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Bertrand Competition in Markets with Network Effects and Switching Costs*

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Abstract

We analyze Bertrand duopoly competition in markets with network effects and consumer switching costs. Depending on the ratio of switching costs to network effects, our model generates four different market patterns: monopolization and market sharing which can be either monotone or alternating. A critical mass effect, where one firm becomes the monopolist for sure only occurs for intermediate values of the ratio, whereas for large switching costs market sharing is the unique equilibrium. For large network effects both monopoly and market sharing equilibria exist. Our welfare analysis reveals a fundamental conflict between maximization of consumer surplus and social welfare when network effects are large. We also analyze firms' incentives for compatibility and we examine how market outcomes are affected by the switching costs, market expansion, and cost asymmetries. Finally, in a dynamic extension of our model, we show how competition depends on agents' discount factors.

JEL Classification: L13, D43, L41

Keywords: Network Effects, Switching Costs, Bertrand Competition

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1 Introduction

Competition in many parts of modern economies, and in particular, in so-called high tech industries is increasingly characterized by technologies which give rise to pronounced network effects and by switching costs consumers have to forego when they change the technology (for recent surveys, see Klemperer, 2005, and Farrell and Klemperer, 2007).[1] Technologies are typically either completely or at least partially incompatible.[2] Though products may be differentiated as usual, its importance for consumers' purchasing decisions is often negligible when compared with their preference for *compatible* products.[3] Both switching costs and network effects have attracted concerns in competition policy circles about the effectiveness of competition (see, e.g., FTC, 1996, and OECD, 1997).[4] While switching costs have been alleged to ease the competitive pressure among firms, network effects have raised concerns that persistent monopolies are inevitable. Both market forces have been studied intensively, though virtually the entire literature focused on one of the two forces exclusively (we present the relevant literature below).

We observe strikingly different market outcomes when incompatible technologies compete against each other and *both* network effects and switching costs are essential features of the market. In many instances, competition between technologies leads to a persistent monopoly outcome where one technology becomes the de facto standard. In other instances, market sharing outcomes prevail such that incompatible standards compete head-to-head. Moreover, markets with network effects often exhibit the so-called "critical mass" effect such that a firm which

[1] The competitive forces in markets with network effects and switching costs have been described in an increasing number of business and market studies; see, for instance, Grindley (1995), Shapiro and Varian (1998), Rohlfs (2001), and Gawer and Cusumano (2002).

[2] Incompatibilities are the norm when firms start to market new products and technologies are protected by business secrets and/or property rights (patents or copyrights).

[3] Not surprisingly, there are numerous stories about alleged "market failures" when consumers have a desire for compatibility. To mention some examples, the QWERTY keyboard standard, Microsoft's operating system MS DOS, or the videocassette recorder standard VHS have all been proscribed as inferior to their losing rivals, namely, Dvorak (see David, 1985, and Liebowitz and Margolis, 1990, for an opposing view), Apple (see, e.g., Shapiro and Varian, 1998), and Beta (see Cusumano, Mylonadis, and Rosenbloom, 1992), respectively.

[4] Policy implications are also discussed in the surveys of Klemperer (1995), Gandal (2002), and Farrell and Klemperer (2007).

reaches the critical mass first completely monopolizes the market thereafter.[5] The market for compact disks and CD players provides an example where the standard introduced by Phillips and Sony in 1983 rapidly became the de facto standard in the industry. Monopoly was also the outcome in the VCR standards battle between VHS sponsored by JVC and Beta sponsored by Sony (see, Cusumano, Mylonadis, and Rosenbloom, 1992). A market sharing outcome between different standards is documented in Augereau, Greenstein, and Rysman (2006) who studied the adoption of 56K modems by internet service providers in the US in the late nineties. The coexistence of different standards in wireless telephone networks (namely, CDMA, TDMA and GSM) in the US (see Gandal and Salant, 2003) is also an example of a market sharing outcome.

Another feature of markets with network effects and switching costs is related to asymmetries in firms' market shares and the possibility that dominance may alternate in a market sharing equilibrium.[6] With respect to the first property we distinguish between monopolization and market sharing patterns such that firms' market shares become more (less) asymmetric in the former (latter) case. Both patterns can be either monotone (if dominance does not alternate) or alternating. Market dominance alternated in the early years of the famous rivalry between Apple's and Microsoft's operating systems. Another example illustrative for alternating dominance is competition between AM and FM standards in radio broadcasting (Besen, 1992). More recently, Toshiba decided to pull out of the HD DVD business so that the rival format Blu-ray sponsored by Sony is expected to dominate that market.[7] Toshiba held a larger installed base than Sony at the time of announcing its withdrawal. The associated market pattern mirrors an alternating monopolization outcome. The market for videogame consoles is currently shared between three major producers (Nintendo, Sony, and more recently, Microsoft). Dominance has alternated in the videogame industry. Nintendo held a dominant position in the eighties and nineties, then lost its dominance while, most recently, it appears to have strengthened its market position relative to its rivals.[8]

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[5] See Rohlfs (1974) and Shapiro and Varian (1998) for the role of the critical mass in markets with network effects.

[6] Incidentally, network markets have been described as "unpredictable" (see Arthur, 1989).

[7] See "Toshiba is Set to Cede DVD-format Fight," Wall Street Journal Europe, February 18, 2008, p. 3.

[8] See "Wii and DS Turn Also-Run Nintendo Into Winner in Videogame Business," Wall Street Journal online, April 19, 2007 (http://online.wsj.com).

In this paper we develop a model of duopolistic competition to analyze how the interplay between network effects and switching costs shapes competitive outcomes. Firms' products are incompatible and each technology gives rise to proprietary network effects which are linearly increasing in the number of buyers. Initially, each firm has an installed base of consumers. Consumers have to bear switching costs if they switch the technology. Switching costs increase symmetrically and linearly over the set of consumers of each technology. Firms compete in prices under given consumer expectations about firms' market shares and we solve for fulfilled expectations Bertrand Nash equilibria.

We find that market outcomes critically depend on two elements: *first*, firms' installed bases and, *second*, a single parameter which measures the relative importance of switching costs compared to the intensity of network effects. For the considered parameter space we obtain the described above market outcomes and the four possible patterns in the market sharing equilibrium. When switching costs are large relative to network effects, then a unique (market sharing) equilibrium exists, while in the opposite case (i.e., network effects are large relative to switching costs) multiple equilibria prevail. In both cases market shares become more balanced in the market sharing equilibrium and follow either a monotone pattern (for large switching costs) or an alternating pattern (for large network effects).

