking, or food-delivery app. We denote by $\mathcal{N}^s \subseteq \{1, \ldots, n\}$ the set of firms offering service $s \in \{a, b\}$, define $n^s := |\mathcal{N}^s|$, and assume $n^s \geq 2$. We call a firm serving one market a single-market firm, and a firm serving both markets an ecosystem. Prices are fixed at zero, and a firm's profit equals its total demand across the two markets.[9] We normalize the size of the potential market a to 1.

In each market s, consumers are steered toward the product of one firm, which we call the default firm. Firm i obtains exogenously given demand $q_{ij}^s \geq 0$ in market s when firm j is the default in market s. Throughout the paper, we maintain:

Assumption 0 (Steering)**.** For all i and $j \neq i$, we have $q_{ii}^s > q_{ij}^s$.

Assumption 0 can be thought of as the definition of steering or default effects in our context — that having the default position is beneficial for a firm.

For now, we assume that the default in market a is assigned exogenously, and endogenize this selection in Section 4. At the same time, firms' probabilities of becoming the default in market b depend in the following way on the outcome in market a. First, any ecosystem is allocated a probability equal to its market-a demand. Second, the probability that remains is divided equally between all firms in market b (including ecosystems). These assumptions capture the cross-market leverage that results if consumers use service b after service a. For a consumer who uses an ecosystem in market a, the ecosystem sets itself as the default in market b. For a consumer who uses a single-market firm in market a, no firm in market b is systematically favored.

Discussion In Appendix A, we provide microfoundations for Assumption 0 based on the main mechanisms for default effects described in the literature. These mechanisms presume that con-

[9] This is consistent with the common situation in digital markets, where a firm benefits not from direct sales to consumers, but from collecting data and/or showing ads.

sumers choose between products imperfectly or in a context-dependent way. First, consistent with a strict notion of defaults, a consumer may receive the default option unless she actively avoids it. Then, the default matters because searching for or switching to something else is costly. Second, in a generalization of default effects, the default option may attract a consumer's attention. Then, the default matters because it is more likely to enter the consumer's consideration set, to become her reference point, or to be taken by her as a recommendation.

For transparency and tractability, our model makes some unrealistic simplifying assumptions. First, while we have posited just one access point and one follow-on market, actual online journeys are varied and often more complex. For example, there may be multiple possible access points, multiple possible follow-on markets after a given access point, and further follow-on markets after the first ones. In addition, consumers may use the same services in different orders. These alternatives add an obvious and realistic qualification to our results: that ecosystems will consist not just of two, but of many connected services. Nevertheless, our basic points continue to hold if there are a few central services that a disproportionate number of consumers start with.

Second, it seems plausible that even beyond sales in market a, a prominent firm in market b is systematically favored. For example, consumers may use Google Maps simply because the brand has become a synonym for search. Our results continue to hold as long as ecosystems can steer consumers across markets.

Third, while it is not inconsistent with them, our model ignores two considerations that are commonly considered central for the dominance of large digital firms (see Calvano and Polo, 2021, for a review): network effects and economies of scope in data. Network effects — that more users make the service better — apply almost exclusively within a market, while all of our results are driven by cross-market effects. Economies of scope in data — that combining consumer data from different markets may increase conversions for advertisers (e.g., Ullrich et al., 2024) — strengthen incentives for mergers in general, but they do not explain our main results regarding the direction of ecosystem growth or the tremendous effort ecosystems expend to secure access points.[10]

[10] See Rhodes et al. (2025) for a paper studying the effects of "data-driven demand complementarities" — resulting in quality improvements — on pricing and innovation by an ecosystem and its single-market competitors.

2.2 Preliminaries: Default Advantage and Default Externality

As an initial step in our analysis, we decompose steering effects. We define firm i's *default advantage* as its gain in demand when it rather than a randomly chosen firm is the default:

$$\alpha_i^s := q_{ii}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Relatedly, we define firm j's *default externality* on firm i as the change in firm i's demand when firm j rather than a randomly chosen firm is the default:

$$\eta_{ij}^s := q_{ij}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Using Assumption 0, it is easy to establish the following.

Lemma 1 (Default Advantage and Externalities)**.**

I. For any firm $i \in \mathcal{N}^s$, we have $\alpha_i^s > 0$.

II. For any firm $i \in \mathcal{N}^s$, there exists another firm $j \neq i$ such that $\eta_{ij}^s < 0$.

Part I restates the premise that the default position is valuable. Part II says that any firm has a rival that imposes a negative default externality on it. Intuitively, if firm i benefited from each competitor's default position, then the default position would be undesirable, contradicting Assumption 0.

3 The Emergence of Ecosystems

In this section, we show that cross-market leverage leads to the emergence of ecosystems through — primarily — takeovers. Since for now the default in market a is exogenously given, we suppress it in the notation for a firm's market-a demand.

3.1 Takeovers in the Shadow of Cross-Market Leverage

Suppose that at the outset, all firms are single-market firms. Then, one firm in the access-point market a, firm G, can make sequential take-it-or-leave-it offers to the firms in market b until one accepts or all reject. We look for subgame-perfect equilibrium outcomes.

As a point of comparison, notice that a target t's "standalone value," i.e., its demand in the absence of a takeover, is

$$V_t^b := \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{t\ell}^b.$$

Without cross-market leverage, this is how much G would value t, and also how much t would be willing to sell for. Hence, in that case the profit from a takeover would be zero.

Proposition 1 (Takeover Discounts and Profits)**.** *In any subgame-perfect equilibrium of the takeover game, there are $t_1, t_2 \in \mathcal{N}^b$ such that (i) G takes over t_1 at a price $V_{t_1}^b - q_G^a |\eta_{t_1 t_2}^b|$, thereby making a profit of $q_G^a(\alpha_{t_1}^b + |\eta_{t_1 t_2}^b|)$; and (ii) t_1 and t_2 solve $\max_{t_1', t_2' \in \mathcal{N}^b} \alpha_{t_1'}^b - \eta_{t_1' t_2'}^b$.*

The proposition states that G makes a profitable takeover. Its profit is due to two considerations that arise from cross-market leverage. First, if G acquires a target t_1, then it can use its cross-market leverage to raise t_1's demand. Hence, G values t_1 above t_1's standalone value. Second, G obtains a "takeover discount" — it can buy t_1 below t_1's standalone value. In equilibrium, t_1 knows that if it did not sell to G, then another target t_2 chosen by the firm would. Furthermore, by Lemma 1, t_2 can be chosen to harm t_1, creating a threat that weakens t_1's bargaining position. In particular, t_1 and t_2 maximize G's takeover profit.[11]

We now compare the profit different firms can earn in a takeover game of the type above, where they make sequential take-it-or-leave-it offers to firms in the other market. For our statement, we call a firm with the highest (pre-takeover) demand in its market a leader.

Proposition 2 (Profits from Forward vs. Backward Integration)**.** *Take any firm i in market b.*

I. The highest profit it can earn from a takeover is through taking over a leader ℓ in market a. It can do so at a price V_ℓ^a, making an additional profit of $q_\ell^a \alpha_i^b$.

[11] Our proof establishes the last result by noting that G can first make low-ball offers to all firms but t_1 and t_2, and once these offers are rejected, approach t_1. Then, t_1 knows that if it rejected the offer, G would take over t_2. This logic relies on the assumption that G cannot approach firms that previously rejected it, effectively granting it commitment power to its next target if t_1 rejects it. Other sources of such commitment, for instance having to go through costly and time-consuming due diligence with potential targets, could generate the same commitment power and outcome. Furthermore, under a weak additional assumption (complementarity below), there is a subgame-perfect equilibrium in which G obtains the same profit as in Proposition 1. In this equilibrium, G approaches t_1 first and t_2 second, which are also its most and second-most favorite targets, and both accept its offer.

II. A leader ℓ in market a can earn strictly more from a takeover than $q_\ell^a \alpha_i^b$, and it can earn weakly more than any other firm in market a.

Part I identifies the profits from "backward integration" — when a firm in market b expands into market a. Cross-market leverage allows the firm to direct some of the target's consumers to itself, so it makes a profit from such a takeover. In doing so, however, the firm does not obtain a takeover discount: taking over the target's competitor does not decrease the target's demand, so it is not threatening. Accordingly, Part II says that the profit from backward integration is lower than that from "forward integration" — when a firm in market a expands into market b.

Under plausible conditions, the profitability of different types of mergers has implications for who makes takeovers. As a simple formalization of these implications, suppose that one firm from either market is randomly selected to play the above takeover game, and the selected firm must pay an extra transaction cost (drawn from a fixed distribution with full support) to complete the acquisition. By Proposition 2, the leader in market a is then most likely to make a takeover. Hence, our model implies that market leaders at access points, like Google or Microsoft, are most prone to growing into ecosystems through profitable mergers.

Both specific examples and general observations are consistent with our story. For instance, Google took over Youtube and added it to its services, Google took over Where 2 Technologies and developed it into Google Maps, Google took over ITA Software to create Google Flights, and Microsoft took over Hotmail and turned it into Outlook.[12] More generally, despite the growing importance of acquisitions by financial firms elsewhere (e.g., Vild and Zeisberger, 2014), strategic mergers are common in digital markets (Eisfeld, 2025).[13] Furthermore, consistent with the idea

[12] See, for example, `https://t1p.de/lsvbk` (YouTube), `https://t1p.de/86x05` (Google Maps), `https://t1p.de/aia1s` (Google Flights), and `https://t1p.de/w3dhr` (Outlook). Other examples include Google taking over Upstartle and 2Web Technologies to create Google Docs and Google Sheets (`https://t1p.de/med6m`); Apple buying the MP3 player SoundJam MP to create iTunes (`https://t1p.de/vfuyv`); Microsoft extending its range of services by taking over Skype and LinkedIn (`https://t1p.de/1dddj` and `https://t1p.de/g9bwj`); Amazon acquiring Audible, LoveFilm (now Amazon Prime Video), and Twitch (`https://t1p.de/52vaf`, `https://t1p.de/7cpmu`, and `https://t1p.de/4p5ue`); Meta taking over WhatsApp, Instagram, and Within (`https://t1p.de/g98rr` and `https://t1p.de/f068f`). All links accessed on June 6, 2024.

[13] The observation that digital ecosystems grow from access points into adjacent markets through takeovers is not restricted to western societies. The two dominant ecosystems in China, Alibaba and Tencent, grew out of Alibaba's dominant marketplace (Taobao) and Tencent's dominant social media and messaging service (WeChat) into ecosystems that offer a broader range of services (Prüfer et al., 2024).

of cross-market leverage, digital firms tend to acquire more "consumer-facing firms" than do other investors (Jin et al., 2023) — who have no consumers to steer. Finally, consistent with our takeover discount, digital firms "tend to acquire companies at lower prices" than financial investors (Eisfeld, 2025, Appendix G, p. 23) — who have no threatening outside option.

The logic of our model also allows us to make further observations. First, suppose that for exogenous reasons, firms in market b are not willing to sell, but the leader G in market a can internally develop service b. G's development cost is c_{int}, while an outsider can develop a service with the same demand at cost c_{ext}. For simplicity, we impose that the outsider makes the development decision first, and if it does not develop, then G can do so. In this situation, the outsider develops the service if its standalone value exceeds c_{ext}. Otherwise it does not, in which case G develops the service if the same standalone value plus G's default advantage exceeds c_{int}. Hence, G may develop the service even if $c_{\text{int}} > c_{\text{ext}}$. In this sense, G has a larger incentive than other firms to develop new services in adjacent markets. Without cross-market leverage, in contrast, G would never be the one developing the service if $c_{\text{int}} > c_{\text{ext}}$. Consistent with our prediction, for instance, Microsoft spent tremendous resources to develop and improve its browser (first Internet Explorer, now Edge), Google developed its email service Gmail in house, and Uber developed its food delivery service UberEats.[14]

Second, notwithstanding its incentive to self-develop service b if necessary, G often prefers to acquire the service through a takeover instead. In particular, suppose that G could hire a team and self-develop the service, or let the team start a company offering the service and take over the company. Firm G prefers the takeover if c_{int} is above the takeover price. Hence, G may use a takeover even if c_{int} is well below the profits it can make with the service. Again, without cross-market leverage this would never be the case. In addition, even if firm G makes a takeover in equilibrium, its option to self-develop the product remains relevant. Much like taking over a competitor, this can serve as a threatening outside option that lowers the preferred target's takeover price.[15]

[14] See `https://t1p.de/l6y2x` and `https://t1p.de/a5egg` (Microsoft), (`https://t1p.de/86x05`) (Google), and `https://t1p.de/2u84f` (Uber). Notably, also Apple and Google developed their own web browsers (`https://t1p.de/pqiz6` and `https://t1p.de/69unz`). All links accessed June 6, 2024.

[15] Relatedly, Katz (1987) argues that large downstream buyers can more credibly threaten to enter an input market

Third, consider the possibility that (for exogenous reasons) forward integration is not feasible, but backward integration is. This may be the case, for instance, when there are already ecosystems in place, and a new access point with small single-market firms emerges. For practical reasons, such as financing or know-how, it seems implausible that a small startup would take over an ecosystem like Google or Meta. Then, since backward integration is profitable (Proposition 2, Part I), it takes place instead. Consistent with this prediction, Google took over Fitbit and Meta acquired Oculus VR.[16]

Note that we have abstracted from the possibility that firms use contracts or auctions, rather than takeovers, to coordinate steering across markets. Our motivation is similar to that of many previous researchers studying vertical relations: that designing and enforcing contracts — especially in highly dynamic digital markets — can be difficult, rendering takeovers the superior option. If steering is based on choice architecture, for example, firms would have to agree every time the service in market a adapts its customer interface. Similarly, a firm improving its service in market b would face a hold-up problem unless the steering price is credibly fixed. Furthermore, in the plausible scenario with further markets following market b, a contracting solution would have to balance the interests of three or more parties.

Nevertheless, when the above problems are not severe, or a merger is infeasible, an existing ecosystem may, consistent with the advertising part of digital markets, sell the attention of its access-point consumers. As an illustration, suppose that (for reasons outside of our model) the ecosystem G no longer offers the service b that benefits most from having the default position. For example, an entrant e might have developed a new service b, and a second — this time horizontal — merger between G and b would not be approved. This entrant would be willing to pay $q_G^a\big(\alpha_e^b - \eta_{eG}^b\big)$ for the attention of G's access-point consumers, and selling this attention to e reduces G's demand in market b by $q_G^a\big(\alpha_G^b - \eta_{Ge}^b\big)$. If the latter is smaller, G is willing to sell advertisements to e.

in the presence of scale economies, and hence get offered a lower input price even absent entry.

[16] See `https://t1p.de/o9wui` (Google) and `https://t1p.de/p8kvb` (Meta), both accessed June 19, 2024.

3.2 Multiple Acquirers and Takeover Prices

Above, we have assumed that a single firm can make takeover bids. We now show through an example that the presence of multiple potential acquirers — which one would hope introduces competition for takeovers — does not necessarily protect targets from low takeover prices.

Consistent with Proposition 2, we focus on forward integration. Market-a firms in $\mathcal{A} \subseteq \mathcal{N}^a$ can make takeovers, and they differ in their market-a demands. All market-b firms are potential takeover targets, and they offer equally good services with standalone value V^b and default externality η^b. We denote by $\bar{\mathcal{A}}(x)$ the x firms in $\mathcal{A}$ with the highest market-a demands, and set $\bar{\mathcal{A}}(x) = \mathcal{A}$ for $x > |\mathcal{A}|$. In every round of the game, one of the remaining targets is offered for sale, and each potential acquirer that has not yet made a takeover can submit a takeover offer. The target can accept any offer. The game ends when all potential targets have been up for sale.

We characterize subgame-perfect equilibrium outcomes satisfying a recursive variant of the convention that takeover offers are cautious, i.e., not above the profits from the takeover. Formally, if a subgame following current offers has a unique cautious subgame-perfect equilibrium outcome, we require firms to use iteratively weakly undominated strategies in the reduced game in which these subgames are replaced by their equilibrium payoffs.[17]

Proposition 3 (Multiple Acquirers)**.** *In any equilibrium with cautious offers, firms in $\bar{\mathcal{A}}(n^b)$, but no other firms, make a takeover. The equilibrium takeover price is $f^* < V^b$ satisfying*

$$f^* = \begin{cases} V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b - 1)} q_i^a & \text{if } \; n^b \geq |\mathcal{A}|, \\ V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b - 1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b - 1)} q_j^a & \text{if } \; n^b < |\mathcal{A}|. \end{cases} \tag{1}$$

Proposition 3 says that even with multiple acquirers, the acquirers can always take over their targets at a price below the targets' standalone value. This means that the possibility of takeovers always harms targets. Intuitively, other acquirers not only introduce competition for an acquiring firm, but also affect the value of targets. Specifically, on the equilibrium path, the last target taken over is less valuable to an acquirer, because the ecosystems created in preceding takeovers will

[17] Requiring *iteratively* undominated strategies rules out that takeover bids are driven by the fear that a low-demand firm in a bids above its possible additional profits.

steer consumers away. Furthermore, in equilibrium the price of all takeovers is determined by this marginal acquisition, and hence is low.