Our main contribution is the analysis of an intermediate range of parameters where network effects and switching costs are balanced. In that region market outcomes critically depend on the size of firms' installed bases. There exists a region where a critical mass effect occurs, such that the initially dominant firm becomes the monopolist for sure (i.e., as a result of a unique equilibrium outcome). Moreover, market patterns are markedly different from the previous cases. If a market sharing equilibrium exists, then it is always given by a monopolization pattern, which can be either monotone or alternating. We conclude that the asymmetry in firms' market shares in the market sharing equilibrium is amplified only when network effects and switching costs are balanced. Both monotone and alternating monopolization patterns are absent when either network effects or switching costs dominate each other. Our analysis reveals that the interplay between switching costs and network effects gives rise to new results, absent in the previous works that focused on either one of both market forces (see literature review below).

We also provide a stability analysis of the identified equilibria and show that when network effects increase, equilibrium outcomes become less stable and more dependent on consumer expectations, while the role of the installed bases vanishes. We also analyze how the type of equilibrium (market sharing or monopoly) affects consumer surplus and social welfare, where we show that a fundamental conflict arises between both welfare goals. While positive network effects require consumers to coordinate on a single technology, consumer surplus is generally higher when both firms compete head-to-head.

We consider several extensions of our basic market model. We analyze firms' preferences for making their products compatible and examine firms' incentives to increase switching costs. We also consider the cases of market expansion and asymmetric costs. Finally, we examine a two-period extension where consumers bear switching costs in the second period only and can freely choose between the products in the first period. In our analysis both firms and consumers are forward-looking and maximize the discounted sum of their payoffs in the two periods.

Our paper contributes to the literature that deals with imperfect competition in markets with network effects and switching costs. There is a large literature on both market forces. Besides few exceptions (e.g., Farrell and Shapiro, 1988), the literature has been focusing either on network effects or switching costs exclusively.[9] With regard to network effects, our paper builds on the seminal paper by Katz and Shapiro (1985) which incorporates network effects into the Cournot oligopoly model. We adopt their concept of a fulfilled expectations Nash equilibrium to our model of Bertrand competition. Katz and Shapiro (1985) obtain multiple equilibria (symmetric and asymmetric) for the case of incompatible products. We get qualitatively similar results, whenever network effects dominate switching costs. However, we also consider installed base effects (which are absent in Katz and Shapiro, 1985), which are crucial for the analysis of market outcomes when network effects and switching costs are more balanced.

The dynamics of markets with network effects has attracted a lot of attention in the literature. Those works focused on markets where consumers enter sequentially and make irreversible adoption decisions. Intertemporal network effects and consumer lock-in typically lead to a monopolization outcome and several dynamic inefficiencies; most notably, excess inertia and excess

[9] As we focus in our literature review on those contributions most closely related to our model we do not touch on important related issues, as, e.g., price discrimination or price commitments that are not part of our analysis (again, we refer to the survey by Farrell and Klemperer, 2007).

momentum (see, Farrell and Saloner, 1986, Katz and Shapiro, 1986, and Arthur, 1989). The dynamics are mainly driven by asymmetries between technologies (in particular, in the form of product differentiation, technological progress, and different times of arrival in the market place). In contrast, in our basic model firms' products are inherently symmetric (i.e., in terms of their network-independent utilities, production costs, and arrival dates), but may differ with respect to their installed bases. Moreover, Farrell and Saloner (1986) as well as Arthur (1989) only analyze consumers' adoption decisions while product supply is perfectly competitive. Duopolistic price competition in a two-period model where different consumer cohorts enter sequentially and intertemporal network externalities occur, has been analyzed in Katz and Shapiro (1986). That model assumes perfect consumer lock-in, so that switching incentives are not analyzed.

Mitchell and Skrzypacz (2006) consider a dynamic duopoly with network effects. When products are not vertically differentiated, there is a Markov-perfect equilibrium where firms' market shares converge to equal market shares if network effects are sufficiently low giving rise to a monotone market sharing pattern. For larger network effects numerical calculations yield a monotone monopolization pattern. While Mitchell and Skrzypacz (2006) analyze only the case where network effects are not too large, we provide analytical solutions for the entire parameter range.

Klemperer (1987a/b) are seminal contributions to the switching costs literature that examine (besides many other things) the "bargains-then-ripoffs" incentives in a two-period market environment with consumer switching costs. Switching costs tend to reduce competition, and thereby, may also benefit firms to the expense of consumers. In a dynamic setting with a cohort of new consumers entering the market in every period a "fat-cat" effect results from switching costs, which gives rise to a monotone market sharing pattern as shown in Beggs and Klemperer (1992). To (1996) analyzes a similar model where consumers live for just two periods. He shows the existence of a unique Markov-perfect equilibrium with the alternating market sharing pattern; a result similar to the one obtained in Farrell and Saloner (1988). The fat cat effect has also been analyzed in Farrell and Shapiro (1988), where it is also shown that the result is robust vis-à-vis (not too large) network effects. Their model gives rise to a rather extreme pattern where the entering cohort of consumers always buys from the entrant firm.[10] While in the cited

[10] As we will show below, such an extreme alternating pattern (where firms interchange market shares) is also

literature consumers are locked-in in equilibrium, in our model there is switching.[11]

Our paper proceeds as follows. In Section 2 we present the model and in Section 3 we derive and characterize the equilibria. In Section 4 we provide welfare results. In Section 5 we consider extensions of our basic model. Finally, Section 6 concludes.

2 The Model

We consider two firms, $i = A, B$, that produce incompatible products, A and B, respectively. We normalize production costs to zero. Firms compete in prices, p_i ($i = A, B$), which they determine simultaneously. Given p_A and p_B, consumers make their purchasing decisions. All consumers have the same valuation of the stand-alone value of the products, $v \geq 0$, which we assume to be sufficiently high such that the market is always covered. The consumption of a product creates positive network effects for users of the same product. We suppose that consumer utility is linearly increasing in network size with coefficient $b > 0$.

We assume a continuum of consumers with a mass of one. We suppose that at the beginning of the period each consumer belongs to the installed base of either firm A or B.[12] Hence, before price competition occurs, each firm already holds an exogenously given market share, $\alpha_i^0 \in [0, 1]$. As we assume that the market is always covered, market shares must add up to unity; i.e., $\alpha_A^0 + \alpha_B^0 = 1$. While at the beginning of the period each consumer belongs to either of the installed bases of the firms, he can switch to the other firm's product. However, switching is costly, whereas buying the prior technology again does not create similar costs.[13]

We build on the well-known Hotelling model of product differentiation to account for switching costs. Consumers are uniformly distributed on the unit interval such that each consumer

an equilibrium outcome in our model which occurs for a particular parameter constellation.

[11] A notable exception is Caminal and Matutes' (1990) analysis of loyalty discounts.

[12] Overall, uncertainty in markets for network goods is large and small events (David, 1985, and Arthur, 1989) may induce consumers to decide for one of the products without foreseeing the implications entirely. An exogenous installed base may also be the result of several promotional activities (e.g., targeted sales or free test products) of the firms. Below we consider a two-period extension with endogenous installed bases.

[13] There are many reasons for consumer switching costs as, for example, technology-specific learning effects or sunk investments into complementary equipment which is incompatible with other brands (see Klemperer, 1995, for a comprehensive list of the many sources of consumer switching costs).

obtains an address $x \in [0,1]$. Both firms are located at the ends of the Hotelling line; firm A at $x_A = 0$ and firm B at $x_B = 1$. All consumers with addresses $x < \alpha_A^0$ belong to the installed base of firm A and all remaining consumers (with $x \geq \alpha_A^0$) are part of the installed base of firm B.

A consumer located at $x \geq \alpha_A^0$ ($x < \alpha_A^0$) who buys product A (B) incurs switching costs tx ($t(1-x)$) which are linearly increasing in the distance between the consumer's address and the location of the product. If a consumer does not switch and buys the product of his installed base, then no such costs arise.[14] We further specify that the costs of switching from product j to product i ($j \neq i$, $j = A, B$) are linearly decreasing with slope t in product i's installed base, α_i^0. We can explain that relationship by learning effects (e.g., how to use a software) which become more pronounced when the number of experienced users (who form the installed base) increases.[15] The total costs of switching for a consumer x who belongs to the installed base of product A (B) and buys product B (A) are, therefore, given by the expression $t|\alpha_A^0 - x|$.

Our approach implies two convenient properties: *Firstly*, there is always a consumer with zero switching costs (which avoids discontinuities), and *secondly,* switching costs increase symmetrically and linearly over both installed bases.[16] We denote firms' market shares at the end

[14] That is, we use the Hotelling set-up to specify the level of switching costs of a single consumer. If we abstract from network effects and switching costs, then both products are perfectly substitutable.

[15] See also Henkel and Block (2006) for peer-effects which help new consumers to join a network. Another advantage of a larger installed base may originate from past purchases of the good which increase total (direct or indirect) network effects "today" (see, for instance, Farrell and Saloner, 1986, and Mitchell and Skrzypacz, 2006). As in our setting all consumers of the installed base are "active" in the period under consideration, we incorporate the competitive advantage associated with a larger installed base via its impact on consumer switching costs.

[16] See Klemperer (1987a) for a discussion of different specifications of consumer switching costs. There are, of course, different functional specifications of switching costs depending on a consumer's address (determining individual gross switching costs) and a product's installed base conceivable. For example, a more general approach would be to assume switching costs of the form $t_1 \cdot x - t_2 \cdot \alpha_A^0$ in case of switching from B to A. We assumed $t_1 = t_2 = t$ which guarantees that consumer utilities are continuous in x. Our results remain largely valid for $t_1 \geq t_2$ but may change if $t_1 < t_2$ (i.e., for a relatively large installed base effect). In the latter case, switching is excessively attractive (a consumer's utility may increase with switching) such that switching in both directions can occur. Assuming $t_1 < t_2$ is, however, not sensible as this implies "switching benefits" for some consumers.

of the period by α_i^1. The utility of consumer x from buying product i can then be written as[17]

$$U_x^i = \begin{cases} v + b\alpha_i^1 - p_i & \text{if} \quad x \in \alpha_i^0 \\ v + b\alpha_i^1 - p_i - t\left|\alpha_A^0 - x\right| & \text{if} \quad x \in \alpha_j^0, \end{cases} \tag{1}$$

for $i, j = A, B$ and $i \neq j$. Thus the utility of a consumer who is loyal and stays with product i, is the sum of the stand-alone value of the product, v, and the network utility, $b\alpha_i^1$, minus the product price, p_i, while a consumer x who switches technologies has to bear additional switching costs, $t\left|\alpha_A^0 - x\right|$. Firm i's new market share at the end of the period, α_i^1, may differ from its installed base, α_i^0, if consumers switch.

For our analysis it is convenient to define the ratio of switching costs to network effects by $k := t/b$, with $k \in (0, \infty)$. Parameter k measures how important network effects are relative to switching costs. For relatively small k, network effects (switching costs) are more (less) important than switching costs (network effects), whereas for relatively large k, the opposite holds.

The timing of the market game is as follows: In the first stage, consumers form expectations about firms' market shares which we denote by α_i^e, for $i = A, B$. In the second stage, firms set prices, p_i, simultaneously so as to maximize their profits. Then, consumers observe firms' prices and make their purchasing decisions, which yield firms' new market shares, $\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0)$. We solve the game for fulfilled expectations Bertrand equilibria (which we define below).

3 Equilibrium Analysis and Main Results

We first derive the demand function. For given expectations and prices every consumer chooses the product which provides him the highest utility. We assume v to be sufficiently large, so that the market is always covered in equilibrium.[18] Setting $U_x^A = U_x^B$ and solving for the marginal consumer who is indifferent between the products of the two firms, yields

$$\alpha_A^1(p_A, p_B, \alpha_A^e; \alpha_A^0) = \min\{\max\{0, \alpha_A^0 + [p_B - p_A + b(2\alpha_A^e - 1)]/t\}, 1\}.$$

[17] With some abuse of notation let α_i^0 also denote the set of consumers on the unit interval which forms the installed base of firm i ($i = A, B$); i.e., $\alpha_A^0 = \{x | 0 \leq x \leq \alpha_A^0\}$ and $\alpha_B^0 = \{x | \alpha_A^0 \leq x \leq 1\}$.

[18] We state the condition for market coverage below.