The above logic implies that with an abundance of targets ($n^b > |\mathcal{A}|$), targets are actually made worse off with an increase in the number of acquirers. In that case, other acquirers lower a target's value without inducing competition. Even if there are more acquirers than targets ($n^b < |\mathcal{A}|$), the decrease in value due to other takeovers outweighs the price-increasing effect of competition. To see this, notice that the takeover price equals the marginal non-acquirer's willingness to pay for a target. But since non-acquirers have lower market-a demand than acquirers, and would thus steer fewer consumers towards their potential target, the premium they are willing to pay above the standalone value is lower than the decrease in value other acquisitions have caused.

4 Default Setting at the Access Point

Having studied the emergence of ecosystems, we now investigate the implications of ecosystems for steering incentives at the access point. To do so, we endogenize default determination in market a.

For simplicity, we suppose that there are two firms in market a, G and M, of which G is an ecosystem, and M may be a single-market firm or an ecosystem. Both G and M offer services that impose a negative default externality on each other. The default position in market a is sold through a sealed-bid second-price auction. We look for cautious Nash equilibria, which requires all firms to use weakly undominated strategies.[18] For stating our results, we define $\Delta^a_{ij} := q^a_{ii} - q^a_{ij} \in (0,1)$, which is a single-market firm i's willingness to pay to replace j as the default in market a.

Proposition 4 (Default Auction)**.**

I. Suppose that M is a single-market firm. If $\Delta^a_{GM}\big(1+\alpha^b_G\big) > \Delta^a_{MG}$, then G wins the auction for the default position with probability 1, and pays Δ^a_{MG}.

II. Suppose that M is an ecosystem. If

$$\Delta^a_{GM}\big(1+\alpha^b_G+\eta^b_{MG}\big) > \Delta^a_{MG}\big(1+\alpha^b_M+\eta^b_{GM}\big),$$

[18] It is well-known that the sealed-bid second-price auction has a continuum of equilibria in weakly dominated strategies both with independent private values (Blume and Heidhues, 2004) as well as known values.

then G wins the auction for the default position with probability 1, and pays

$$\Delta^a_{MG}\big(1+\alpha^b_M\big)+\Delta^a_{GM}|\eta^b_{MG}|.$$

By standard arguments, G wins the auction if it has a higher valuation for the default position than M does; and if G wins, it pays M's valuation. To understand the proposition, therefore, we discuss G's valuation; if M is a single-market firm, its valuation is simply Δ^a_{MG}, and if it is an ecosystem, then its valuation is analogous to G's.

If G rather than M becomes the default in market a, its market-a demand rises by Δ^a_{GM}. Because of cross-market leverage, this increases its profit in market b by $\Delta^a_{GM}\alpha^b_G$, where α^b_G is its default advantage in market b. Hence, if M is a single-market firm (Part I), G's total valuation for the default position in market a is $\big(1+\alpha^b_G\big)\Delta^a_{GM}$. Relative to a single-market firm, therefore, G's valuation is increased by the "default multiplier" $1+\alpha^b_G$. In an extension of our model in which G offers services in many markets, it earns extra profits in all those markets too. We can capture such a situation in reduced form by assuming that G's default advantage α^b_G is large, leading to a large default multiplier.[19] Consequently, G may far outbid a firm that in market a is similar to it, i.e., even when $\Delta^a_{GM}\approx\Delta^a_{MG}$.

If M is another ecosystem (Part II), then G's valuation for the default position in market a is even higher. In that case, if M wins the auction and thereby raises its market-a demand, its

[19] More concretely, suppose that, in addition to the access point, there are markets $b_1,\dots,b_S$, and the ecosystem G is active in all of these markets. First, suppose that each consumer wants to use exactly one service b_s upon leaving the access point, but different consumers demand different services. For example, some consumers search for vacation spots (market a) and book a flight afterwards (market b_1) while others search for yesterday's game (market a) to watch its highlights (market b_2). Let $w_i\in(0,1)$ be the share of consumers interested in service b_i. Firm G's willingness to pay for the default position in market a is then

$$\Big(1+\sum_{s=1}^{S}w_s\alpha_G^{b_s}\Big)\Delta^a_{GM}.$$

Alternatively, suppose that upon leaving the access-point market, all consumers want to use all other services in a (for simplicity) fixed and identical sequence. For example, a consumer may search for an interview about yesterday's game (market a) that she translates to her mother tongue (market b_1) and then emails the translation to a friend (market b_2). In this case, G's willingness to pay for the default in market a is

$$\Big(1+\sum_{s=1}^{S}\prod_{m=1}^{s}\alpha_G^{b_m}\Big)\Delta^a_{GM}.$$

Either way, firm G's willingness to pay for the default strictly increases in the number of services it offers.

cross-market leverage hurts G in market b through M's default externality on G. Preventing this provides an extra motive for G to snatch the default position in market a from M.

The above results imply that G may make an extremely large payment for the default position. For a single-market firm M, this is the case if the default effect in market a is large (i.e., for high Δ^a_{GM} and Δ^a_{MG}). For an ecosystem M, the same is the case also if cross-market steering is very effective and default externalities are large.

These results explain why digital firms make large payments or use other direct means to secure access-point defaults. A striking example (mentioned in the introduction) is Google's astronomical payments for the default position in the market for search, where it competes, for instance, against Microsoft.[20] Beyond payments, Google enforces its default position on Android devices through a nexus of usage contracts for its operating system and "must-have" apps such as Google Play Store and Google Maps (Heidhues et al., 2023a).

More broadly, the increased value of controlling access points gives ecosystems an incentive to secure them in less direct ways. A firm may, for instance, spend extra resources to develop a new access-point service and thereby reap a first-mover advantage. This provides one reason why Google might devote tremendous resources to self-driving cars or its own AI services, which are considered major new access points in the making. Similar incentives apply to services on smart wearables, AI apps that help users control their phones, and smart TV platforms that bring together streaming services, live TV, and other apps.[21]

Proposition 4 analyzes what happens if G competes either against a single-market firm or against an ecosystem. An interesting additional issue arises if G competes against both types of firms at the same time, and the single-market firm offers the service a that benefits most from the default position. Then, G's willingness to bid more when competing against another ecosystem can generate multiple equilibria. In one equilibrium, the ecosystems bid high in an effort to prevent the

[20] See, for example, `https://t1p.de/r9drd` (accessed June 7, 2024). The importance of such payments is also clear from Google's internal communications. Chris Barton (Google) writes: "We need to incentivize carriers to ship Google by using the same approach we at Google have used for many years: 'We will pay you revenue share in return for exclusive default placement.' This contract is an exchange." (internal email, April 2011, `https://www.justice.gov/d9/2023-09/416302.pdf`).

[21] For Google's investment in self-driving cars, see, for example, `https://t1p.de/uo5sm`. Google has also developed — partly through takeovers — a smart TV platform that brings together multiple services (see `https://t1p.de/1aetd` and `https://t1p.de/khbht`). All links accessed July 3, 2024.

other ecosystem from winning. Then, an ecosystem wins and pays a lot. In the other equilibrium, the single-market firm wins, and the ecosystems bid less aggressively exactly because they are not worried about the other ecosystem winning. Then, the winner pays a lower amount.

The above logic implies that despite having no direct interest in either market, an OEM may want to exclude single-market competitors from bidding for the default position at the access point. By doing so, the OEM guarantees that the ecosystems compete against each other, ensuring itself the profits from the high-price equilibrium above. The excluded firm may be offering the highest-quality service in market a; for instance, if it is the most appealing firm as defined below, and appeal reflects quality.

5 Complementarity and Systemwide Dominance

Our central results on the formation and behavior of ecosystems above are derived under the minimal assumption that an ecosystem can steer consumers. We now show that if a further plausible assumption holds, an especially large ecosystem develops.

We say that firm i has greater "appeal" than firm j in market s if

$$\sum_{\ell \in \mathcal{N}^s} q^s_{i\ell} > \sum_{\ell \in \mathcal{N}^s} q^s_{j\ell},$$

i.e., i's demand under uniform random default assignment exceeds j's. To simplify exposition, we assume that no two firms are equally appealing. For the rest of the paper, we maintain:

Assumption 1 (Steering Complements Appeal)**.** Let $i, i', j, j' \in \mathcal{N}^s$.

I. If j has greater appeal than j', then $q^s_{ij'} \geq q^s_{ij}$ for all $i \neq j, j'$.

II. If i has greater appeal than i', then $q^s_{ii} - q^s_{ij} > q^s_{i'i'} - q^s_{i'j}$ for all $j \neq i, i'$.

III. If i has greater appeal than j, then $q^s_{ii} - q^s_{ij} > q^s_{jj} - q^s_{ji}$.

Broadly, Assumption 1 imposes that more appealing firms benefit more from holding the default position than less appealing firms. Specifically, a more appealing default service attracts more consumers from all options, therefore lowering any competitor's demand (Part I), gaining more

than a less appealing firm from replacing a given rival as the default (Part II), and benefiting more from replacing a less appealing firm as the default than vice versa (Part III).

In Appendix A, we identify plausible conditions under which our microfoundations lead to consumer behavior satisfying complementarity. Depending on the microfoundation, this is either always the case, or if the default effect is sufficiently weak so that a less appealing service cannot take over most of the market by holding the default position. Intuitively, a more appealing firm often benefits more from drawing a boundedly rational consumer's attention, as consumers are more likely to stick with appealing products.[22] Thus, we think of complementarity as a relatively weak assumption.

We now identify how complementarity sharpens our main results, beginning with Section 4. With only single-market firms, the most appealing firm in market a benefits most from holding the default position, which implies that this firm wins the default auction in any weakly-undominated equilibrium.[23] In line with this observation, we suppose from now on that either the most appealing firm holds the default position in market a or the default firm is randomly determined. Both imply that the most appealing firm is the leader in market a.

Now we turn to the takeover game underlying Proposition 1 in Section 3. We label firms in market b according to their appeal, so that t_j is the jth most-appealing firm.

Corollary 1. *Under Assumption 1, in any subgame-perfect equilibrium G takes over the most appealing firm t_1 in market b at a price $V^b_{t_1} - q^a_G|\eta^b_{t_1t_2}|$, thereby making a profit of $q^a_G(\alpha^b_{t_1} + |\eta^b_{t_1t_2}|)$.*

Intuitively, because the most appealing firm t_1 has the greatest default advantage, taking over t_1 allows G to maximally exploit the default-setting advantage it derives from its customer base in

[22] Consistent with this assumption, Allcott et al. (2025) find that simply drawing the attention of Google users to Bing (and eliminating switching costs by forcing an active choice) is in itself insufficient to increase Bing's market share, because these users perceive Bing as an inferior search engine.

[23] Suppose G is the most appealing firm in market a and M the second most appealing one. Then Assumption 1 implies that $\Delta_{Gj} > \Delta_{jG}$ and $\Delta_{Gj} > \Delta_{ij}$ for all $i, j \neq G$ and $i \neq j$. With two single-market firms, G must win the access-point auction if bidders use weakly undominated strategies and thus bid their willingness to pay since $\Delta_{GM} > \Delta_{MG}$. With more firms the willingness to pay depends on who wins the auction otherwise. Nevertheless, it is easy to see that firm G bidding Δ_{GM} and all rivals $j \neq G$ bidding Δ_{jG} is a weakly undominated equilibrium. In fact, we show in Proposition 4 of a working-paper version of our paper (ECONtribute Discussion Paper, No. 329, `https://t1p.de/6444k`) that G must win the auction with probability 1 in any equilibrium in which bidders use weakly undominated strategies.

market a. In addition, by Part I of Assumption 1, t_1's value as an independent firm drops most when G takes over the second most appealing firm t_2. Thus, the credible threat to buy t_2 is what determines the takeover discount when taking over t_1. And while in principle G could trade a lower default advantage when taking over a firm other than t_1 for a higher takeover discount via the more powerful threat to buy t_1 otherwise, Part III of Assumption 1 implies that this is suboptimal.[24]

Then, combining Corollary 1 with Proposition 2 yields:

Corollary 2. *Under Assumption 1, the most appealing firm (and leader) in market a earns more from a takeover than any other firm, and earns most when taking over the most appealing firm in market b.*

Based on Corollaries 1 and 2, we conclude that an ecosystem combines the most appealing services in markets a and b, magnifying the size dominance of the two firms in the economy. Moreover, if the market leader in a has formed an ecosystem, by Proposition 4 its willingness to pay for the default position increases even further, so it still wins the auction. We conclude that the ecosystem wins everything — both markets and the default position.

6 Policy Discussion

Following high-profile reports in the US (Scott Morton et al., 2019), UK (Furman et al., 2019), and EU (Crémer et al., 2019), policymakers have proposed a variety of laws aiming to curb the power of "big tech." Many of these interventions fall into two broad categories:

1. **Leverage policies** weaken or eliminate an ecosystem's ability to steer consumers to itself. For example, Articles 6(5) and 6(6) of the EU's Digital Market Act (DMA) prohibit large ecosystems (referred to as "gatekeepers") from giving their own complementary services preferential treatment.[25] There has also been discussion of either breaking up big-tech firms or

[24] Since taking over the most appealing firm while threatening with the second-most appealing one is optimal, it is easy to see that under Assumption 1 there exists a subgame-perfect equilibrium in which the acquirer approaches targets in the order of their appeal.

[25] Similarly to the EU, China's drafted guidelines would restrict dominant ecosystems' ability to give its own services preferential treatment (Prüfer et al., 2024).

limiting their ability to engage in further takeovers, which can be seen as an extreme form of preventing leverage.[26]

2. **Access-point policies** prevent an ecosystem from setting its own service as the default at an access point. For instance, Articles 6(3) and 6(4) of the EU DMA limit the use of default settings on operating systems, virtual assistants, web browsers, and app stores. Indeed, EU consumers now often see a choice screen when first using a browser or engaging with a search engine on a new mobile device.[27] Similarly, one function of the interoperability requirements in Article 6(7) is to limit preferential default setting at access points.[28] And in the US, Judge Mehta restricted the duration and form of the contract for Google's exclusive search default positions on Apple devices.[29]

Motivated by these initiatives, we ask how leverage and access-point policies affect ecosystems and market outcomes, maintaining Assumption 1 (complementarity) throughout.

We begin with short-run effects. For services that are valuable to consume and vertically differentiated, it is plausible that higher appeal reflects higher consumer surplus. Indeed, this is a common property of all our microfoundations. Under this interpretation, the arguments in Section 5 imply that the highest-quality firm in market a forms an ecosystem with the highest-quality firm in market b and wins the default position in market a. Hence, our model endogenously predicts a benefit from the equilibrium ecosystem: it defaults consumers to the best service in market a, and then steers them toward the best service in market b. A leverage policy lowers the second of these benefits, while an access-point policy lowers both benefits. As a result, both types of policies carry

[26] The proposal to break up big tech companies has been, for example, a central part of Senator Warren's campaign for US President (e.g., `https://t1p.de/rtm8k`), and the US Justice Department has (so far unsuccessfully) proposed the option to break up Google's ecosystem after a recent court ruling found that Google monopolized the search market (e.g., `https://t1p.de/blll2`). Scott Morton et al. (2019) argue for scrutinizing big tech acquisitions more heavily. On this topic, see also Cabral et al. (2021). Both links accessed August 27, 2024.

[27] Beyond the DMA, in the EU the steering of consumers is also restricted through various other regulations, including the EU's Digital Service Act (e.g., Art. 25), Data Act, AI Act, as well as the Unfair Commercial Practices Directive (for an overview see Busch and Fletcher, 2024). These aim to protect consumers from online choice architecture — sometimes referred to as "dark patterns" — that misleads or otherwise induces the consumer to act in the firm's rather than her own interest. Link accessed May 15, 2026.

[28] For instance, the Commission's preliminary finding is that Google's steering through always-on hotword detection, which allows a user to access their mobile by saying "hey Google" without similar access for others, violates the article (see `https://shorturl.at/rtGCx`, accessed May 15, 2026).

[29] See `https://t1p.de/wtteh` (accessed October 7, 2026).

a welfare cost. Consistent with these predictions, users complained, for example, about the lack of direct steering from Google search to Google Maps in response to the DMA.[30]

There are, however, some important caveats regarding this message. First, if Assumption 1 does not hold, the equilibrium ecosystem need not combine the market leader in a with the most appealing follow-on service.[31] Second, if consumers have heterogeneous tastes for services, then appeal no longer reliably indicates consumer surplus. In particular, being steered to the most appealing service may harm some consumers, and it is easy construct examples in which this loss outweighs the benefit to others. Third, even with homogeneous tastes, appeal and welfare rankings align only for valuable services (e.g., search, maps, or the operating system of a self-driving car) that are optimal for a person to use. The relationship can easily reverse for services (e.g., social networks) that may be overconsumed.[32] Then, a more appealing service can induce more overconsumption. Fourth, and perhaps most importantly, a common concern of policymakers is that the ability to steer entrenches the power of ecosystems and thereby reduces dynamism in the digital economy. We turn to this concern next.