We can now express the demands for firms' products for given expectations, prices, and installed bases as

$$\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0) =$$
$$\begin{cases} 0 & \text{if} \quad p_j - p_i \leq -t\alpha_i^0 - b(2\alpha_i^e - 1) \\ \alpha_i^0 + \frac{p_j - p_i + b(2\alpha_i^e - 1)}{t} & \text{if} \quad -t\alpha_i^0 - b(2\alpha_i^e - 1) < p_j - p_i < t(1-\alpha_i^0) - b(2\alpha_i^e - 1) \\ 1 & \text{if} \quad p_j - p_i \geq t(1-\alpha_i^0) - b(2\alpha_i^e - 1), \end{cases} \tag{2}$$

with $i, j = A, B$ and $i \neq j$.

We solve for fulfilled expectations Bertrand equilibria in which every firm i sets its price given the price of the competitor and consumer expectations about future market shares to maximize its profit, $\pi_i(p_i, p_j, \alpha_i^e; \alpha_i^0) := \alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0)p_i$. We next define the fulfilled expectations Bertrand equilibrium.[19]

Definition 1. *The fulfilled expectations Bertrand equilibrium is a vector of prices and market shares* $(p_A^*, p_B^*, \alpha_A^*, \alpha_B^*)$*, such that each price,* p_i^**, maximizes firm* i*'s profit given consumer expectations,* α_i^**, and the price of the competitor,* p_j^* $(i, j = A, B,\ i \neq j)$*:*

$$p_i^* = \arg\max_{p_i \geq 0} \ \pi_i(p_i, p_j^*, \alpha_i^*; \alpha_i^0).$$

Moreover, consumer expectations are fulfilled:

$$\alpha_i^* = \alpha_i^1(p_i^*, p_j^*, \alpha_i^*; \alpha_i^0).$$

Two types of equilibria are possible: *First*, an interior equilibrium in which both firms serve the market, and *second*, corner solutions where one firm monopolizes the market. We refer to the former equilibrium as the "market sharing equilibrium" and to the latter equilibrium as the "monopoly equilibrium". We start with the analysis of the market sharing equilibrium.

[19] The concept of a fulfilled expectations equilibrium is borrowed from Katz and Shapiro (1985) with the only difference that in our case firms compete in prices and not in quantities. Another approach is to assume that expectations are formed *after* firms set prices. Both approaches generate equilibrium patterns which are qualitatively the same (see also Suleymanova and Wey, 2010 and Grilo, Shy, and Thisse, 2001). The formal analysis of the latter approach is available from the authors on request.

Market sharing equilibrium. In an interior equilibrium firms' first order conditions must be fulfilled for market shares that lie within the unit interval and nonnegative prices. According to (2) the demand for firm i in an interior equilibrium is given by

$$\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0) = \alpha_i^0 + \frac{p_j - p_i + b(2\alpha_i^e - 1)}{t} \text{ for } i = A, B \text{ and } i \neq j. \tag{3}$$

Maximizing $\pi_i(p_i, p_j, \alpha_i^e; \alpha_i^0)$ with respect to p_i we obtain firm i's first order condition

$$\alpha_i^1 - p_i/t = 0, \tag{4}$$

and, hence, its best response function

$$p_i(p_j, \alpha_i^e; \alpha_i^0) = \frac{t\alpha_i^0 + b(2\alpha_i^e - 1) + p_j}{2} \text{ for } i = A, B \text{ and } i \neq j. \tag{5}$$

Solving firms' best response functions and substituting $\alpha_j = 1 - \alpha_i$ ($j \neq i$), yields firms' profit maximizing prices

$$p_i(\alpha_i^e; \alpha_i^0) = \frac{t(\alpha_i^0 + 1) + b(2\alpha_i^e - 1)}{3} \text{ for } i = A, B \text{ and } i \neq j. \tag{6}$$

Substituting (6) for $i, j = A, B$ into Condition (4) and using $k = t/b$ gives the reduced demand functions

$$\alpha_i^1(\alpha_i^e; \alpha_i^0, k) = \frac{k(\alpha_i^0 + 1) + 2\alpha_i^e - 1}{3k} \text{ for } i = A, B \text{ and } i \neq j. \tag{7}$$

In a fulfilled expectations equilibrium it must hold that consumer expectations about market shares are fulfilled; i.e., we require that $\alpha_i^I(\alpha_i^e; \alpha_i^0, k) = \alpha_i^e$ (the index "I" stands for the *interior* equilibrium) holds for $i = A, B$. Applying this condition to Equation (7) yields the equilibrium market share of firm i in the market sharing outcome

$$\alpha_i^I(\alpha_i^0, k) = \frac{k(1 + \alpha_i^0) - 1}{3k - 2} \text{ for } i = A, B. \tag{8}$$

Equation (8) shows that firms' equilibrium market shares only depend on their initial market shares and the parameter k. Existence of the market sharing equilibrium is guaranteed if and only if

$$0 < \alpha_i^I(\alpha_i^0, k) < 1 \tag{9}$$

holds. We are now in a position to state the following lemma.[20,21]

[20] To proceed in a parsimonious way, we rule out $k = 2/3$, where the function $\alpha_i^I(\alpha_i^0, k)$ is not defined. At that point an interior equilibrium exists only for $\alpha_i^0 = 1/2$ (with any $\alpha_i^1 \in (0, 1)$ being an interior equilibrium). Of course, in the following we also consider only the relevant parameter space with $k > 0$ and $\alpha_i^0 \in [0, 1]$, for $i = A, B$.

[21] All proofs are provided in the Appendix.

Lemma 1. *A unique market sharing equilibrium exists, where firms' market shares and prices are given by* $\alpha_i^I(\alpha_i^0, k) = \left[k(1+\alpha_i^0) - 1\right]/(3k-2)$ *and* $p_i^I = t\alpha_i^I$*, respectively, if and only if either* $\alpha_i^0 \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$ *or* $\alpha_i^0 \in (1-\overline{\alpha}^0(k), \overline{\alpha}^0(k))$ *holds* $(i = A, B)$*, with* $\overline{\alpha}^0(k) := 2 - 1/k$*. Moreover,* $\partial\overline{\alpha}^0/\partial k > 0$*,* $\lim_{k\to(2/3)} \overline{\alpha}^0(k) = 1/2$*,* $\overline{\alpha}^0(1) = 1$*, and* $\overline{\alpha}^0(1/2) = 0$*.*

Monopoly equilibrium. In a monopoly equilibrium where one firm gains the entire market (say firm A), it must hold that $\alpha_A^e = \alpha_A^M = 1$ (the index "M" stands for the *monopoly* equilibrium). Clearly, the price of firm A, p_A, then follows from setting $U_1^A = U_1^B$, such that the marginal consumer is located at the other end of the unit interval; i.e., at the point $x = 1$. Otherwise, if $U_1^A > U_1^B$, then firm A could increase its profit by increasing its price and if $U_1^A < U_1^B$, then firm A would not gain the entire market with $\alpha_A^M = 1$. The rival firm B can not do better than setting $p_B = 0$, because for positive prices $p_B > 0$ firm B may increase its profit by lowering its price. Equating U_x^A and U_x^B either at $x = 0$ or $x = 1$ yields the price of firm i $(i = A, B)$ in the monopoly equilibrium

$$p_i^M(\alpha_i^0) = b - t(1-\alpha_i^0), \tag{10}$$

when firm i becomes the monopolist and firm j $(j \neq i)$ is driven off the market. The price $p_i^M(\alpha_i^0)$ (together with $p_j^M = 0$, with $j \neq i$) can only constitute an equilibrium if it is nonnegative, so that

$$k(1-\alpha_i^0) \leq 1 \tag{11}$$

must hold. Moreover, firm i must not have an incentive to increase its price above the price given by (10). By increasing the price firm i faces the demand as given by (2) and its profit is then given by $\pi_i(p_i, 0, 1; \alpha_i^0) = p_i(\alpha_i^0 t - p_i + b)/t$ as $p_j = 0$ and $\alpha_i^e = 1$ must hold in the monopoly equilibrium. We guarantee that firm i does not have an incentive to increase its price if

$$\left.\frac{\partial \pi_i(p_i, 0, 1; \alpha_i^0)}{\partial p_i}\right|_{p_i = p_i^M(\alpha_i^0)} = 2 - \alpha_i^0 - 1/k \leq 0$$

holds. Rewriting this condition gives

$$k(2-\alpha_i^0) \leq 1, \text{ for } i = A, B. \tag{12}$$

Obviously, Condition (12) is binding when compared with Condition (11). Substituting the installed bases, α_i^0 and α_j^0 $(i, j = A, B$ and $i \neq j)$, into (12) we obtain that a monopoly equilibrium exists with firm i (firm j) gaining the whole market, if $\alpha_i^0 \geq \overline{\alpha}^0(k)$ ($\alpha_i^0 \leq 1-\overline{\alpha}^0(k)$) holds. We summarize our results in the following lemma.

Lemma 2. *A monopoly equilibrium with $\alpha_i^M = 1$ ($\alpha_j^M = 1$) exists ($i, j = A, B$ and $i \neq j$), if $\alpha_i^0 \geq \overline{\alpha}^0(k)$ ($\alpha_i^0 \leq 1 - \overline{\alpha}^0(k)$). The monopoly price of the winning firm is given by $p_i^M = b - t(1-\alpha_i^0)$, while the losing firm cannot do better than setting $p_j = 0$. In that area the following constellations emerge:*

i) Multiple monopoly equilibria: If $\alpha_i^0 \in [\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)]$, then both $\alpha_i^M = 1$ and $\alpha_j^M = 1$ ($i \neq j$) are equilibrium outcomes.

ii) Unique monopoly equilibrium: If $\alpha_i^0 > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ or if $\alpha_i^0 = \overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_i^M = 1$ is the unique monopoly equilibrium. If $\alpha_i^0 < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ or if $\alpha_i^0 = 1-\overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_j^M = 1$ ($i \neq j$) is the unique monopoly equilibrium.

Combining Lemmas 1 and 2, we can fully characterize equilibria in the next proposition.[22]

Proposition 1. *The following equilibrium constellations emerge.*

i) Monopoly and market sharing equilibria: If $\alpha_i^0 \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$, then $\alpha_i^M = 1$, $\alpha_j^M = 1$ and $\alpha_i^1 = \alpha_i^I(\alpha_i^0, k)$ for $i = A, B$ and $i \neq j$ are equilibria.

ii) Unique market sharing equilibrium: If $\alpha_i^0 \in (1-\overline{\alpha}^0(k), \overline{\alpha}^0(k))$, then $\alpha_i^1 = \alpha_i^I(\alpha_i^0, k)$ for $i = A, B$ is the unique equilibrium.

iii) Unique monopoly equilibrium: If $\alpha_i^0 > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ or if $\alpha_i^0 = \overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_i^M = 1$ ($i = A, B$) is the unique monopoly equilibrium. If $\alpha_i^0 < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ or if $\alpha_i^0 = 1-\overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$, then $\alpha_j^M = 1$ ($i, j = A, B$ and $i \neq j$) is the unique monopoly equilibrium.

iv) Multiple monopoly equilibria: Both $\alpha_i^M = 1$ and $\alpha_j^M = 1$ ($i, j = A, B$ and $i \neq j$) are the only equilibria, if $\alpha_i^0 \in \{1-\overline{\alpha}^0(k), \overline{\alpha}^0(k)\}$ for all $k \in [1/2, 2/3)$.

It is instructive to interpret Proposition 1 in terms of the switching costs-network effects ratio, k. As k increases with switching costs and decreases with network effects, we can distinguish three cases: *i)* "high switching costs" or "low network effects" for $k > 1$, *ii)* "moderate switching

[22] Proposition 1 allows us to derive a lower bound on v such that the market is indeed always covered in any equilibrium. Examining the equilibrium market shares, we obtain the condition $v \geq (t-b)\left[t(2-\alpha_i^0) - b\right]/(3t - 2b)$. This condition is only relevant for $k > 1$. For values $k \leq 1$ network effects are large enough to guarantee market coverage for any $v \geq 0$.

costs" or "moderate network effects" for $1/2 < k < 1$, and *iii)* "low switching costs" or "large network effects" for $k < 1/2$.[23]