We follow the policy discourse and discuss the effect of ecosystems and the above policies on the contestability of markets — i.e., the ability of non-dominant firms to make sufficient profits to overcome barriers to entry.[33] We assume that an ecosystem G has emerged by combining the most appealing services in markets a and b, and has acquired the default position in market a. Consider a potential entrant e into market $s \in \{a, b\}$. Firm e faces a fixed entry cost, which is drawn from a distribution with full support, and its demand $q^s_{ej} \geq 0$ — conditional on entry and firm $j \in \mathcal{N}^s \cup \{e\}$ being the default — satisfies Assumptions 0 and 1. We say that a policy makes market s *more contestable* if it makes entry into market s more likely.

Proposition 5 (Contestability Concerns)**.** *Suppose Assumption 1 holds.*

[30] See Pape and Rossi (2026) or `https://t1p.de/q9vfi` (accessed September 21, 2026).

[31] Similarly, allowing for the emergence of multiple ecosystems qualifies the convenience benefit more generally. With more than one ecosystem, not all of them can offer — and steer consumers to — the best follow-on service.

[32] See Allcott et al. (2020) for empirical evidence that the average user spends too much time on Facebook, and Bursztyn et al. (2024) for experimental evidence suggesting that a large share of TikTok and Instagram users would be better off if these platforms did not exist in the first place.

[33] Contestability is defined in Recital 32 of the DMA as "[...] ability of undertakings to effectively overcome barriers to entry and expansion and challenge the gatekeeper on the merits of their products and services."

I. An access-point policy makes market a more contestable.

II. Any policy, access-point or leverage, makes market b more contestable.

Part I considers entry into market a. Since G offers the most appealing existing service there, it would be the entrant's strongest competitor. Hence, by Lemma 1 and Assumption 1, G imposes a negative default externality on the entrant. An access-point policy eliminates this externality and thus makes market a more contestable.

Part II addresses contestability in market b. Because G is more likely than its constituent to become the default in market b, and because G imposes a negative default externality on any rival in market b, it reduces e's demand conditional on entry. Mitigating this effect, both an access-point and a leverage policy lower the probability that G becomes the default in market b, raising e's profit conditional on entry and thus improving contestability.

Extending the model in realistic directions reveals additional policy implications of the basic logic underlying Proposition 5. We discuss three sets of extensions. We focus on "good entry" by assuming that e's service is more appealing than the ecosystem's (recall that appeal is defined in the symmetric situation where no firm is favored).

First, consider entry into market a, and suppose that the third party selling the access-point default re-runs the auction upon entry, in which the entrant expects to bid. Then, the logic of Proposition 4 implies that either G or e wins the auction. In particular, if G wins, it draws consumers from an entrant who may be offering a better service. A sufficiently effective leverage policy ensures that e wins the auction and lowers the amount it pays for the default, so it makes market a more contestable.[34] More subtly, an access-point policy also raises contestability unless e has a much more appealing product than G. Even if e wins upon entering, it benefits from the auction only if its default advantage α_e^a compensates for the required payment. This can happen only if e's product is so much more appealing than G's that α_e^a/α_G^a exceeds G's default multiplier $1+\alpha_G^b$.

[34] With only single-market firms, under Assumption 1, the firm offering the most appealing service a wins the auction in any cautious equilibrium. Absent the policy, if $\Delta_{Ge}^a(1+\alpha_G^b) > \Delta_{eG}^a$, there exists a cautious equilibrium in which G wins the auction against a more appealing entrant. And even if the entrant wins the auction absent the policy, it may pay up to $\Delta_{Ge}^a(1+\alpha_G^b)$, while with the policy it pays at most Δ_{Ge}^a in any cautious equilibrium.

Second, in the extension with additional markets connected to b, further implications arise. Since G derives a default advantage in market b, both types of policy lower the probability that G becomes the default in a connected market. As a result, both types of policy encourage entry also in connected markets. This effect is reinforced if e indeed enters market b, taking further market share from G and thus creating a "double dividend" to encouraging entry.

Finally, we discuss contestability of markets in which the ecosystem is not yet active. Consider an entrant e that decides whether to enter a market before G can take over one of the then existing firms. As we have discussed in Section 3.1, an ecosystem has a strong incentive to expand through backward integration into a new access point, so an entrant in such a market may be able to sell expensively. Leverage policies, therefore, initially lower contestability in new access-point markets. This contrasts with entry into a follow-on market. Proposition 1 implies that conditional on entry, the ecosystem G takes over the entrant at a discount that is increasing in G's cross-market leverage. Hence, leverage policies raise contestability in follow-on markets.[35]

7 Conclusion

Given our prediction that digital markets will be characterized by ecosystems with systemwide dominance, one may wonder why the same thing does not happen in the offline world. While we do not have a fully precise answer, several observations may be helpful for answering this question. To start, in some settings offline ecosystems do emerge. For example, tropical hotels often feature an array of services, including airport transfers and a selection of water sports and tours, that keep travelers from looking for other options. Nevertheless, such arrangements do seem less dominant in offline settings. Unlike tropical hotels, for example, hotels in Paris or London are usually not part of an ecosystem. A simple explanation may be that offline sellers are less able to steer consumers than online firms. Even if a hotel in Paris recommends a particular restaurant to a traveler, she may be tempted by the many other restaurants she encounters. Since takeovers are costly, they may make sense only if steering by the acquirer can substantially increase a target's market share, and this

[35] A policy that would prohibit ecosystems from acquiring startups would also increase contestability in the follow-on market in the above setup. Such a policy, however, would give the ecosystem an extra incentive to develop a market-b technology in house.

is simply not the case in most offline settings. Tropical hotels are arguably in a unique position to steer because many travelers are reluctant to venture outside due to their lack of knowledge about the culture or environment. We have, however, not evaluated this hypothesis systematically, and doing so is an important topic for future research.

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A Behavioral Foundations of Default Effects

We provide a series of microfoundations for a firm's benefit of having the default position. Proofs (and more detailed intuitions) for the claims below can be found in Appendix C. Consider a single representative consumer who values the service $s \in \{a, b\}$ offered by firm $i \in \mathcal{N}^s$ at v_i^s, which is drawn from a distribution with continuous cumulative distribution function (CDF) F_i^s. (Alternatively, think of a mass of consumers whose valuations are distributed according to F_i^s.) The consumer's valuations are drawn independently across firms and markets. Within market s, the value distributions can be ranked in terms of (weak) first-order-stochastic dominance (FOSD) and have common support. These rankings correspond to the "appeal rankings" introduced in the main text, and we refer to a more appealing product simply as better. For each microfoundation, we analyze how the demand for the different services changes if i replaces j as the default in market s. (In the interpretation with a mass of consumers, this corresponds to a common change in the default for all consumers.) Throughout, we assume that the consumer chooses a service s based on within-market considerations only. Our qualitative results generalize, however, to a forward-looking consumer who understands, and takes into account, the steering by ecosystems.

Technically Unsavvy Consumer Consider a consumer who is assigned a default product and does not know that this default can be changed or, alternatively, believes that doing so is prohibitively costly. If the value of the default product is above a critical threshold $\underline{v} \in \mathbb{R}$, the consumer uses the default service. Otherwise, she refrains from using any service in market s (i.e., she selects an outside-option activity). This model of a "technically unsavvy consumer" satisfies Assumption 0 if and only if $F_i^s(\underline{v}) < 1$ for all $i \in \mathcal{N}^s$. In addition, it satisfies Assumption 1 if and only if $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ whenever F_i^s first-order stochastically dominates F_j^s. Moreover, for valuable services, consumer welfare is higher with a better default.

To simplify notation, we assume in all remaining applications that the consumer must use one of the services (i.e., has no outside option).

Switching Costs In a less extreme version of the above model, the consumer incurs physical switching costs of $\gamma > 0$ when switching away from the default product. Assumption 0 then follows from the common support and independence of $\{F_i^s\}_{i \in \mathcal{N}^s}$. To also establish Assumption 1, we strengthen our assumptions on the distributions of values: all distributions have full support on $\mathbb{R}$, and can be ranked in terms of their reversed hazard rates.[36] Then, if $n^s \geq 3$ and switching costs are small enough,[37] this model also satisfies Assumption 1. Moreover, under the same distributional assumptions, consumer welfare is higher with a better default.

Sequential Search A consumer observes the value (but not the identity) of the default product for free and can then search for other products at an increasing cost. For simplicity, search costs increase sufficiently fast for the consumer to search at most once. A searching consumer draws one of the non-default products at random.[38] The model implies Assumption 0, and it satisfies Assumption 1 if and only if $n^s \geq 3$ and $F_i^s(v) \neq F_j^s(v)$ for small enough v whenever F_i^s first-order stochastically dominates F_j^s. Consumer welfare is higher with a better default.

Consideration Sets The consumer considers only $k^s \in \{1, \ldots, n^s - 1\}$ offers in market s. The consumer's "consideration set" includes the default product with certainty, and any non-default product with probability $(k^s-1)/(n^s-1)$. From her consideration set, the consumer selects the product with the highest realized value v_i^s (as in Lleras et al., 2017). This model can be thought of as a simultaneous search model in which the consumer (perhaps optimally) chooses the size of her consideration set before her attention is drawn to the default product. For example, the consumer may consider products based on her "mood," and the default firm is the only one being able to predict her mood (e.g., by using data from past interactions) and tailor its product accordingly. The model implies Assumption 0, and it satisfies Assumption 1 if and only if $n^s \geq 3$ and $k^s \geq 2$. Consumer welfare is higher with a better default.

[36] We provide a formal definition of "dominance in reversed hazard rates" in Appendix C.2.

[37] The literal cost of switching to another option in the digital context is typically small.

[38] This microfoundation resembles Armstrong and Zhou (2011)'s work on prominence in search markets, whereby a product is prominent either because of payments (as in our access-point market; their Section 1.2) or cross-market steering (as in our follow-on market; their Section 3).

Loss Aversion Suppose that the default product acts as a reference point, and the consumer dislikes falling short of this reference point. Product i is defined by a pair (v_i^s, d_i^s) with $v_i^s \sim G^s$ and $d_i^s \in \mathbb{R}_{>0}$. We think of d_i^s as an app-design dimension affecting the consumer's utility in an additively separable way (as in most applications of Kőszegi and Rabin, 2006). Let $F_i^s(u) = G^s(u - d_i^s)$ be the distribution of $v_i^s + d_i^s$. We assume that, if the consumer uses the product of firm j instead of the default product offered by firm i, she experiences a loss of λd_i (with $\lambda > 1$) because she cannot enjoy firm i's proprietary design features, and a gain of d_j from using firm j's product instead. In addition, the consumer has some reference point in the service dimension, and we think of the distribution G^s as being "gain-loss adjusted." The model satisfies Assumption 0. To establish Assumption 1, we again assume that any F_i^s and F_j^s can be ranked in terms of their reversed hazard rates. Then, Assumption 1 holds as long as the consumer is not too loss averse. A better default, however, does not necessarily increase the welfare of a loss-averse consumer.

Other Models of As-If Switching Costs From a positive perspective, our loss-aversion-based model of default effects is mathematically equivalent to a model of product-specific switching costs. Similarly, other mechanism give rise to "as if" switching costs, with similar positive (and normative) implications.[39] The consumer may, for example, (mis)interpret the default status of a product as a recommendation, introducing non-physical switching costs. Or a consumer may procrastinate on making a choice, which again introduces non-physical switching costs.

B Proofs

Proof of Lemma 1. Part I. By Assumption 0, we immediately have

$$\alpha_i^s = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} (q_{ii}^s - q_{i\ell}^s) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{(q_{ii}^s - q_{i\ell}^s)}_{>0} > 0.$$

[39] Non-physical switching costs are arguably higher than the physical costs of switching, which makes the restriction to small switching costs — necessary for some of our results — stronger. At the same time, the additional structure implied by these models can help with satisfying Assumption 1.

Part II. Let $i \in \mathcal{N}^s$, and to derive a contradiction, suppose $\eta^s_{ij} \geq 0$ for all $j \neq i$. Then,

$$\alpha^s_i = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (q^s_{ii} - q^s_{i\ell}) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (\alpha^s_i - \eta^s_{i\ell}) \leq \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \alpha^s_i = \frac{n^s - 1}{n^s} \alpha^s_i < \alpha^s_i,$$

where the strict inequality follows from $\alpha^s_i > 0$; a contradiction. □

Proof of Proposition 1. Part I. We solve the game backwards. If G takes over t, it earns additional gross profits of $V^b_t + q^a_G \alpha^b_t$, which by Lemma 1 (Part I), is greater than t's standalone value V^b_t. Standard ultimatum-bargaining game analysis, hence, implies that in any subgame in which there is a single target t left, G must offer V^b_t and t always accepts that offer.

Now consider any subgame in which a target t' receives an offer and there is one remaining target $t \neq t'$ left thereafter. By subgame perfection, t' accepts the offer only if it is greater than $V^b_{t'} + q^a_G \eta^b_{t't}$, and whenever it accepts the offer in equilibrium, by standard arguments, the offer cannot be strictly greater than $V^b_{t'} + q^a_G \eta^b_{t't}$. When an offer of $V^b_{t'} + q^a_G \eta^b_{t't}$ is accepted, G earns net profits of $q^a_G(\alpha^b_{t'} - \eta^b_{t't})$ while otherwise it earns $q^a_G \alpha^b_t$. Hence, G earns at most $q^a_G \max\{\alpha^b_{t'} - \eta^b_{t't}, \alpha^b_t\}$. Similarly, if firm G instead approached t first, it would earn at most $q^a_G \max\{\alpha^b_t - \eta^b_{tt'}, \alpha^b_{t'}\}$. Therefore, in any subgame with just two targets left, G earns at most

$$\max_{t_1, t_2 \in \mathcal{N}^b} q^a_G \left(\alpha^b_{t_1} - \min\left\{0, \eta^b_{t_1 t_2}\right\} \right) = \max_{t_1, t_2 \in \mathcal{N}^b} q^a_G \left(\alpha^b_{t_1} - \eta^b_{t_1 t_2} \right) =: \pi, \tag{2}$$

where the equality follows because by Part II of Lemma 1, for every t_1 there exists some t'_2 such that $\eta^b_{t_1 t'_2} < 0$. Because we have a finite number of targets, the maximum in (2) exists.

Next, we observe that in any subgame with two or more remaining targets in which t accepts G's offer, G must offer at least $V^b_t + q^a_G \min_{t' \in \mathcal{N}^b \setminus \{t\}} \min\{0, \eta^b_{tt'}\}$; for otherwise the target is better of rejecting the offer no matter what happens in later subgames. This, however, implies that G's profits from the takeover in any subgame are at most

$$\max_{t' \in \mathcal{N}^b} q^a_G \left(\alpha^b_t - \min\left\{0, \eta^b_{tt'}\right\} \right) \leq \max_{t_1, t_2 \in \mathcal{N}^b} q^a_G \left(\alpha^b_{t_1} - \min\left\{0, \eta^b_{t_1 t_2}\right\} \right) = \pi.$$

Finally, let t'_1 and t'_2 be targets for which the maximum in (2) is attained. By standard arguments, in any subgame with only t'_1 and t'_2 left, G's offer must be accepted by the target it first approaches. By approaching all targets other than t'_1 and t'_2 beforehand and offering them

zero, which must be rejected, G can induce the subgame in which only t_1' and t_2' remain. In the subgame-perfect equilibrium of this subgame, G earns π from the takeover. Since G cannot earn more than π in any other subgame, offering zero to all other targets first and playing a subgame-perfect equilibrium strategy when only t_1' and t_2' remain is part of a subgame-perfect equilibrium strategy for G. Since doing so is always a possible deviation, furthermore, G must earn π from the takeover in any subgame-perfect equilibrium. Existence of a subgame perfect equilibrium follows from standard arguments. Since G must offer any target t at least $V_t^b + q_G^a \min_{t' \in \mathcal{N}^b \setminus \{t\}} \eta_{tt'}^b$, for G to earn π, it must take over a target t_1 from a pair t_1 and t_2 that attains the maximum in (2) at the price specified in the proposition. □

Proof of Proposition 2. Part I. Suppose firm $i \in \mathcal{N}^b$ plays the takeover game, and wants to take over a firm $j \in \mathcal{N}^a$. Because firm i can not direct consumers into market a, it has to pay j's standalone value for j to accept the offer. Upon taking over firm j, firm i can direct j's consumers in market a to its own product in market b, generating a default advantage α_i^b. Hence, firm i earns additional profits of $q_j^a \alpha_i^b > 0$ from taking over firm j. By definition, these profits are highest when taking over a market leader ℓ because $q_\ell^a \geq q_k^a$ for all $k \in \mathcal{N}^a$.

Part II. By Proposition 1, a market leader ℓ in market a earns additional profits of

$$q_\ell^a \left(\alpha_i^b + |\eta_{ii^*}^b|\right) > q_\ell^a \alpha_i^b$$

from taking over firm i in market b (where i^* is i's strongest rival). By definition, a market leader ℓ generates more profits from taking over i than any other firm $j \in \mathcal{N}^a$ does. □

Sketch of the Proof of Proposition 3. We only sketch the proof here, and provide the full argument in Appendix D. Conceptually, the proof proceeds by backward induction in the usual way. A crucial, but tedious, step is to show the existence of a cautious equilibrium in which acquirers use iteratively weakly undominated strategies in the reduced game where future subgames with a unique cautious equilibrium outcome are replaced by that outcome.