In the area $k < 1/2$ switching costs are low and network effects dominate which gives rise to multiple equilibria.[24] Depending on consumer expectations both a monopoly outcome and a market sharing outcome are possible. This result does not depend on the size of firms' installed bases. A large installed base does not "tip" the market necessarily into the monopoly outcome; if consumers do not expect a firm to monopolize the market. We obtain qualitatively the same pattern as in Katz and Shapiro (1985), where the coexistence of symmetric and asymmetric equilibria has been shown for the case of Cournot competition between incompatible technologies. When switching costs are high ($k > 1$), market sharing constitutes the unique equilibrium. This result shows that the *relative* importance of network effects and switching costs is critical to understand market outcomes. A preoccupation with network effects alone can lead one to conclude that the market behaves "tippy" (see Shapiro and Varian, 1998) and is likely to be monopolized by one of the technologies, while it actually remains in a market sharing equilibrium because of high switching costs. Similar to Beggs and Klemperer (1992) and Mitchell and Skrzypacz (2006), large switching costs evoke a fat-cat effect that works in favor of a market sharing outcome. A dominant firm prefers to exploit its installed base and allows the rival firm to gain market shares. We can conclude that our model nests two important views on markets with network effects and switching costs: First, if network effects dominate ($k < 1/2$), then similar results as derived in Katz and Shapiro (1985) emerge, while for cases where switching costs dominate ($k > 1$) results from the switching costs literature (Beggs and Klemperer, 1992) are valid.

In the intermediate range $1/2 < k < 1$ network effects and switching costs are more balanced. In that region we obtain strikingly different market outcomes, neither captured in the network effects nor in the switching costs literature. In that area the installed base plays a crucial role in determining the market outcome. Proposition 1 allows us to derive an important result on the contentious issue of consumer lock-in which is also closely related to the so-called critical mass

[23] To simplify, we do not discuss the somehow special cases with $\alpha_i^0 \in \{\overline{\alpha}^0, \overline{\overline{\alpha}}^0\}$ for $1/2 \leq k < 2/3$, where only the two monopoly equilibria emerge.

[24] By introducing uncertainty about the quality of firms' products and solving a global game with correlated private values Argenziano (2008) derives a unique equilibrium of the game.

effect emerging in network industries. A critical mass effect occurs when a firm gets a market share so large that consumers become inevitably trapped in that technology. The following corollary states our result concerning the existence of a critical mass, $\widetilde{\alpha}_i^0$, for firm i, such that the unique equilibrium outcome is the monopoly outcome with $\alpha_i^M = 1$.

Corollary 1. *One firm holds a critical mass of consumers, $\widetilde{\alpha}_i^0$, and therefore, becomes the monopolist, with $\alpha_i^M = 1$, for sure (as a unique equilibrium outcome) either if $\widetilde{\alpha}_i^0 > 1 - \overline{\alpha}^0(k)$ or $\widetilde{\alpha}_i^0 < \overline{\alpha}^0(k)$ for all $k \in [1/2, 2/3)$, or if $\widetilde{\alpha}_i^0 \geq \overline{\alpha}^0(k)$ or $\widetilde{\alpha}_i^0 \leq 1 - \overline{\alpha}^0(k)$ for all $k \in (2/3, 1]$. The critical mass always fulfills $\widetilde{\alpha}_i^0 > 1/2$, with $i = A, B$.*

As much of the literature on technology adoption in markets with network effects assumes perfect lock-in of consumers (see, e.g., Farrell and Saloner, 1986, or Arthur, 1989), one may argue that a critical mass effect only occurs for very large switching costs. In contrast, our analysis of the interplay of network effects and switching costs reveals that rather small (but not too small) switching costs are more likely to create a critical mass effect than large switching costs. Assuming perfect lock-in as a proxy for switching costs can, therefore, lead to false conclusions. On the other hand, in order to lock-in consumers for sure, large network effects ($k < 1/2$) alone cannot make it. If network effects are large and the costs of switching are negligible, then consumers can always afford to switch to the other firm. Our analysis, therefore, shows that network effects are an important driver that leads to the monopolization of markets. However, consumer lock-in can only occur in the presence of switching costs such that both market forces remain balanced.

If none of the firms has reached the critical mass, then the type of equilibrium under moderate switching costs depends on the exact value of k. If switching costs are rather low (i.e., $1/2 < k < 2/3$ holds), then multiple equilibria prevail as in the case of small switching costs ($k < 1/2$). For larger switching costs (with $2/3 < k < 1$) the equilibrium is similar to the case of high switching costs, such that market sharing prevails.

Proposition 1 allows us to discuss how likely equilibria are when switching costs or network effects change. The increase of switching costs appears to be a typical phenomenon in markets with pronounced network effects. In the early stages of market development switching costs are often less important. As consumers invest into product-specific complementary assets and achieve learning effects by using the technology, switching costs are likely to increase. Starting in the region where network effects are large ($k < 1/2$) our model predicts that an increase of switching costs increases the likelihood of a monopoly outcome if the intermediate parameter region is reached, where one of the firms obtains a critical mass.[25] However, if the increase in switching costs is very large, then the region with a unique market sharing equilibrium may be reached ($k > 1$), so that a monopolization of the market can be ruled out.

Another scenario concerns the increase of network effects. Suppose we are in a market where switching costs are substantial ($k > 1$). For example, this may be the case in so-called two-sided market environments, as e.g., online trading platforms. Our model then predicts that those markets are likely to be driven into the "intermediate" parameter region, where it is highly likely that one of the two products obtains a critical mass leading to monopoly.[26] If however, the increase in network effects is rather drastic, we may also end up in the region $k < 1/2$ (large network effects) where a definite prediction of the market outcome becomes impossible.

Let us now have a closer look at how market shares change in the market sharing equilibrium. First, we are interested whether the initially dominant firm keeps its dominance. Second, we are interested in the asymmetry of market shares; namely, is the difference in firms' market shares increases or decreases? With respect to the first property we distinguish between monotone and alternating market patterns, where the former (latter) case refers to an outcome where the dominant firm keeps (loses) its dominant position. With respect to the second property we distinguish monopolization and market sharing patterns+, where the former (latter) case means that the difference in market shares widens (narrows).

Proposition 2. *Consider the parameter range where market sharing is an equilibrium outcome and assume $\alpha_i^0 \neq 1/2$. We can then distinguish four different market patterns:*

[25] Interestingly, in the case of typewriters the advance of touch typing was identified by David (1985) as the main reason why the QWERTY keyboard design became the industry standard. Touch typing is, of course, a keyboard specific skill which creates substantial switching costs. Recently, Toshiba decided to pull out of the HD DVD business so that the rival format Blu-ray sponsored by Sony obtains a monopoly position in that market (see "Toshiba is Set to Cede DVD-format Fight," Wall Street Journal Europe, February 18, 2008, p. 3.). The decision was announced by Toshiba just after Time Warner decided to support exclusively Blu-ray. Time Warner's decision can be interpreted as an increase in (expected) switching costs.

[26] Such a case can be seen in E-Bay's success. E-Bay uses a reputation system where users evaluate sellers' performances. Such a reputation system creates positive network effects which may have grown over time.

i) Monotone market sharing. If $k > 1$, *then the initially dominant firm, i, loses market share but keeps its dominant position; i.e.,* $\alpha_i^0 > \alpha_i^I > 1/2 > \alpha_j^I > \alpha_j^0$, *for* $i, j = A, B$ *and* $i \neq j$.

ii) Monotone monopolization. If $k \in (2/3, 1)$, *then the market share of the initially dominant firm, i, increases; i.e.,* $\alpha_i^I > \alpha_i^0 > 1/2 > \alpha_j^0 > \alpha_j^I$, *for* $i, j = A, B$ *and* $i \neq j$.

iii) Alternating monopolization. If $k \in (1/2, 2/3)$, *then the initially dominant firm, i, loses its dominant position and the share of the rival firm, j, is larger than the initial share of the dominant firm; i.e.,* $\alpha_j^I > \alpha_i^0 > 1/2 > \alpha_j^0 > \alpha_i^I$, *for* $i, j = A, B$ *and* $i \neq j$.

iv) Alternating market sharing. If $0 < k < 1/2$, *then the initially dominant firm, i, loses its dominant position and the share of the rival firm, j, is smaller than the initial share of the dominant firm; i.e.,* $\alpha_i^0 > \alpha_j^I > 1/2 > \alpha_i^I > \alpha_j^0$, *for* $i, j = A, B$ *and* $i \neq j$.

Moreover, if $\alpha_i^0 = 1/2$, *then* $\alpha_i^I = 1/2$, *with* $i = A, B$. *If* $k = 1/2$ *and* $\alpha_i^0 > 0$, *then* $\alpha_i^I = 1 - \alpha_i^0$, *for* $i = A, B$. *If* $k = 1$ *and* $\alpha_i^0 > 0$, *then* $\alpha_i^I = \alpha_i^0$ *for* $i = A, B$.

Proposition 2 shows that changes in firms' market shares in the market sharing equilibrium are determined by the ratio of switching costs to network effects, k. Moreover, in the market sharing equilibrium firm i's price is given by $p_i^I = t\alpha_i^I$, where α_i^I is firm i's equilibrium market share. Hence, each firm's equilibrium price is proportional to the switching costs parameter and its equilibrium market share.

If switching costs are high ($k > 2/3$), then the initially dominant firm keeps its dominance even though it sets a higher price than the rival firm. That is, the dominant firm's installed base induces less aggressive pricing which is also called a fat-cat effect.[27] The fat-cat effect tends to reduce the dominant firm's market share. At the same time network effects tend to increase its market share. When network effects are relatively small ($k > 1$), the fat-cat effect dominates network effects, so that its market share decreases. If, however, network effects become larger (i.e., $2/3 < k < 1$), the fat-cat effect of charging a high price is dominated by network effects, so that the dominant firm increases its market share.[28]

When switching costs are small ($k < 2/3$), then consumers cannot expect in equilibrium

[27] This result is similar to Beggs and Klemperer (1992) who show that a market with consumer switching costs should converge monotonically towards a stable market sharing outcome.

[28] Mitchell and Skrzypacz (2006) also obtain a monotone monopolization pattern for relatively large network effects and monotone market sharing pattern for small network effects. Interestingly, they do not obtain patterns where dominance is reversed which may be due to their numerical analysis.

that the initially dominant firm keeps its dominant position. Suppose the opposite. Then the initially dominant firm would always get a larger market share than the expected one due to the importance of network effects in the region where switching costs are small, which is inconsistent with initial expectations. When switching costs are relatively large (i.e., $1/2 < k < 2/3$), then the consistency of consumer expectations in equilibrium requires that the new dominant firm obtains a larger market share than the initially dominant firm's installed base. That is, large network effects of the new dominant firm must compensate for the relatively large switching cost.[29] When switching costs become very small ($k < 1/2$), then dominance is also reversed, but as network effects are now also large, the new dominant firm's market share must be smaller than the installed base of the initially dominant firm. If, in contrast, consumers expected that the initially smaller firm gets a larger market share, then because of large network effects and very small switching costs it would get even a larger market share than the expected one (which is inconsistent with original expectations).

Our results imply that strong network effects do not necessarily lead to an amplified imbalance of firms' market shares. Only if strong network effects are combined with sufficiently large switching costs, then a market sharing equilibrium exists in which firms' market shares become more asymmetric. When network effects are strong and switching costs are negligible, then in the market sharing equilibrium firms' market shares become more symmetric and follow an alternating pattern.[30]

Understanding market patterns also helps to explain the existence of the market sharing equilibrium. The region where a market sharing equilibrium exists increases in k for $k > 2/3$, while it also increases when k becomes smaller than $2/3$. To understand this, first notice that for values $k > 2/3$ (but smaller than $k = 1$) the initially dominant firm increases its market

[29] If market dominance is amplified but reversed, then a firm may have a strategic incentive to *reduce* its original market share to increase the likelihood of becoming the dominant firm tomorrow. This is, a firm may strategically "underinvest" to look "lean and hungry." However, because of the critical mass effect, a firm also has a strong incentive to strategically increase its installed base.

[30] Proposition 2 also shows that in a particular case (precisely, $k = 1/2$ and $\alpha_i^0 > 0$) firms may interchange market shares, a pattern similar to the alternating dominance outcome in Farrell and Shapiro (1998). Moreover, if firms are symmetric ex ante (i.e., $\alpha_i^0 = 1/2$), then the market remains in the equal market sharing equilibrium. This result is also suggested in Katz and Shapiro (1985), where firms are assumed to be symmetric, and hence, obtain equal market shares in the symmetric (interior) equilibrium.

share. When we increase network effects in that region, then a market sharing equilibrium can only be sustained if the initially dominant firm is small enough. Hence, when k approaches 2/3 from above (i.e., network effects increase) the region where the market sharing equilibrium exists shrinks. If the initially dominant firm's installed base is too large, only a monopoly equilibrium is possible (i.e., the market is tipped into the monopoly outcome).