We start by considering subgames with more potential acquirers than targets. We first show that with only a single target left, there exists a cautious equilibrium in which the potential acquirer with the greatest market-a demand takes over the target at the price specified in the proposition.

We then argue that this is the outcome in any cautious equilibrium with a single target left. Next, using backward induction, we move on to subgames with $k \geq 2$ remaining targets and at least $k+1$ remaining acquirers. We begin by establishing the following necessary conditions for a cautious equilibrium: (i) no firm with a market-a demand below the kth highest among the remaining acquirers makes a takeover in equilibrium; (ii) the takeover price must be the one specified in the proposition; and (iii) the firm with the kth highest market-a demand does not take over a target other than the last one. Finally, we construct a cautious equilibrium that implements the outcome specified in the proposition.

Having solved the subgames with more targets than acquirers allows us to proceed by backward induction for the case in which there are currently as many targets as potential acquirers. The proof proceeds similarly as above. Finally, we turn to the case in which there are currently less acquirers than targets, again proceeding by backward induction. □

Proof of Proposition 4. It is well-known that with two bidders, it is weakly dominant for each to bid its valuation. Hence, to prove the claim, we simply calculate these valuations.

Part I. Suppose M is a single-market firm. Hence, M's willingness to pay to replace G as the default is Δ^a_{MG}. If G replaces M as the default in market a, it earns additional profits of Δ^a_{GM} in that market, and it increases the probability of being the default in market b by Δ^a_{GM}. Hence, G values the default position in market a at $\Delta^a_{GM}(1+\alpha^b_G)$. The claim follows.

Part II. Suppose M is an ecosystem. We start by deriving G's willingness to pay to replace M as the default in a. When firm G is the default in market a, it makes profits of

$$q^a_{GG} + q^a_{GG}q^b_{GG} + q^a_{MG}q^b_{GM} + (1 - q^a_{GG} - q^a_{MG})\frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q^b_{G\ell} = q^a_{GG} + q^a_{GG}\alpha^b_G + q^a_{MG}\eta^b_{GM} + \frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q^b_{G\ell}.$$

When firm M is the default in market a, firm G earns

$$q^a_{GM} + q^a_{GM}q^b_{GG} + q^a_{MM}q^b_{GM} + (1 - q^a_{GM} - q^a_{MM})\frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q^b_{G\ell} = q^a_{GM} + q^a_{GM}\alpha^b_G + q^a_{MM}\eta^b_{GM} + \frac{1}{n^b}\sum_{\ell \in \mathcal{N}^b} q^b_{G\ell}.$$

Using $q^a_{GG} - q^a_{GM} = \Delta^a_{GM}$ and $-(q^a_{MM} - q^a_{MG}) = -\Delta^a_{MG}$, we obtain

$$\text{WTP}_{GM} = \Delta^a_{GM}(1+\alpha^b_G) - \Delta^a_{MG}\eta^b_{GM} = \Delta^a_{GM}(1+\alpha^b_G) + \Delta^a_{MG}|\eta^b_{GM}|,$$

where the last equality follows from the fact that $\eta^b_{GM} < 0$. Similarly, because $\eta^b_{MG} < 0$,

$$\text{WTP}_{MG} = \Delta^a_{MG}(1 + \alpha^b_M) + \Delta^a_{GM}|\eta^b_{MG}|.$$

The claims follows by observing that

$$\text{WTP}_{GM} > \text{WTP}_{MG} \quad \text{ if and only if } \quad \Delta^a_{GM}(1+\alpha^b_G+\eta^b_{MG}) > \Delta^a_{MG}(1+\alpha^b_M+\eta^b_{GM}). \qquad \square$$

Proof of Corollary 1. By Proposition 1, G takes over a firm t_1, while threatening to otherwise take over t_2, such that t_1 and t_2 solve $\max_{t'_1,t'_2\in\mathcal{N}^b} \alpha^b_{t'_1} - \eta^b_{t'_1t'_2}$. By definition, $\alpha^b_i - \eta^b_{ij} = q^b_{ii} - q^b_{ij}$. Applying Parts III, II, and I of Assumption 1, respectively, yields

$$q^b_{t_1t_1} - q^b_{t_1t_2} > q^b_{t_2t_2} - q^b_{t_2t_1} \geq q^b_{jj} - q^b_{jt_1} \geq q^b_{jj} - q^b_{jj'}$$

for all $j, j' \neq t_1, t_2$. Hence, under Assumption 1, G takes over the most appealing service b. $\square$

Proof of Proposition 5. Part I. Absent an access-point policy, conditional on entering market a, firm e earns q^a_{eG} net of entry cost. With the policy, e could earn $\frac{1}{n^a+1}\sum_{j\in\mathcal{N}^a\cup\{e\}} q^a_{ej}$ net of cost. Hence, an access-point policy raises e's (net) profit conditional on entry if and only if

$$q^a_{eG} - \frac{1}{n^a+1}\sum_{j\in\mathcal{N}^a\cup\{e\}} q^a_{ej} = \eta^a_{eG} < 0.$$

Because G offers the most appealing existing service a, by Part I of Assumption 1, $q^a_{eG} \leq q^a_{ej}$ for all $j \in \mathcal{N}^a$. But then, by Part II of Lemma 1, we must have $\eta^a_{eG} < 0$, which yields the claim.

Part II. Absent an access-point policy, conditional on entering market b, firm e earns

$$q^a_{GG}q^b_{eG} + (1 - q^a_{GG})\frac{1}{n^b+1}\sum_{j\in\mathcal{N}^b\cup\{e\}} q^b_{ej} = q^a_{GG}\eta^b_{eG} + \frac{1}{n^b+1}\sum_{j\in\mathcal{N}^b\cup\{e\}} q^b_{ej}$$

net of entry cost. With an access-point policy, e could earn a net profit of

$$\left(\frac{1}{n^a}\sum_{j\in\mathcal{N}^a} q^a_{Gj}\right)q^b_{eG} + \left(1 - \frac{1}{n^a}\sum_{j\in\mathcal{N}^a} q^a_{Gj}\right)\frac{1}{n^b+1}\sum_{j\in\mathcal{N}^b\cup\{e\}} q^b_{ej} = \left(\frac{1}{n^a}\sum_{j\in\mathcal{N}^a} q^a_{Gj}\right)\eta^b_{eG} + \frac{1}{n^b+1}\sum_{j\in\mathcal{N}^b\cup\{e\}} q^b_{ej}.$$

Hence, an access-point policy raises e's (net) profit conditional on entry if and only if

$$\left(q^a_{GG} - \frac{1}{n^a}\sum_{j\in\mathcal{N}^a} q^a_{Gj}\right)\eta^b_{eG} = \alpha^a_G\eta^b_{eG} < 0.$$

By the same arguments as in Part I, because G offers the most appealing existing service b, we must have $\eta^b_{eG} < 0$. Moreover, by Part I of Lemma 1, $\alpha^a_G > 0$. This proves the first claim.

Finally, consider a leverage policy. With such a policy, e could earn $\frac{1}{n^b+1}\sum_{j\in\mathcal{N}^b\cup\{e\}} q^b_{ej}$ net of cost. Combining this with e's potential profit absent a policy as derived above, a leverage policy raises e's (net) profit conditional on entry if and only if $q^a_{GG}\eta^b_{eG} < 0$. Because $q^a_{GG} > 0$ by assumption, the claim follows from the fact that $\eta^b_{eG} < 0$ (as repeatedly argued above). □

Online Appendix

C Proofs of the Microfoundations

C.1 Technically Unsavvy Consumer

Consider market s, and let $n^s \geq 2$. The consumer is assigned a default service i, which she uses if its value v_i is above a $\underline{v} \in \mathbb{R}$. Otherwise, the consumer does not use service s.

Lemma 2 (Technically Unsavvy Consumer)**.**

I. Assumption 0 holds if and only if $F_i^s(\underline{v}) < 1$ for all i.

II. Assumption 1 holds if and only if $F_i^s(\underline{v}) < 1$ and $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ for all i and all $j \neq i$.

Proof. Part I. The consumer never uses a non-default product. Hence, for any i and $j \neq i$,

$$q_{ii}^s - q_{ij}^s = 1 - F_i^s(\underline{v}).$$

Hence, Assumption 0 holds if and only if $F_i^s(\underline{v}) < 1$. This proves Part I.

Part II. Part I of Assumption 1 holds because for any i and $j \neq i$, $q_{ii}^s = 1 - F_i^s(\underline{v}) \geq 0$ and $q_{ij}^s = 0$. Next, we observe that for any i, i', and $j \neq i, i'$,

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = F_{i'}^s(\underline{v}) - F_i^s(\underline{v}) = (q_{ii}^s - q_{ii'}^s) - (q_{i'i'}^s - q_{i'i}^s)$$

which is positive if and only if $F_{i'}^s(\underline{v}) > F_i^s(\underline{v})$. Because any two distribution F_i^s and $F_{i'}^s$ can be ranked in terms of FOSD, Parts III and IV of Assumption 1 thus hold if and only if $F_i^s(\underline{v}) \neq F_{i'}^s(\underline{v})$ for all i and i' with $i \neq i'$. This completes the proof of Part II. □

Consumer Welfare Suppose $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ for all i and all $j \neq i$, and in line with Section 6, let $\underline{v} \geq 0$; that is, the service is valuable in that the consumer "buys reasonably" (Heidhues et al., 2023b). If i is the default, consumer welfare is $\int_{\underline{v}}^{\infty} v \, dF_i^s(v)$. Since all F_ℓ^s have the same support, we have

$$\int_{\underline{v}}^{\infty} v \, dF_i^s(v) - \int_{\underline{v}}^{\infty} v \, dF_j^s(v) = \underline{v}\big(F_j^s(\underline{v}) - F_i^s(\underline{v})\big) + \int_{\underline{v}}^{\infty} F_j^s(v) - F_i^s(v) \, dv.$$

Because $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$ and $\underline{v} \geq 0$, the above is positive if F_i^s first-order stochastically dominates F_j^s. Hence, for valuable services consumer welfare is higher with a better default.

C.2 Switching Costs

Consider market s, and let $n^s \geq 2$. We study a more general model than the one described in Appendix A in that we allow for product-specific switching costs. Specifically, we assume that the consumer observes the value of all products, but incurs a switching cost $\gamma_i > 0$ when i's product is the default and she chooses the product of $j \neq i$. Moreover, we assume that, if F_i^s first-order stochastically dominates F_j^s, then $\gamma_i \geq \gamma_j$. The consumer uses one service.

Lemma 3 (Switching Costs: Steering)**.** *Assumption 0 holds.*

Proof. When firm i's product is the default, it is chosen if and only if $v_i + \gamma_i \geq \max_{\ell \in \mathcal{N}^s \setminus \{i\}} v_\ell$. This happens with probability

$$q_{ii}^s = \int \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v).$$

When firm j's product is the default, the consumer buys the product of firm i with probability

$$q_{ij}^s = \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v).$$

We obtain

$$\begin{aligned} q_{ii}^s - q_{ij}^s &= \int F_j^s(v + \gamma_i) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v) - \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v) \\ &> \int \Big(F_j^s(v + \gamma_i) - F_j^s(v - \gamma_j) \Big) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v), \end{aligned}$$

which is positive because $\gamma_i, \gamma_j > 0$. □

To also establish Assumption 1, we impose more structure. First, we assume that all value distributions have full support on $\mathbb{R}$, which implies that F_i^s admits a density f_i^s.[40] Second, we assume that any two distributions can be ranked in terms of their reversed hazard rates, which is

[40] If v_i^s has full support on $\mathbb{R}$, then F_i^s is strictly increasing and thus almost everywhere differentiable.

stronger than a ranking in terms of first-order stochastic dominance. Formally, F_i^s dominates F_j^s in the reversed hazard rate if $f_i^s(v)/F_i^s(v) > f_j^s(v)/F_j^s(v)$ for all $v \in \mathbb{R}$.

Lemma 4 (Switching Costs: Appeal-Steering Complementarity)**.** *Suppose that F_i^s dominates F_j^s in the reversed hazard rate for all $j \in \mathcal{N}^s$. Then, for any $n^s \geq 3$ and any such $\{F_\ell^s\}_{\ell \in \mathcal{N}^s}$, there exists some $\bar{\gamma} > 0$, such that for any $\gamma_i < \bar{\gamma}$, Assumption 1 holds.*

Proof. First, consider j and j' with F_j^s dominating $F_{j'}^s$ in the reversed hazard rate. Then,

$$\begin{aligned} q_{ij'}^s - q_{ij}^s &= \int \left(F_{j'}^s(v-\gamma_{j'})F_j^s(v) - F_j^s(v-\gamma_j)F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j,j'\}} F_\ell^s(v)\ dF_i^s(v) \\ &\geq \int \left(F_{j'}^s(v-\gamma_j)F_j^s(v) - F_j^s(v-\gamma_j)F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j,j'\}} F_\ell^s(v)\ dF_i^s(v), \end{aligned}$$

where the inequality follows from $\gamma_j \geq \gamma_{j'}$ and $F_{j'}^s$ being increasing. Moreover, because F_j^s dominates $F_{j'}^s$ in the reversed hazard rate, we have

$$\frac{\partial}{\partial v} \frac{F_j^s(v)}{F_{j'}^s(v)} \propto f_j^s(v)F_{j'}^s(v) - f_{j'}^s(v)F_j^s(v) > 0.$$

This implies, in turn, that

$$\frac{F_j^s(v)}{F_{j'}^s(v)} > \frac{F_j^s(v-\gamma_j)}{F_{j'}^s(v-\gamma_j)} \quad \text{or, equivalently,} \quad F_{j'}^s(v-\gamma_j)F_j^s(v) - F_j^s(v-\gamma_j)F_{j'}^s(v) > 0. \tag{3}$$

Hence, $q_{ij'}^s > q_{ij}^s$, which means that Part I of Assumption 1 holds.

Second, consider i and j with F_i^s dominating F_j^s in the reversed hazard rate. To verify Part III of Assumption 1, we define $G^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v)$ and observe

$$\begin{aligned} q_{ii}^s - q_{ij}^s &= \int F_j^s(v+\gamma_i)G^s(v+\gamma_i)\ dF_i^s(v) - \int F_j^s(v-\gamma_j)G^s(v)\ dF_i^s(v) \\ &= 1 - \int F_i^s(u-\gamma_i)G^s(u)\ dF_j^s(u) - \int F_j^s(u)F_i^s(u-\gamma_i)\ dG^s(u) \\ &\quad - \left(1 - \int F_i^s(u+\gamma_j)G^s(u+\gamma_j)\ dF_j^s(u) - \int F_j^s(u-\gamma_j)F_i^s(u)\ dG^s(u) \right) \\ &= q_{jj}^s - q_{ji}^s + \int \left(F_j^s(u-\gamma_j)F_i^s(u) - F_j^s(u)F_i^s(u-\gamma_i) \right) dG^s(u) \\ &\geq q_{jj}^s - q_{ji}^s + \int \left(F_j^s(u-\gamma_j)F_i^s(u) - F_j^s(u)F_i^s(u-\gamma_j) \right) dG^s(u), \end{aligned}$$

where the inequality follows from $\gamma_i \geq \gamma_j$ and F_i^s being increasing. Hence, by (3), it follows that $q_{ii}^s - q_{ij}^s > q_{jj}^s - q_{ji}^s$ and thus Part III of Assumption 1.

Third, consider i and i' with F_i^s dominating $F_{i'}^s$ in the reversed hazard rate, and denote as $\widetilde{G}^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i,i',j\}} F_\ell^s(v)$. Then, since $F_i^s(v)$ and $F_{i'}^s(v)$ admit a density by assumption,

$$\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = &\int F_j^s(v+\gamma_i)\widetilde{G}^s(v+\gamma_i)F_{i'}^s(v+\gamma_i)f_i^s(v)\ dv \\
&- \int F_j^s(v+\gamma_{i'})\widetilde{G}^s(v+\gamma_{i'})F_i^s(v+\gamma_{i'})f_{i'}^s(v)\ dv \\
&- \int F_j^s(v-\gamma_j)\widetilde{G}^s(v)\Big[F_{i'}^s(v)f_i^s(v) - F_i^s(v)f_{i'}^s(v)\Big]\ dv \\
\geq &\int F_j^s(v+\gamma_i)\widetilde{G}^s(v+\gamma_i)\Big[F_{i'}^s(v+\gamma_i)f_i^s(v) - F_i^s(v+\gamma_i)f_{i'}^s(v)\Big]\ dv \\
&- \int F_j^s(v-\gamma_j)\widetilde{G}^s(v)\Big[F_{i'}^s(v)f_i^s(v) - F_i^s(v)f_{i'}^s(v)\Big]\ dv,
\end{aligned}$$

where inequality follows from $\gamma_i \geq \gamma_{i'}$, and F_j^s, $F_{i'}^s$ and $\widetilde{G}^s$ being increasing.

We distinguish two cases. First, let $\gamma_j = \gamma_i + \epsilon$ for $\epsilon > 0$. Then, at $\gamma_i = 0$, the above is

$$\int \widetilde{G}^s(v)\Big[F_j^s(v) - F_j^s(v-\epsilon)\Big]\Big[F_{i'}^s(v)f_i^s(v) - F_i^s(v)f_{i'}^s(v)\Big]\ dv > 0,$$

where the inequality follows from $\epsilon > 0$ and F_j^s having full support, and F_i^s and $F_{i'}^s$ satisfying the reverse hazard rate. By continuity, there exists some $\bar{\gamma}_{ii'j} > 0$, so that for any $\gamma_i < \bar{\gamma}_{ii'j}$, we have $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$. Note that the constraint is getting tighter for smaller ϵ.

Second, let $\gamma_j = \gamma_i - \epsilon$ for some $\epsilon \in [0, \gamma_i]$. Hence, if $\gamma_i = 0$, then also $\gamma_j = 0$. It follows that $q_{ii}^s - q_{ij}^s = q_{i'i'}^s - q_{i'j}^s$ if $\gamma_i = 0$. We now take, for a fixed ϵ, the partial derivative of the lower bound on $(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s)$ derived above with respect to γ_i. Clearly, both integrands are bounded from above and from below. Hence, by Lebesgue's dominated convergence theorem, we can exchange differentiation and integration. This implies that this partial derivative is given by

$$\begin{aligned}
&\int \Big[F_{i'}^s(v+\gamma_i)f_i^s(v) - F_i^s(v+\gamma_i)f_{i'}^s(v)\Big]\frac{\partial}{\partial \gamma_i}\Big(F_j^s(v+\gamma_i)\widetilde{G}^s(v+\gamma_i)\Big)\ dv \\
&\quad + \int \Big[f_{i'}^s(v+\gamma_i)f_i^s(v) - f_i^s(v+\gamma_i)f_{i'}^s(v)\Big]F_j^s(v+\gamma_i)\widetilde{G}^s(v+\gamma_i)\ dv \\
&\quad + \int f_j^s(v-\gamma_j)\widetilde{G}^s(v)\Big[F_{i'}^s(v)f_i^s(v) - F_i^s(v)f_{i'}^s(v)\Big]\ dv.
\end{aligned}$$

Evaluating this partial derivative at $\gamma_i = 0$, we obtain

$$\int \left[F^s_{i'}(v) f^s_i(v) - F^s_i(v) f^s_{i'}(v) \right] \left(2 f^s_j(v) \widetilde{G}^s(v) + F^s_j(v) \tilde{g}(v) \right) dv,$$

which is strictly positive by the reversed hazard rate condition. Since $q^s_{ii} - q^s_{ij} = q^s_{i'i'} - q^s_{i'j}$ at $\gamma_i = 0$, this implies that again, there exists some $\bar{\gamma}_{ii'j} > 0$, such that for any $\gamma_i < \bar{\gamma}_{ii'j}$, $q^s_{ii} - q^s_{ij} > q^s_{i'i'} - q^s_{i'j}$. Here, the constraint gets tighter as ϵ gets bigger. Noticing that there are only finitely many combinations of i, i', and j, we conclude that $\bar{\gamma} := \min_{i,i',j \in \mathcal{N}^s} \bar{\gamma}_{ii'j} > 0$. Hence, if $\gamma_i < \bar{\gamma}$ for all i, also Part II of Assumption 1 holds. This completes the proof. □

Consumer Welfare Suppose switching costs are symmetric: $\gamma_\ell \equiv \gamma$ for all $\ell \in \mathcal{N}^s$. Denote as ν^s_i the value (net of switching costs) of the product *chosen* by the consumer when product i is the default. The value ν^s_i is drawn from the cumulative distribution function

$$\mathbb{P}[\nu^s_i \leq v] = F^s_i(v) \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F^s_\ell(v + \gamma).$$

Notice that

$$\mathbb{P}[\nu^s_i \leq v] - \mathbb{P}[\nu^s_j \leq v] = \left[F^s_i(v) F^s_j(v+\gamma) - F^s_i(v+\gamma) F^s_j(v) \right] \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F^s_\ell(v + \gamma).$$

Hence, if F^s_i dominates F^s_j in the reversed hazard rate, then ν^s_i first-order stochastically dominates ν^s_j. As a result, consumer welfare is higher with a better default.

Large Switching Costs To see that the restriction to small enough switching costs has bite, let $n^s = 3$, $\gamma_i = \gamma$, and assume that v^s_i is drawn from a normal distribution with a mean μ_i and a variance of 1. As illustrated in Figure 1 for the case of $\mu_1 = \frac{1}{2}$, $\mu_2 = 0$, and $\mu_3 = -\frac{1}{2}$, Part II of Assumption 1 does *not* hold in general. In this particular example, the upper bound on the switching cost γ for Assumption 1 to hold is given by $\bar{\gamma} \approx 2.15$.

C.3 Sequential Search

Consider market s, and let $n^s \geq 2$. The consumer observes the value, but not the identity of the default product for free and can then search at a cost. The first search costs $c_1 > 0$, the second

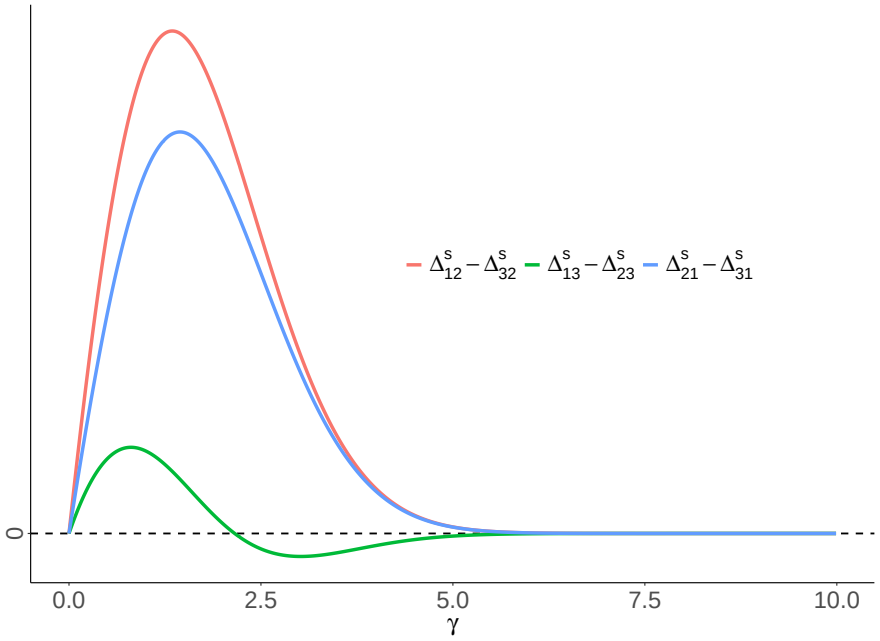

Figure 1: *Violation of Assumption 1 (Part II) with normal values. Note:* $\Delta^s_{ij} = q^s_{ii} - q^s_{ij}$.

search costs $c_2 \geq c_1$, and the kth search costs $c_k \geq c_{k-1}$. For simplicity, we set $c_2 = \infty$, thereby effectively assuming that the consumer searches at most once. If the consumer searches for another product, she randomly draws one of the non-default products.

Suppose that firm i's product is the default. The consumer searches if and only if the realized value v^s_i lies below some cutoff $\hat{v}^s_i \in \mathbb{R}$. Moreover, because the consumer does not learn the identity of the default firm, she does not learn anything about the value of the non-default firms and thus about the value of search. Formally, $\hat{v}^s_i = \hat{v}^s$. Then:

Lemma 5 (Sequential Search)**.**

I. Assumption 0 holds.

II. Assumption 1 holds if and only if $n^s \geq 3$ *and for all* $i, i' \in \mathcal{N}^s$*, there exists some* $v \leq \hat{v}^s$*, such that* $F^s_{i'}(v) \neq F^s_i(v)$.

Proof. Preliminaries. First, if firm i's product is the default in s, its profit in market s is

$$\begin{aligned} q_{ii}^s &= \mathbb{P}\big[v_i^s > \hat{v}^s\big] \;+\; \mathbb{P}\big[v_i^s \leq \hat{v}^s\big] \;\frac{1}{n^s-1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s \leq \hat{v}^s\big] \\ &= \mathbb{P}\big[v_i^s > \hat{v}^s\big] \;+\; \frac{1}{n^s-1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \Big\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] - \mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s > \hat{v}^s\big] \Big\} \\ &= \frac{1}{n^s-1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \Big\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big] \Big(1 - \mathbb{P}\big[v_i^s > v_\ell^s | v_i^s > \hat{v}^s\big] \Big) \Big\} \\ &= \frac{1}{n^s-1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{\Big\{ \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s\big] \Big\}}_{=:\; \mathcal{A}_{i\ell}}, \end{aligned} \tag{4}$$

where the second equality holds by the law of total probability.

Second, suppose firm j's product is the default. Firm i makes a sale only if the consumer searches *and* firm i is the random firm that she finds. Firm i's profit in market s then is

$$\begin{aligned} q_{ij}^s &= \frac{1}{n^s-1} \mathbb{P}\big[v_j^s \leq \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_j^s | v_j^s \leq \hat{v}^s\big] \\ &= \frac{1}{n^s-1} \underbrace{\Big\{ \mathbb{P}\big[v_i^s > v_j^s\big] - \mathbb{P}\big[v_j^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s > v_j^s | v_j^s > \hat{v}^s\big] \Big\}}_{=:\; \mathcal{B}_{ij}}, \end{aligned} \tag{5}$$

where the second equality again follows from the law of total probability.

Third, suppose that the default is determined at random. Using Eq. (4) and (5), we conclude that firm i's profit in market s is then given by

$$\frac{1}{n^s(n^s-1)} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \Big\{ \mathcal{A}_{i\ell} + \mathcal{B}_{i\ell} \Big\}.$$

Fourth, we express the probabilities above in terms of the underlying CDFs. We observe

$$\mathbb{P}\big[v_i^s > v_\ell^s\big] = \int_{-\infty}^{\infty} F_\ell^s(v) \; dF_i^s(v),$$

and

$$\mathbb{P}\big[v_i^s > \hat{v}^s\big] \mathbb{P}\big[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s\big] = \mathbb{P}\big[v_i^s \leq v_\ell^s, v_i^s > \hat{v}^s\big] = 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_\ell^s(v) \; dF_i^s(v).$$

This implies that

$$\begin{aligned}\mathcal{A}_{i\ell} &= \mathbb{P}\big[v_i^s > v_\ell^s\big] + \mathbb{P}\big[v_i^s > \hat{v}^s\big]\mathbb{P}\big[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s\big] \\ &= \int_{-\infty}^{\hat{v}^s} F_\ell^s(v)\ dF_i^s(v) + 1 - F_i^s(\hat{v}^s) \\ &= 1 - F_i^s(\hat{v}^s) + F_\ell^s(\hat{v}^s)F_i^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v)\ dF_\ell^s(v),\end{aligned} \tag{6}$$

where the last equality follows from integration by parts, as well as

$$\begin{aligned}\mathcal{B}_{ij} &= \mathbb{P}\big[v_i^s > v_j^s\big] - \mathbb{P}\big[v_j^s > \hat{v}^s\big]\mathbb{P}\big[v_i^s > v_j^s | v_j^s > \hat{v}^s\big] \\ &= \int_{-\infty}^{\infty} F_j^s(v)\ dF_i^s(v) - \left(1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v)\right) \\ &= 1 - \int_{-\infty}^{\infty} F_i^s(v)\ dF_j^s(v) - 1 + F_j^s(\hat{v}^s) + \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\ &= F_j^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v)\ dF_j^s(v),\end{aligned} \tag{7}$$

where the second equality again follows from integration by parts.

Finally, we define

$$\begin{aligned}\mathcal{C}_{ij} &:= \mathcal{A}_{ij} - \mathcal{B}_{ij} \\ &= 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_j^s(v)\ dF_i^s(v) + 1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\ &= 2 - F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s) - \big(1 - F_j^s(\hat{v}^s)F_i^s(\hat{v}^s)\big) + \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) - \int_{\hat{v}^s}^{\infty} F_i^s(v)\ dF_j^s(v) \\ &= 1 - F_i^s(\hat{v}^s)\big(1 - F_j^s(\hat{v}^s)\big) - F_j^s(\hat{v}^s),\end{aligned} \tag{8}$$

where the second equality follows from integration by parts. This also means that $\mathcal{C}_{ij} = \mathcal{C}_{ji}$.

Part I. Subtracting (5) from (4) gives

$$q_{ii}^s - q_{ij}^s = \frac{1}{n^s - 1}\left(\mathcal{C}_{ij} \quad + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \mathcal{A}_{i\ell}\right).$$

Because all F_ℓ^s have common support, we have $\mathcal{A}_{i\ell}, \mathcal{C}_{ij} > 0$. Hence, Assumption 0 holds.

Part II. Suppose F_i^s first-order stochastically dominates F_j^s. Since $\mathcal{C}_{ij} = \mathcal{C}_{ji}$, we have

$$(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) = \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \{\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell}\}.$$

Using Eq. (6), we conclude

$$\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell} = \big(F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s)\big)\big(1 - F_\ell^s(\hat{v}^s)\big) + \int_{-\infty}^{\hat{v}^s} \big(F_j^s(v) - F_i^s(v)\big)\, dF_\ell^s(v) \geq 0,$$

holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Moreover, if $n^s = 2$, $q_{ii}^s - q_{ij}^s = q_{jj}^s - q_{ji}^s$. Combining both, we conclude that Part III of Assumption 1 holds if and only if $n^s \geq 3$ and for all $i, i' \in \mathcal{N}^s$, there exists some $v \leq \hat{v}^s$, so that $F_{i'}^s(v) \neq F_i^s(v)$.

Next, consider three firms i, i', and $j \neq i, i'$. We have

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = \frac{1}{n^s - 1}\bigg(\mathcal{C}_{ij} - \mathcal{C}_{i'j} \quad + \quad \mathcal{A}_{ii'} - \mathcal{A}_{i'i} \quad + \sum_{\ell \in \mathcal{N}^s \setminus \{i,i',j\}} \big\{\mathcal{A}_{i\ell} - \mathcal{A}_{i'\ell}\big\}\bigg).$$

To verify Part II of Asssumption 1, let F_i^s first-order stochastically dominate $F_{i'}^s$. By (8),

$$\mathcal{C}_{ij} - \mathcal{C}_{i'j} = \big(F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s)\big)\big(1 - F_j^s(\hat{v}^s)\big) \geq 0,$$

holding with a strict inequality if and only if $F_i^s(\hat{v}^s) > F_i^s(\hat{v}^s)$. Moreover, by (6), we obtain

$$\mathcal{A}_{ii'} - \mathcal{A}_{i'i} = F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s) + \int_{-\infty}^{\hat{v}^s} F_{i'}^s(v)\, dF_i^s(v) - \int_{-\infty}^{\hat{v}^s} F_i^s(v)\, dF_{i'}^s(v) \geq 0,$$

holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Finally, by the same argument as above, we have $\mathcal{A}_{i\ell} \geq \mathcal{A}_{i'\ell}$, holding with a strict inequality if and only if $F_{i'}^s(v) \neq F_i^s(v)$ for some $v \leq \hat{v}^s$. Hence, Part II of Assumption 1 holds if and only if for all $i, i' \in \mathcal{N}^s$, there exists some $v \leq \hat{v}^s$, so that $F_{i'}^s(v) \neq F_i^s(v)$.

Finally, for any j, j', and $i \neq j, j'$, we obtain

$$q_{ij'}^s - q_{ij}^s = \frac{1}{n^s - 1}\bigg(\mathcal{C}_{ij} - \mathcal{C}_{ij'} + \mathcal{A}_{ij'} - \mathcal{A}_{ij}\bigg).$$

If F_j^s first-order stochastically dominates $F_{j'}^s$, then, by (8),

$$\mathcal{C}_{ij} - \mathcal{C}_{ij'} = \big(1 - F_i^s(\hat{v}^s)\big)\big(F_{j'}^s(\hat{v}^s) - F_j^s(\hat{v}^s)\big) \geq 0,$$

holding with a strict inequality if and only if $F_{j'}^s(\hat{v}^s) > F_j^s(\hat{v}^s)$. Similarly, by (6),

$$\mathcal{A}_{ij'} - \mathcal{A}_{ij} = \int_{-\infty}^{\hat{v}^s} \big\{F_{j'}^s(v) - F_j^s(v)\big\}\, dF_i^s(v) \geq 0,$$

holding with a strict inequality if and only if $F_{j'}^s(v) \neq F_j^s(v)$ for some $v \leq \hat{v}^s$. Hence, Part I of Assumption 1 holds if and only if $F_{j'}^s(v) \neq F_j^s(v)$ for some $v \leq \hat{v}^s$. $\square$

Consumer Welfare If product i is the default product, then consumer welfare equals

$$\begin{aligned}
&\mathbb{P}\big[v_i^s \geq \hat{v}^s\big]\mathbb{E}\big[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\big] + \big(1 - \mathbb{P}\big[v_i^s \geq \hat{v}^s\big]\big)\frac{1}{n^s - 1}\sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{E}\big[\max\{v_i^s, v_\ell^s\} - c_1 | v_i^s \leq \hat{v}^s\big] \\
&= \int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \frac{1}{n^s - 1}\bigg[\int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v)\bigg] \\
&= \frac{n^s - 2}{n^s - 1}\int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \frac{1}{n^s - 1}\bigg[\int_{\hat{v}^s}^{\infty} v \; dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i,j\}} \int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v)\bigg],
\end{aligned}$$

where $G_{i\ell}^s$ is the CDF of $\max\{v_i^s, v_\ell^s\}$. Because v_i^s and v_ℓ^s are independent, we have

$$G_{i\ell}^s(v) = \mathbb{P}\Big[\big\{\max\{v_i^s, v_\ell^s\} \leq v\big\} \cup \big\{v_i^s \leq \hat{v}^s\big\}\Big] = \begin{cases} F_i^s(v)F_\ell^s(v) & \text{if } v < \hat{v}^s, \\ F_i^s(\hat{v}^s)F_\ell^s(v) & \text{if } v \geq \hat{v}^s, \end{cases}$$

and the corresponding density is given by

$$g_{ij}^s(v) = \begin{cases} f_i^s(v)F_j^s(v) + F_i^s(v)f_j^s(v) & \text{if } v < \hat{v}^s, \\ F_i^s(\hat{v}^s)f_j^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

First, we observe that, for any $\ell \neq i, j$,

$$G_{i\ell}^s(v) - G_{j\ell}^s(v) = \begin{cases} \big(F_i^s(v) - F_j^s(v)\big)F_\ell^s(v) & \text{if } v < \hat{v}^s, \\ \big(F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s)\big)F_\ell^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

Hence, if F_i^s first-order stochastically dominates F_j^s, then $G_{i\ell}^s$ first-order stochastically dominates $G_{j\ell}^s$ for all $\ell \neq i, j$, and as a consequence, also

$$\int_{-\infty}^{\infty} v - c_1 \; dG_{i\ell}^s(v) > \int_{-\infty}^{\infty} v - c_1 \; dG_{j\ell}^s(v) \quad \text{for all } \ell \neq i, j.$$

Second, we observe that

$$\begin{aligned}
&\int_{-\infty}^{\infty} v - c_1 \; dG_{ij}^s(v) - \int_{-\infty}^{\infty} v - c_1 \; dG_{ji}^s(v) \\
&\qquad = F_i^s(\hat{v}^s)\int_{\hat{v}^s}^{\infty} v - c_1 \; dF_j^s(v) - F_j^s(\hat{v}^s)\int_{\hat{v}^s}^{\infty} v - c_1 \; dF_i^s(v) \\
&\qquad = F_i^s(\hat{v}^s)\big(1 - F_j^s(\hat{v}^s)\big)\mathbb{E}\big[v_j^s - c_1 | v_j^s \geq \hat{v}^s\big] - F_j^s(\hat{v}^s)\big(1 - F_i^s(\hat{v}^s)\big)\mathbb{E}\big[v_i^s - c_1 | v_i^s \geq \hat{v}^s\big],
\end{aligned}$$

and thus

$$\left(\int_{\hat{v}^s}^{\infty} v\ dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1\ dG_{ij}^s(v)\right) - \left(\int_{\hat{v}^s}^{\infty} v\ dF_j^s(v) + \int_{-\infty}^{\infty} v - c_1\ dG_{ji}^s(v)\right)$$
$$= \big(1 - F_i^s(\hat{v}^s)\big)\big(1 - F_j^s(\hat{v}^s)\big)\left(\mathbb{E}\big[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\big] - \mathbb{E}\big[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\big]\right) + c_1\left(F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s)\right).$$

Notice that

$$\mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] = \begin{cases} \frac{F_i^s(v)}{1 - F_i^s(\hat{v}^s)} & \text{if } v \geq \hat{v}^s \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for any $v \geq \hat{v}^s$, if F_i^s first-order stochastically dominates F_j^s, then

$$\begin{aligned}
&\mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] - \mathbb{P}[v_j^s \leq v | v_j^s \geq \hat{v}^s] \\
&\qquad = \frac{1}{\big(1 - F_i^s(\hat{v}^s)\big)\big(1 - F_j^s(\hat{v}^s)\big)}\left[F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_j^s(\hat{v}^s)F_i^s(v)\right] \\
&\qquad \leq \frac{1}{\big(1 - F_i^s(\hat{v}^s)\big)\big(1 - F_j^s(\hat{v}^s)\big)}\left[F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_i^s(\hat{v}^s)F_i^s(v)\right] \\
&\qquad = -\frac{F_j^s(v) - F_i^s(v)}{1 - F_j^s(\hat{v}^s)},
\end{aligned}$$

where the inequality follows from $F_j^s(v) \geq F_i^s(v)$. We conclude that the dominance relation is preserved when truncating the distributions of v_i^s and v_j^s at $\hat{v}^s$. This, in turn, implies that if F_i^s first-order stochastically dominates F_j^s, then $\mathbb{E}\big[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\big] \geq \mathbb{E}\big[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\big]$.

Combining both observations, if F_i^s first-order stochastically dominates F_j^s, then consumer welfare increases by more than

$$\frac{n^s - 2}{n^s - 1}\left[\mathbb{P}\big[v_i^s \geq \hat{v}^s\big]\mathbb{E}\big[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s\big] - \mathbb{P}\big[v_j^s \geq \hat{v}^s\big]\mathbb{E}\big[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s\big]\right] > 0$$

if product i replaces j as the default. Hence, a better default improves consumer welfare.

C.4 Consideration Sets

Consider market s, and let $n^s \geq 2$. The consumer considers $k^s \in \{1, 2, \ldots, n^s - 1\}$ offers, and selects one with the highest realized value v_i^s among the considered ones. The consumer considers the default product with certainty. Any non-default product enters the consideration set with probability of $(k^s - 1)/(n^s - 1)$; that is, attention to non-default products is random.

Lemma 6 (Consideration Sets)**.**

I. Assumption 0 holds.

II. Assumption 1 holds if and only if $n^s \geq 3$ *and* $k^s \geq 2$.

Proof. Preliminaries. Consider firm i, and refer to $r_{ii}^s \in [0,1]$ as its (average) "conversion rate" conditional on being in the consideration set and facing $k^s - 1$ random competitors. This is exactly what we call in the main text the firm's demand conditional on being default.

To see how the conversion rate depends on the value that firm i and its rivals offer, we denote as $\mathcal{N}^s(x)$ a subset of $x \geq 1$ firms that are active in market s. We introduce the CDF

$$q_{\mathcal{N}^s(x)}(v) \ := \prod_{\ell \in \mathcal{N}^s(x)} F_\ell^s(v);$$

that is, the probability that the products of all firms in the set $\mathcal{N}^s(x)$ have a realized value less or equal to v. Using this notation, we can write the conversion rate of firm i as

$$r_{ii}^s \ = \sum_{\mathcal{N}^s(k^s-1):\ i \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v). \tag{9}$$

If i is in the consideration set, all $n^s - 1$ rivals are equally likely to be drawn into one of the $k^s - 1$ remaining spots. Hence, there are $\binom{n^s-1}{k^s-1}$ different sets of competitors, each of which occurs with probability $1/\binom{n^s-1}{k^s-1}$. Fixing a set of competitors $\mathcal{N}^s(k^s-1)$, the consumer uses i's service if and only if it offers the highest value; that is, $v_i^s > v_j^s$ for all $j \in \mathcal{N}^s(k^s-1)$. This happens with probability $\int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v)$. Because all distributions F_j^s have common support, we have $r_{ii}^s > 0$. By the same argument, we get $r_{ii}^s < 1$ if and only if $k^s \geq 2$.

Now consider firms i and j with F_i^s first-order stochastically dominating F_j^s. Then,

$$\begin{aligned} r_{ii}^s - r_{jj}^s \quad &\propto \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \right\} \\ &\quad + \sum_{\substack{\mathcal{N}^s(k^s-1):\ i\notin\mathcal{N}^s(k^s-1),\\ j\in\mathcal{N}^s(k^s-1)}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) - \sum_{\substack{\mathcal{N}^s(k^s-1):\ j\notin\mathcal{N}^s(k^s-1),\\ i\in\mathcal{N}^s(k^s-1)}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \\ &\geq \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \right\} \\ &\qquad + \sum_{\substack{\mathcal{N}^s(k^s-1):\ i\notin\mathcal{N}^s(k^s-1),\\ j\in\mathcal{N}^s(k^s-1)}} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \right\} \\ &\geq 0, \end{aligned}$$

where the first inequality holds because $F_j^s(v) \geq F_i^s(v)$ and the second one follows from the fact that $q_{\mathcal{N}^s(k^s-1)}(v)$ is increasing in v together with F_i^s first-order stochastically dominating F_j^s. Moreover, whenever $k^s \geq 2$, the second inequality is strict, which in turn implies $r_{ii}^s > r_{jj}^s$.

Similarly, we refer to $r_{ij}^s \in [0,1]$ as the (average) conversion rate of firm i conditional on being in the consideration set and facing j as well as $k^s - 2$ random competitors. Notice that this conversion rate is *not* the same as the demand of firm i when firm j's product is the default. Using the same notation as above, we can write this conversation rate as

$$r_{ij}^s \quad = \sum_{\substack{\mathcal{N}^s(k^s-1):\ i\notin\mathcal{N}^s(k^s-1),\\ j\in\mathcal{N}^s(k^s-1)}} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v). \tag{10}$$

This implies, in particular, that

$$r_{ij}^s - r_{ij'}^s \quad = \sum_{\mathcal{N}^s(k^s-2):\ i,j,j'\notin\mathcal{N}^s(k^s-2)} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-2)}(v)\left[F_j^s(v) - F_{j'}^s(v)\right]\ dF_i^s(v).$$

Hence, if F_j^s first-order stochastically dominates $F_{j'}^s$ and $k^s \leq n^s - 1$, then $r_{ij}^s < r_{ij'}^s$.

Part I. For any i and $j \neq i$, we have

$$q_{ii}^s - q_{ij}^s = r_{ii}^s - \frac{k^s-1}{n^s-1} r_{ij}^s \quad = \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v). \tag{11}$$

Since all F_ℓ^s have the same support, the above is postive. Hence, Part Assumption 0 holds.

Part II. Suppose F_j^s first-order stochastically dominates $F_{j'}^s$. Then,

$$\begin{aligned} q_{ij'}^s - q_{ij}^s &= \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i,j'\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1),\\ j'\in\mathcal{N}^s(k^s-1)}} \int \left(F_{j'}^s(v) - F_j^s(v)\right) \prod_{\ell\in\mathcal{N}^s(k^s-1)\setminus\{j'\}} F_\ell^s(v)\ dF_i^s(v) > 0, \end{aligned}$$

where the second equality follows from the definition of $q_{\mathcal{N}^s(k^s-1)}(v)$ and the inequality holds (weakly) as F_j^s first-order stochastically dominates $F_{j'}^s$. The inequality is strict since all F_ℓ^s have no mass points and common support. Hence, if $k^s \geq 2$, Part I of Assumption 1 holds.

Next, consider firms, i and i', with F_i^s first-order stochastically dominating $F_{i'}^s$. Then,

$$\begin{aligned} (q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) &= \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i',j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_{i'}^s(v) \\ &\geq \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &\qquad - \sum_{\mathcal{N}^s(k^s-1):\ i',j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1),\\ i'\in\mathcal{N}^s(k^s-1)}} \int \left(F_{i'}^s(v) - F_i^s(v)\right) \prod_{\ell\in\mathcal{N}^s(k^s-1)\setminus\{i'\}} F_\ell^s(v)\ dF_i^s(v) \\ &\geq 0, \end{aligned}$$

where the first inequality follows from $q_{\mathcal{N}^s(k^s-1)}(v)$ being monotonically increasing in v and F_i^s first-order stochastically dominating $F_{i'}^s$ while the second one simply follows from first-order stochastic dominance. Whenever $k^s \geq 2$, the inequalities are strict because the distributions have no mass points and common support. Hence, if $k^s \geq 2$, Part II of Assumption 1 holds.

Finally, let F_i^s first-order stochastically dominate F_j^s, and observe that

$$\begin{aligned}(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) = &\sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &- \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_j^s(v) \\ \geq &\sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ &- \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v)\ dF_i^s(v) \\ = &\ 0,\end{aligned}$$

where the inequality holds since $q_{\mathcal{N}^s(k^s-1)}(v)$ is increasing and F_i^s first-order stochastically dominates F_j^s, and it is strict if $k^s \geq 2$. Hence, if $k^s \geq 2$, Part III of Assumption 1 holds. □

Consumer Welfare Denote as ν_i^s the value of the product *chosen* by the consumer when product i is the default. The value ν_i^s is drawn from the cumulative distribution function

$$\mathbb{P}[\nu_i^s \leq v] = F_i^s(v) \sum_{\mathcal{N}^s(k^s-1):\ i\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).$$

Notice that

$$\mathbb{P}[\nu_i^s \leq v] - \mathbb{P}[\nu_j^s \leq v] = \Big(F_i^s(v) - F_j^s(v)\Big) \sum_{\mathcal{N}^s(k^s-1):\ i,j\notin\mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).$$

Hence, if F_i^s first-order stochastically dominates F_j^s, then ν_i^s first-order stochastically dominates ν_j^s, and consumer welfare is higher with a better default.

C.5 Loss Aversion

Consider market s, and let $n^s \geq 2$. Each product i is defined by a pair (v_i^s, d_i^s) with $v_i^s \sim G^s$ and $d_i^s \in \mathbb{R}_{>0}$. We think of d_i^s as an app-design dimension, which affects the consumer's utility in an additively separable way. Let $F_i^s(u) = G^s(u - d_i^s)$ be the distribution of $v_i^s + d_i^s$. We assume that, if the consumer uses the product of firm j instead of the default product i, she experiences a loss

of λd_i (with $\lambda > 1$) because she cannot enjoy firm i's proprietary design features, and a gain of d_j from using firm j's product instead. In addition, she has some reference point in the service dimension, and we think of G^s as being "gain-loss adjusted."

Lemma 7 (Loss Aversion)**.**

I. Assumption 0 holds.

II. For any $n^s \geq 3$ and any $\{F^s_\ell\}_{\ell \in \mathcal{N}^s}$ that can be ranked in terms of the reversed hazard rate, there exists some $\bar{\lambda} > 1$, such that for any $\lambda \in (1, \bar{\lambda})$, Assumption 1 holds.

Proof. We first show that the model is mathematically equivalent to the switching-cost model in Section C.2. Let product i be the default. Because v^s_i is gain-loss adjusted, and we assume that the reference point in this dimension is independent of a product's default status, the consumer chooses the product of firm i over that of a rival j if and only if

$$v_i > v_j + d_j - \lambda d_i \quad \text{or, equivalently,} \quad v_i + d_i > v_j + d_j - \underbrace{(\lambda - 1) d_i}_{=:\, \gamma_i}.$$

The claim now follows immediately from Lemmas 3 and 4. □

Consumer Welfare As argued above, with gain-loss utility, our model reduces to a (heterogenous) switching-cost model. The counterexample in Figure 1 for large switching costs already indicates that welfare need not improve with a better default. If, in addition, one follows some authors who declare gain-loss utility as irrelevant for welfare, it can indeed be optimal to choose a purposefully bad default in order to induce active choice (see also Goldin and Reck, 2022). To see this most clearly, suppose that there exists a product $w \in \mathcal{N}^s$ with

$$\mathbb{P}[v_w + d_w + \gamma_w < v_j + d_j] \approx 1$$

for all $j \neq w$. Then, if w is the default, the consumer makes an active choice with almost probability 1, and thus almost certainly chooses the welfare-maximizing option. Hence, with loss-aversion-based default effects, it can be theoretically optimal to force choice by setting a bad default.

D Proof of Proposition 3

Since all targets are identical, for any $t_1, t_2 \in \mathcal{N}^b$, we write η^b instead of $\eta^b_{t_1 t_2}$. Similarly, for any $t \in \mathcal{N}^b$, we write α^b and V^b instead of α^b_t and V^b_t. For any round of bidding, we distinguish between a *bidding subgame* in which all remaining acquirers decide what bid to submit, and an *acceptance subgame* in which the target that is currently up for sale decides whether to accept one of the bids. We will subsequently derive the equilibrium takeover price in the following three cases: (a) $n^b < |\mathcal{A}|$, (b) $n^b = |\mathcal{A}|$, and (c) $n^b > |\mathcal{A}|$.

Case (a). Let $n^b < |\mathcal{A}|$. We solve the game backwards and proceed by induction over the number of remaining targets.

Induction hypothesis: *Consider any bidding subgame with $k \geq 1$ remaining targets. Denote as $A(k) \subseteq \mathcal{A}$ the set of potential acquires in this subgame that have not yet taken over a target, and note that $|A(k)| \geq k+1$ because there are more potential acquirers than targets (i.e., $n^b < |\mathcal{A}|$). (In slight abuse of notation, we suppress the dependence of this subgame on the prior history, which determines the identity of potential acquirers. This is without loss of generality because these identities are fixed from this point onward.) Denote by $\ell_i \in A(k)$ the potential acquirer with the ith highest market share in market a among the firms in $A(k)$. Define as $\bar{A}(k,x) := \{\ell_1, \ldots, \ell_x\}$ the set of the x firms with the highest market shares in $A(k)$, and set $\bar{A}(k,0) = \emptyset$. We hypothesize that all firms in $\bar{A}(k,k)$ take over one of the remaining targets at a price*

$$f^*(k) := V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k)} q^a_i \; + \sum_{i \in \bar{A}(k,k-1)} q^a_i \Bigg) + \; q^a_{\ell_{k+1}} \alpha^b. \tag{12}$$

Induction anchor: Consider a bidding subgame with a single target left ($k = 1$). We write $f^* = f^*(1)$. The induction hypothesis states that ℓ_1 *takes over the last target at a price of*

$$f^* = V^b \; - \; \big|\eta^b\big| \sum_{i \in \mathcal{A} \setminus A(1)} q^a_i \; + \; q^a_{\ell_2} \alpha^b. \tag{13}$$

We verify the induction hypothesis in two steps.

1. *Step: equilibrium construction.* We construct a cautious equilibrium in which ℓ_1 takes over

the last target at a price of f^*. Denote the target's adjusted standalone value by

$$\nu := V^b - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus A(1)} q_i^a.$$

and the takeover revenue of a potential acquirer $j \in A(1)$ by

$$\pi_j := \nu + q_j^a \alpha^b.$$

We argue that the target accepting any highest bid at or above ν (breaking ties randomly), ℓ_1 bidding f^*, ℓ_2 submitting a bid by uniformly randomizing over $[f^* - \epsilon, f^*]$, and any other firm $j \neq \ell_1, \ell_2$ bidding ν is a cautious equilibrium of this subgame for small enough ϵ.[41]

We first verify that the above is a subgame perfect equilibrium. If the target rejects the highest bid, it earns ν. Hence, it is optimal to accept any highest bid at or above ν.

Firm ℓ_1 has no incentive to bid more than f^* as then it would take over the target at a strictly higher price, and it does not want to bid less than $f^* - \epsilon$ as then it would lose and earn nothing in market b instead of $\left(q_{\ell_1}^a - q_{\ell_2}^a\right)\alpha^b > 0$. If firm ℓ_1 deviates to $f \in (f^* - \epsilon, f^*)$, it earns at most

$$\frac{f - (f^* - \epsilon)}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right). \tag{14}$$

At $f = f^*$, the profit in (14) equals $\left(q_{\ell_1}^a - q_{\ell_2}^a\right)\alpha^b$; that is, the candidate equilibrium profit. Moreover, the derivative of (14) with respect to f is

$$\frac{1}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right) - \left(1 - \frac{f^* - f}{\epsilon}\right) > \frac{1}{\epsilon}\left(\nu + q_{\ell_1}^a \alpha^b - f\right) - 1 > \frac{\left(q_{\ell_1}^a - q_{\ell_2}^a\right)\alpha^b}{\epsilon} - 1 > 0,$$

which holds for all $f \in (f^* - \epsilon, f^*)$ if ϵ is small enough. Hence, firm ℓ_1 has no incentive to deviate. In fact, for ϵ small enough bidding f^* is firm ℓ_1's strict best reply.

Finally, no firm $j \neq \ell_1$ has an incentive to bid above f^* — a bid the target would accept — as then they would earn negative profits from the takeover, and they are indifferent between bids (weakly) below the takeover price since independently of the size of such a bid they will lose for certain and earn zero profits in b. Hence, no firm $j \neq \ell_1$ has an incentive to deviate.

It remains to be shown that all firms play iteratively weakly undominated strategies in the reduced bidding game in which all acceptance subgames with a unique equilibrium payoff are replaced by the corresponding equilibrium payoffs. This follows from the lemma below.

[41] The equilibrium we construct is inspired by Blume (2003).

Lemma 8 (Iteratively Weakly Undominated Strategies)**.**

I. Bidding ν is iteratively weakly undominated for any potential acquirer.

II. For any potential acquirer $\ell_j \neq \ell_1$, bidding $f \in (\nu, \pi_{\ell_j})$ is iteratively weakly undominated, and for firm ℓ_1, bidding $f \in (\nu, \pi_{\ell_2})$ is iteratively weakly undominated.

III. For firm ℓ_2, any mixed strategy with support in $(\nu, \pi_{\ell_2}) = (\nu, f^*)$ and a continuous CDF is iteratively weakly undominated.

IV. For firm ℓ_1, bidding $\pi_{\ell_2} = f^*$ is iteratively weakly undominated.

Proof. Preliminary. Note that all acceptance subgames following *multiple* highest bids at or above ν do *not* have a unique equilibrium payoff, as the equilibrium payoffs depend on whom the target selects, and are thus identical in the original and the reduced game; that is, the reduced game is still an extensive-form game. In particular, for any j making a highest bid, the target accepting j's bid with probability 1 is an equilibrium of the acceptance subgame.

Part I. We argue that in the reduced bidding game, for any potential acquirer, it is iteratively weakly undominated to bid ν. We do so by induction over the rounds of elimination of weakly dominated strategies. Bidding ν is trivially undominated in the first round of elimination; for example, it is the strict best reply to every rival bidding strictly below ν. Now suppose that ν is weakly undominated for all potential acquirers after k rounds of elimination, and consider round $k+1$. For every potential acquirer j, bidding ν is a strict best response to all its rivals bidding ν and the target accepting j's offer with probability one. Hence, for every potential acquirer, bidding ν is weakly undominated in the $(k+1)$th round of elimination. This proves the claim.

Part II. Again, we proceed by induction, starting with the potential acquirer $\ell_j \neq \ell_1$ that has the lowest demand in a. Suppose that after k rounds of elimination, bidding $f \in (\nu, \pi_{\ell_j})$ is iteratively weakly undominated for all $\ell_{j'}$ with $j' \leq j$, and note this is trivially true after zero rounds of elimination. We now argue that it then is also weakly undominated to bid $f \in (\nu, \pi_{\ell_j})$ after $k+1$ rounds of elimination for all potential acquirers $\ell_{j'}$ with $j' \leq j$. By Part I, it is iteratively weakly undominated to bid ν for all acquirers, and from now on presume that all potential acquirers $\ell_{j''}$ with $j'' > j$ do so. For any potential acquirer $j' \leq j$, bidding f is a strict best response to all rivals with an index weakly below j bidding f and the target accepting j''s offer with probability

one. Hence, for any j', bidding f is weakly undominated in the $(k+1)$th round of elimination.

Part III. Suppose that firm ℓ_1 bids $f' \in [\nu, \pi_{\ell_2})$ and all firms $j \neq \ell_1, \ell_2$ bid ν, which is weakly undominated by Parts I and II. Now consider two (potentially) mixed strategies of firm ℓ_2 with support in (ν, π_{ℓ_2}), and denote the corresponding CDFs as G and H with $G \neq H$. Assume that G is continuous. When bidding according to G, firm ℓ_2's additional profits in market b are

$$\big(1 - G(f')\big)\big(\pi_{\ell_2} - \mathbb{E}_G[f|f \geq f']\big) = \big(1 - G(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \; dG(f).$$

Similarly, when bidding according to H, firm ℓ_2 earns additional profits of

$$\big(1 - H(f')\big)\big(\pi_{\ell_2} - \mathbb{E}_H[f|f \geq f']\big) = \big(1 - H(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \; dH(f).$$

For the sake of a contradiction, suppose that the (potentially) mixed strategy with CDF H weakly dominates the mixed strategy with CDF G. Then, it follows from the last two equations that for any $f' \in [\nu, \pi_{\ell_2})$,

$$\big(G(f') - H(f')\big)\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f \; dH(f) + \int_{f'}^{\pi_{\ell_2}} f \; dG(f) \geq 0. \tag{15}$$

Integration by parts implies that (15) is equivalent to

$$-\big(H(f') - G(f')\big)(\pi_{\ell_2} - f') + \int_{f'}^{\pi_{\ell_2}} H(f) - G(f) \; df \geq 0. \tag{16}$$

We derive a contradiction in three steps. First, evaluating (15) at $f' = \nu$, we conclude

$$\mathbb{E}_H[f] \leq \mathbb{E}_G[f].$$

Hence, H cannot first-order stochastically dominate G, so that $G(f) < H(f)$ for some $f \in [\nu, \pi_{\ell_2})$. Define $\bar{f} := \sup\{\arg\sup \; H(f) - G(f)\}$, and notice that $H(\bar{f}) - G(\bar{f}) > 0$.

Second, because H is right-continuous and G is continuous, also $H - G$ is right-continuous. This in turn implies that the left-hand side of (16) is right-continuous. Hence, if $\bar{f} < \pi_{\ell_2}$,

$$-\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f) \; df \geq 0.$$

Third, to establish a contradiction, we distinguish the following two cases: $\bar{f} < \pi_{\ell_2}$ and $\bar{f} = \pi_{\ell_2}$. If $\bar{f} < \pi_{\ell_2}$, $H(f) - G(f)$ cannot be constant on $(\bar{f}, \pi_{\ell_2})$. Hence, there exists some $\hat{f} \in (\bar{f}, \pi_{\ell_2})$ and

some $\zeta > 0$, such that $\sup_{f \in (\hat{f}, \pi_{\ell_2})} H(f) - G(f) < H(\bar{f}) - G(\bar{f}) - \zeta$. Thus,

$$\begin{aligned}
-\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f)\ df & \\
&< -\big(H(\bar{f}) - G(\bar{f})\big)(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(\bar{f}) - G(\bar{f})\ df - \int_{\hat{f}}^{\pi_{\ell_2}} \zeta\ df \\
&= -\zeta(\pi_{\ell_2} - \hat{f}) \\
&< 0;
\end{aligned}$$

a contradiction. If $\bar{f} = \pi_{\ell_2}$, $H(f)$ jumps at $f = \pi_{\ell_2}$. Since G is continuous, and H is right-continuous and does not first-order stochastically dominate G, there exists $f^s < \bar{f}$, so that $H(f^s) = G(f^s)$ and $H(f) < G(f)$ for $f \in (f^s, \pi_{\ell_2})$. At $f' = f^s$, the left-hand side of (15) is

$$\underbrace{\big(G(f^s) - H(f^s)\big)}_{=0} \pi_{\ell_2} - \int_{f^s}^{\pi_{\ell_2}} f\ dH(f) + \int_{f^s}^{\pi_{\ell_2}} f\ dG(f) = \mathbb{E}_G[f | f \geq f^s] - \mathbb{E}_H[f | f \geq f^s] < 0,$$

where the inequality holds as H FOSD-dominates G on (f^s, π_{ℓ_2}); a contradiction.

Part IV. Suppose that all of ℓ_1's rivals bid according the candidate equilibrium strategies, which are weakly undominated by Parts I and III. Then, as we have argued above, bidding f^* is firm ℓ_1's strict best reply and thus weakly undominated. □

2. Step: unique equilibrium outcome. We show that ℓ_1 takes over the last target at f^* in any cautious equilibrium. First, we argue that in any cautious equilibrium the infimum $\underline{f}$ of the highest equilibrium takeover bid is at least f^*. Suppose not, i.e. $\underline{f} < f^*$. We show that then both firms ℓ_1 and ℓ_2 bid weakly above $\underline{f}$ with probability one. If one of them does not, it must in equilibrium with positive probability submit bids below f^* that loose with probability one. This firm could then deviate and move this probability mass to a bid $(\underline{f} + f^*)/2$ and win with positive probability thereby earning positive expected profits. Furthermore, if the distribution of the highest equilibrium takeover bid has no mass point at $\underline{f}$, both ℓ_1 and ℓ_2 must bid above $\underline{f} + \epsilon'$ for some $\epsilon' > 0$ with probability one; for otherwise, the expected profits as $f \to \underline{f}$ from above approach zero while those of deviating and bidding $(\underline{f} + f^*)/2$ do not; a contradiction. This, however, contradicts that $\underline{f}$ is the infimum of the highest bid. Thus, with positive probability both ℓ_1 and ℓ_2 bid $\underline{f}$, and win when doing so. But since they earn strictly positive profits when winning at this price, either

of them would be better of moving the probability mass from $\underline{f}$ to $\underline{f} + \epsilon''$ for some small enough $\epsilon'' > 0$. We conclude that the infimum of the highest equilibrium bid is at least f^*.

Second, we argue that the supremum of the highest equilibrium bid in a cautious equilibrium is f^*. Bidding at or above f^* is weakly dominated for all firms other than ℓ_1. Hence, firms other than ℓ_1 must bid strictly below f^* with probability one. Thus, firm ℓ_1 wins with probability one when bidding f^*, and hence cannot bid strictly above f^* either. Hence, the highest equilibrium bid must be f^* in any cautious equilibrium with probability one; and it must be submitted by ℓ_1, and thus ℓ_1 wins with probability one in any cautious equilibrium.

Induction step: Suppose that the induction hypothesis holds for any bidding subgame with $k \geq 1$ remaining targets. Consider a bidding subgame with $k+1$ remaining targets, and note that there are at least $k+2$ remaining acquirers. Define $A(k+1)$ and $\bar{A}(k+1, x)$ analogously to the above.

The induction step follows in five steps: in steps one to four, we verify that the induction hypothesis must hold in any cautious equilibrium of a bidding subgame with $k+1$ remaining targets; in step five, we then construct a cautious equilibrium of this bidding subgame.

1. Step: no firm $\ell_j \in A(k+1)$ with $j \geq k+2$ takes over the current target. Recall that we rank acquirers in terms of their market shares in market a, with a lower index j indicating a higher market share. Let ℓ_w be the firm buying target $k+1$, and let $\pi_j(w)$ be firm j's additional subgame perfect equilibrium profit in the bidding subgame with k remaining targets as a function of w.

For the sake of a contradiction, suppose firm ℓ_j with $j \geq k+2$ takes over the current target with positive probability. Then, by the induction hypothesis the remaining targets are acquired the firms in $\bar{A}(k+1, k)$. Hence, ℓ_j's gross profits in market b are

$$\pi_j(j) = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k)} q_i^a \Bigg) + \; \alpha^b q_{\ell_j}^a.$$

Using the induction hypothesis, we construct a reduced game by replacing any continuation subgame following the current acceptance subgame (i.e. following the target's acceptance decision) by its unique equilibrium payoffs, and replacing all possible acceptance subgames in which the target is not indifferent (between accepting and rejecting some offer) by its unique equilibrium payoffs.

If ℓ_j does not take over the current target, it will not make a takeover; thus, its profits from

the takeover market will be zero. If it takes over the current target, its profits will be $\pi_j(j)$. In the reduced game, thus, it is weakly dominated for firm ℓ_j to bid at or above $\pi_j(j)$. Hence, because ℓ_j takes over the current target with positive probability, and because it plays a weakly undominated strategy in any cautious equilibrium, the infimum $\underline{f}$ of the highest bid must be strictly below $\pi_j(j)$.

We next argue that any firm $\ell_{j'} \in \bar{A}(k+1,k)$ makes greater profits when taking over the current target at $\underline{f}$ than when not doing so, irrespective of which rival ℓ_w would take over the current target otherwise. To see why, let $f^* = f^*(k+1)$, and consider three cases. First, if $w \leq k$, by the induction hypothesis, any firm $\ell_{j'} \in \bar{A}(k+1,k)$ takes over a remaining target at a price

$$f^* = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k)} q_i^a \Bigg) + \; \alpha^b q_{\ell_{k+2}}^a \geq \pi_j(j),$$

and following the takeover in market b earns additional gross profits of

$$\pi_{j'}(w) = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k+1) \setminus \{\ell_{j'}\}} q_i^a \Bigg) + \; \alpha^b q_{\ell_{j'}}^a.$$

This implies additional net profits of $\alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > 0$. If instead firm $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} = \pi_{j'}(w) - \underline{f} > \pi_{j'}(w) - \pi_j(j) \geq \pi_{j'}(w) - f^* = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a),$$

where the first equality holds since ℓ_w would take over one of the remaining targets.

Second, if $w = k+1$, then by the induction hypothesis any firm $\ell_{j'} \in A(k+1,k)$ takes over a remaining target at a price

$$f' = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k-1)} q_i^a + q_{\ell_{k+1}}^a \Bigg) + \; \alpha^b q_{\ell_{k+2}}^a > f^*,$$

and earns additional gross profits of $\pi_{j'}(k+1) = \pi_{j'}(j')$. The additional net profits are $\pi_{j'}(j') - f'$. If $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* > \pi_{j'}(j') - f'.$$

Third, if $w \geq k+2$, then by the induction hypothesis a firm $\ell_{j'} \in A(k+1,k)$ takes over a remaining target at a price

$$f'' = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k-1)} q_i^a + q_{\ell_w}^a \Bigg) + \; \alpha^b q_{\ell_{k+1}}^a,$$

and earns additional gross profits of

$$\pi_{j'}(w) = V^b \; - \; |\eta^b| \Bigg(\sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1,k) \setminus \{\ell_{j'}\}} q_i^a + q_{\ell_w}^a \Bigg) + \; \alpha^b q_{\ell_{j'}}^a .$$

and additional net profits of

$$\pi_{j'}(w) - f'' = \alpha^b \big(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a \big) + |\eta^b| \big(q_{\ell_{j'}}^a - q_{\ell_k}^a \big).$$

If $\ell_{j'}$ takes over the current target at $\underline{f}$, by the induction hypothesis, it earns

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* = \alpha^b (q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b| (q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a).$$

Hence, because

$$\alpha^b (q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b| (q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > \alpha^b \big(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a \big) + |\eta^b| \big(q_{\ell_{j'}}^a - q_{\ell_k}^a \big),$$

firm $\ell_{j'}$ strictly prefers winning at $\underline{f}$ also in this last case.

Observe that there are at least two firms in $\bar{A}(k+1,k)$. Since any $\ell_{j'} \in \bar{A}(k+1,k)$ strictly prefers winning to loosing when bidding $\underline{f}$, by a similar argument than in the induction anchor (2. Step), it cannot be the case that two or more firms in $\bar{A}(k+1,k)$ have a mass point at $\underline{f}$. Furthermore, if no rival has a mass point at $\underline{f}$, as the bid of a firm $\ell_{j'}$ approaches $\underline{f}$ from above its profits approach those of losing for certain. In that case, however, by a similar argument than in the induction anchor, there exists an $\epsilon' > 0$ such that firm $\ell_{j'}$ is strictly better of moving any probability mass from below $\underline{f} + \epsilon'$ to $\underline{f} + \epsilon'$. Hence, any firm in $\bar{A}(k+1,k)$ has an incentive to bid strictly above $\underline{f}$ with probability one. In this case, however, firm $\ell_{j'}$ lowest bid is bounded below by $\underline{f} + \epsilon'$, contradicting that $\underline{f}$ is the infimum of the highest bid. Hence, a firm in $A(k+1,k+1)$ takes over the current target.

2. Step: the infimum $\underline{f}$ of the highest equilibrium bid for the current target is weakly above f^*. Suppose otherwise. By bidding in the interval $(\underline{f}, f^*)$, firm ℓ_{k+2} can earn positive profits. Hence, in equilibrium, firm ℓ_{k+2} must win with positive probability, contradicting the first step above.

3. Step: the supremum $\bar{f}$ of the highest equilibrium bid for the target currently on sale is weakly below f^*. Suppose otherwise. First, recall that bidding at or above f^* is weakly dominated for

every firm ℓ_j with $j \geq k+2$. Eliminating bids weakly above f^* for all $j \geq k+2$, we now argue that it is weakly dominated to bid above f^* also for firm ℓ_{k+1}. To see why, notice that if firm ℓ_{k+1} bids above f^* it either wins or looses to a firm $j \leq k$. In either case, by the induction hypothesis, the remaining $k+1$ targets are bought by the firms $\bar{A}(k+1, k+1)$. Furthermore, if ℓ_{k+1} looses, by the induction hypothesis, it pays f^*. Hence, when bidding f^* firm ℓ_{k+1} always earns $\alpha^b(q^a_{\ell_{k+1}} - q^a_{\ell_{k+2}})$, and bidding strictly above f^* is weakly dominated for firm ℓ_{k+1} in the reduced bidding game.

Now consider a firm that attains the supremum; that is, a firm that bids in $(^{(\bar{f}+f^*)}/_2, \bar{f}]$ and wins with positive probability. If the firm moves the probability mass from $(^{(\bar{f}+f^*)}/_2, \bar{f}]$ to $^{(\bar{f}+f^*)}/_2$ instead, it either wins at a lower price or looses to another firm in $\bar{A}(k+1, k)$ and, by the induction hypothesis, takes over another target at f^*. In either case it earns higher profits, and hence has an incentive to deviate; a contradiction.

4. Step: firm ℓ_{k+1} cannot take over the current target with positive probability. Suppose it does. Then, it must bid f^* with positive probability, since $\underline{f} = \bar{f} = f^*$. By the induction hypothesis, if ℓ_{k+1} takes over the current target, in the following subgame the firms in $\bar{A}(k+1, k)$ take over the remaining targets at $f' > f^*$, and hence earn strictly less than when taking over the current target at f^*. But if a firm in $\bar{A}(k+1, k)$ deviates and bids $f^* + \epsilon'$ for $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small ϵ'.

5. Step: equilibrium construction. We sketch a cautious equilibrium of the bidding subgame with $k+1$ remaining targets in which all firms in $\bar{A}(k+1, k+1)$ take over a target at f^*. Using the induction hypothesis, following the current round of bidding a cautious equilibrium with the desired properties exist for any bidding subgame with k remaining targets. We thus specify behavior only in the current round of bidding. By an argument analogous to that in the induction anchor (1. Step and in particular, Lemma 8), the current target accepting any highest bid above

$$V^b \; - \; |\eta^b| \Big(\sum_{i \in \mathcal{A} \setminus A(k)} q^a_i + \sum_{i \in \bar{A}(k,k-1)} q^a_i \Big)$$

with probability one (breaking ties randomly), every firm in $\bar{A}(k+1, k)$ bidding f^*, firm ℓ_{k+1} uniformly randomzing over $(f^* - \epsilon, f^*)$ for small enough $\epsilon > 0$, and any other firm j bidding

$$V^b \; - \; |\eta^b| \Big(\sum_{i \in \mathcal{A} \setminus A(k)} q^a_i + \sum_{i \in \bar{A}(k,k-1)} q^a_i \Big)$$

is a weakly undominated equilibrium of the reduced game in which all subgames with a unique equilibrium outcome are replaced by the corresponding equilibrium payoffs. Hence, it is part of a cautious equilibrium of this subgame.

Finally, we argue that $f^* < V^b$. Let $\underline{q} := \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a$. By definition, for any $i \in \bar{\mathcal{A}}(n^b-1)$, we have $q_i^a > \underline{q}$. Thus, because all targets are symmetric, we have

$$f^* = V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a \; < \; V^b - \underline{q}\Big(|\eta^b|(n^b-1) - \alpha^b\Big).$$

To establish $f^* < V^b$, it suffices to show that $|\eta^b|(n^b-1) = \alpha^b$. Denote by $\overline{Q}^b$ a target's demand when being the default and by $\underline{Q}^b$ a target's demand when not. By Assumption 0, $\overline{Q}^b > \underline{Q}^b$, and by symmetry of the targets,

$$\alpha^b = \frac{n^b - 1}{n^b}(\overline{Q}^b - \underline{Q}^b) \quad \text{and} \quad \eta^b = -\frac{1}{n^b}(\overline{Q}^b - \underline{Q}^b).$$

Hence, $(n^b - 1)|\eta^b| = \alpha^b$.

Case (b). Let $n^b = |\mathcal{A}|$. We solve the game backwards. Consider first any subgame with one potential acquirer j and one target, which implies that all other firms in $\mathcal{A}$ have taken over another target before. Since j makes a take-it-or-leave-it offer, it must offer the amount at which the target is indifferent between accepting and rejecting, and since there is a strictly positive gain of trade the target must accept in equilibrium. Hence, the takeover price is

$$V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{j\}} q_i^a.$$

We proceed by induction over the number of remaining targets, focusing on subgames in which there are as many targets as potential acquirers. (Notice that we can use Case (a) to deal with subgames in which there are more acquirers than targets.)

Induction hypothesis: *Consider any subgame in which there are $k \geq 2$ potential acquirers and k targets left. Let ℓ_k be the potential acquirer with the lowest market share in market a among the k remaining potential acquirers. Then, in any cautious equilibrium of this subgame, a potential acquirer other than ℓ_k takes over the current target at a price of*

$$f^*(k) := V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

Induction anchor: Consider any subgame with two potential acquirers ℓ_1 and ℓ_2 and two targets. Let $q^a_{\ell_1} > q^a_{\ell_2}$. We have to show that ℓ_1 takes over the current target at $f^* = f^*(2)$.

First, we observe that if the current target is not taken over, by Case (a), firm ℓ_1 takes over the remaining target in the next round. Anticipating this takeover, the current target's anticipated standalone value is f^*. Hence, the current target accepts any highest bid strictly above f^*, and rejects any highest bid strictly below f^*; at f^* the current target is indifferent.

Second, we show that the infimum $\underline{f}$ of the highest equilibrium bid is at least f^*. Suppose not. Then, by our first observation, with positive probability, the current target is not taken over. Anticipating that ℓ_1 would take over the last remaining target in the following subgame, ℓ_2 could earn net profits of $q^a_{\ell_2}\alpha^b$ from taking over the current target at f^*. Hence, for small enough $\epsilon' > 0$, shifting probability mass from $(\underline{f}, f^*)$ to $f^* + \epsilon'$ is a profitable deviation for ℓ_2; a contradiction.

Third, we argue that in any cautious equilibrium the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose otherwise. Because the target accepts all bids above f^*, both ℓ_1 and ℓ_2 must attain $\bar{f}$ for otherwise the firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. To establish a contradiction, we observe that for firm ℓ_2 it is weakly dominated to assign positive mass to bids in $((f^*+\bar{f})/2, \bar{f}]$. If ℓ_1 submits a bid weakly below $(f^*+\bar{f})/2$, then firm ℓ_2 is strictly better of bidding $(f^*+\bar{f})/2$ with probability one, as this way it pays a lower price, both when losing and taking over the last target at f^* or when winning at this price. If firm ℓ_1 submits a bid above $(f^*+\bar{f})/2$, firm ℓ_2 is (weakly) better of bidding $(f^*+\bar{f})/2$ with probability one and losing for certain, in which case it takes over the remaining target at f^* in the following subgame. Hence, assigning positive mass to bids in $((f^*+\bar{f})/2, \bar{f}]$ is weakly dominated for ℓ_2 in the reduced bidding game; a contradiction.

Fourth, we show that firm ℓ_1 takes over the current target with probability 1. Note that firm ℓ_2 cannot take over the current target with positive probability. Suppose it does. Then, it must bid f^* with positive probability. If ℓ_2 takes over the current target, in the following subgame ℓ_1 takes over the remaining target at a price of $f' > f^*$, and hence earns strictly less than when taking over the current target at f^*. But if ℓ_1 deviates and bids $f^* + \epsilon'$ for any $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small $\epsilon' > 0$. Similarly, the target must accept a bid of f^* by firm ℓ_1 with probability 1, for otherwise ℓ_1 could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. Since if ℓ_2 takes over the current target ℓ_1 pays a strictly higher price in the next round, similar arguments as in Case (a) imply that firm ℓ_1 bidding f^* and firm ℓ_2 uniformly randomizing over $(f^* - \epsilon, f^*)$ is part of a cautious equilibrium of this subgame for small enough $\epsilon > 0$.

Induction step: Suppose the induction hypothesis holds for any subgame with $k \geq 2$ remaining targets and acquirers. Consider a subgame with $k+1$ remaining targets and acquirers. Denote $f^* = f^*(k+1)$. We follow the same five steps as in the induction anchor.

First, we observe that if the current target is not taken over, by Case (a), all firms except for ℓ_{k+1} take over one of the remaining targets in the following rounds. Anticipating this, the current target's anticipated standalone value is f^*. The current target thus accepts any highest bid strictly above f^*, and rejects any highest bid strictly below f^*; at f^* the current target is indifferent.

Second, we show that the infimum $\underline{f}$ of the highest equilibrium bid is at least f^*. Suppose not. Then, with positive probability, the current target is not taken over. Anticipating that it would not make a takeover otherwise, firm ℓ_{k+1} could earn net profits of $q^a_{\ell_{k+1}}\alpha^b$ from taking over the current target at f^*. Hence, for small enough $\epsilon' > 0$ shifting probability mass from $(\underline{f}, f^*)$ to $f^* + \epsilon'$ is a profitable deviation for ℓ_{k+1}; a contradiction.

Third, we argue that in any cautious equilibrium the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose not. Because the target accepts all bids above f^*, at least two firms must attain $\bar{f}$ for otherwise the only firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. Notice that, by the same argument as in the induction anchor, assigning positive probability mass to bids above $(f^*+\bar{f})/2$ is weakly dominated for ℓ_{k+1}, so it cannot be ℓ_{k+1} that attains $\bar{f}$. Now consider two firms ℓ_i and ℓ_j with $i, j \neq k+1$ that attain $\bar{f}$. Both firms have an incentive to shift probability mass from $((f^*+\bar{f})/2, \bar{f}]$ to $(f^*+\bar{f})/2$ because they either win at a lower price or — by the induction hypothesis — take over one the remaining targets at f^* in the following rounds; a contradiction.

Fourth, we show that the current target is taken over with probability 1 by a firm other than ℓ_{k+1}. Note that ℓ_{k+1} cannot take over the current target with positive probability. Suppose it does. Then, it must bid f^* with positive probability. If ℓ_{k+1} takes over the current target, in the following

subgame the remaining acquirers take over the remaining targets at a price of $f' > f^*$, and hence earn strictly less than when taking over the current one at f^*. If either of these firms deviates and bids $f^* + \epsilon'$ for any $\epsilon' > 0$, it wins for certain, which is a profitable deviation for sufficiently small $\epsilon' > 0$; a contradiction. Similarly, the target must accept an equilibrium bid of f^* with probability 1, for otherwise a firm other than ℓ_{k+1} could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. By similar arguments as in Case (a), firm $\ell_1, \ldots, \ell_k$ bidding f^* and firm ℓ_{k+1} uniformly randomizing over $(f^* - \epsilon, f^*)$ is part of a cautious equilibrium of this subgame for small enough $\epsilon > 0$.

Case (c). Let $n^b > |\mathcal{A}|$. We solve the game backwards, and begin by solving subgames with one more acquirer than targets. (Other relevant subgames are captured by (a) and (b).)

Induction hypothesis: *Consider a subgame with $k \geq 1$ remaining potential acquirers and $k+1$ remaining targets. The current target is taken over at a price of*

$$f^* := V^b \; - \left|\eta^b\right| \sum_{i \in \mathcal{A}} q_i^a.$$

Induction anchor: Let $k = 1$, so that there are one acquirer ℓ and two targets left. If the current target is not taken over, by Case (b), ℓ takes over the last remaining target at a price

$$f' = V^b \; - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus \{\ell\}} q_i^a.$$

The current target's profits in that case are f^*. Thus the target must reject any bid strictly less than f^*, and accept any bid strictly above f^*. Hence, firm ℓ has a strict incentive to lower any bid strictly above f^*. For the sake of a contradiction, suppose the target rejects f^*. Then, ℓ pays $f' > f^*$ for the takeover, and thus has an incentive to deviate by bidding $(f^*+f')/2$, a bid that the current target accepts. Hence, ℓ takes over the current target at f^*.

Induction step: Suppose that the induction hypothesis holds for k remaining acquirers and remaining $k+1$ targets. Now consider a subgame with $k+1$ remaining acquirers and $k+2$ targets. Let ℓ_k be the remaining acquirer with the lowest market share in market a.

First, if the current target is not taken over, by Case (b), all other targets are bought at

$$f'' = V^b \; - \left|\eta^b\right| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

The current target's profits in that case are f^*. Thus, it must reject any bid strictly less than f^*, and accept any bid strictly above f^*; at f^* the current target is indifferent.

Second, we argue that the supremum $\bar{f}$ of the highest equilibrium bid is at most f^*. Suppose not. Since the target accepts all bids above f^*, at least two firms must attain $\bar{f}$ for otherwise the only firm attaining $\bar{f}$ would have an incentive to lower $\bar{f}$. Either firm has an incentive to shift probability mass from $((f^*+\bar{f})/2, \bar{f}]$ to $(f^*+\bar{f})/2$ since they either win at a lower price or by the induction hypothesis, take over a remaining target at f^* in the following rounds; a contradiction.

Third, we show that the target accepts a bid of f^*. Suppose otherwise. Then, by Case (b), all potential acquirers take over one of the remaining targets at a price f''. Hence, for small enough $\epsilon' > 0$, by bidding $f^* + \epsilon'$ for the current target, each potential acquirer could make a takeover at a lower price; a contradiction.

Fourth, by an argument similar to Lemma 8, bidding f^* is weakly undominated for every potential acquirer. Hence, potential acquirers bidding f^* is part of a cautious equilibrium.

We finally argue that any acquirer takes over a target at f^* in any subgame with $k \geq 1$ potential acquirers and $k + x$, for $x \geq 2$, targets left. First, we note that no target would sell strictly below f^* since even if all acquirers buy a rival it earns f^* upon rejecting an offer. Hence, no target can be bought at a price strictly below f^*. Second, no acquirer would bid more than f^* since such an acquirer could deviate and not bid until a subgame is reached in which there are k' acquirers and $k' + 1$ targets left. In such a subgame, it can take over a target at f^*. Hence, in equilibrium an acquirer takes over a target at f^*. □