If network effects increase even further (i.e., k becomes smaller than 2/3), then a market sharing equilibrium is only possible if market dominance is reversed; as otherwise (for any other expectations which do not assume a reversion of dominance), expectations are not fulfilled. If market dominance is reversed, then of course, switching costs are important for existence (as there is a lot of switching in equilibrium). Hence, the existence of a market sharing equilibrium below $k = 2/3$ becomes more likely the lower switching costs become; or, similarly, when firms' installed bases are more symmetric. This implies that for $1/2 < k < 2/3$ the installed base of the dominant firm can become larger as switching costs decrease (i.e., k gets smaller) which explains the expansion of the market sharing region for $k < 2/3$.

Our results are quite robust to alternative forms of switching costs. Notice that the exact form of the switching costs distribution should become more important for relatively large values of switching costs among consumers. Formally, this is the case when $k > 1$. Exactly in that area our model reproduces the well-known results on the monotone market sharing pattern driven by a fat-cat effect which has been identified in several other papers: the initially dominant firm exploits its installed base and loses market shares.

The exact form of switching costs may play a role when network effects and switching costs are more balanced. In this case our model predicts the emergence of the critical mass effect. The exact value of the critical mass should certainly depend on the exact formulation of the switching costs. Yet, the very effect seems to us to be robust and quite intuitive. If a firm holds a large installed base of consumers it should become a monopolist when network effects and switching costs are more balanced. Finally, when network effects are much more important than switching costs the exact form of the switching costs function should become less important for the qualitative characterization of equilibria and the pattern of market shares.

We finally characterize the stability of the equilibria stated in Proposition 1. We define a stable equilibrium as an equilibrium which is robust to small perturbations in firms' market shares. Formally, if $a_i(k)$ is firm i's market share in a stable equilibrium, then two conditions are fulfilled: *First*, if $\alpha_i^0 = a_i(k)$, then in the unique equilibrium firm i's market share is $\alpha_i^1(\alpha_i^0, k) = a_i(k)$. *Second*, there exists $\epsilon(k) > 0$ such that if $\alpha_i^0 \in (\max\{0, a_i(k) - \epsilon(k)\}; \min\{a_i(k) + \epsilon(k), 1\})$ and $\alpha_i^0 \neq a_i(k)$, then the unique equilibrium fulfills $\left|\alpha_i^1(\alpha_i^0, k) - a_i(k)\right| < \left|\alpha_i^0 - a_i(k)\right|$. Both requirements together imply that a small perturbation of a stable equilibrium leads to a new unique equilibrium market share of firm i, which is closer to the stable equilibrium than the initial one. Proposition 3 states our results.

Proposition 3. *Depending on k the following equilibria are stable.*

i) With low network effects ($k > 1$) firms share the market equally in the unique stable equilibrium.

ii) With moderate network effects ($1/2 < k < 1$) two stable equilibria emerge in which one firm serves the whole market.

iii) With large network effects ($k \leq 1/2$) and if $k = 1$ there are no stable equilibria.

According to Proposition 3, an equilibrium is less likely to be stable, when network effects increase. When network effects are small ($k > 1$), there exists a unique stable equilibrium. When network effects are moderate ($1/2 < k < 1$) both monopoly equilibria are stable, while for large network effects ($k \leq 1/2$) none of the equilibria is stable. As network effects become larger (or, switching costs decrease), consumer expectations play a more important role and drive market outcomes. In contrast, the role of the installed base in determining market outcomes decreases.

4 Welfare Results

We now examine the social welfare and consumer surplus consequences of our model. We compare consumer surplus and social welfare under the monopoly equilibria and the market sharing equilibrium when both equilibria coexist. We show that a fundamental conflict between social welfare and consumer surplus maximization prevails. The next proposition summarizes our results.

Proposition 4. *Consider the parameter region where both the market sharing equilibrium and the monopoly equilibria coexist. Then, social welfare is always higher in the monopoly equilibria when compared with the market sharing equilibrium. For the comparison of consumer surplus* *we obtain the following cases:*

i) If $k \leq 1/2$*, then consumer surplus is always higher in the market sharing equilibrium when compared with both monopoly equilibria.*

ii) Let $k \in (1/2, 2/3)$ *and suppose that either one of the firms becomes the monopolist in the monopoly equilibrium. Then there exists a unique threshold value* $\widehat{\alpha}^0(k)$ *(*$1 - \widehat{\alpha}^0(k)$*), with* $\widehat{\alpha}^0(k) := [k(13 - 10k) - 4]/k^2$*, such that consumer surplus is higher in the market sharing equilibrium when compared with the monopoly equilibrium where* $\alpha_i^M = 1$ *(*$\alpha_j^M = 1$*) if* $\alpha_i^0 > \widehat{\alpha}^0(k)$ *(*$\alpha_i^0 < 1 - \widehat{\alpha}^0(k)$*), with* $i, j = A, B$ *and* $i \neq j$*. The opposite holds if* $\alpha_i^0 < \widehat{\alpha}^0(k)$ *(*$\alpha_i^0 > 1 - \widehat{\alpha}^0(k)$*), while indifference holds for* $\alpha_i^0 = \widehat{\alpha}^0(k)$ *(*$\alpha_i^0 = 1 - \widehat{\alpha}^0(k)$*). Moreover,* $\widehat{\alpha}^0(k)$ *(*$1 - \widehat{\alpha}^0(k)$*) is strictly concave (convex) over* $k \in (1/2, 2/3)$*, reaches its maximum (minimum) at* $k = 8/13$ *with* $\widehat{\alpha}^0(8/13) = 9/16$ *(*$1 - \widehat{\alpha}^0(8/13) = 7/16$*), while* $\widehat{\alpha}^0(1/2) = 0$ *and* $\lim_{k \to 2/3} \widehat{\alpha}^0(k) = 1/2$ *hold.*

iii) Let $k \in (1/2, 2/3)$ *and suppose that both firms* $i = A, B$ *may become the monopolist in the monopoly equilibrium. Then, for all* $\alpha_i^0 \in (\widehat{\alpha}^0(k), 1 - \widehat{\alpha}^0(k))$ *which implies* $k \in (1/2, 4/7)$*, consumer surplus is higher in the market sharing equilibrium when compared with both monopoly equilibria, while in all other instances either one of the monopoly equilibria gives rise to a higher consumer surplus when compared with the market sharing equilibrium. Moreover, if* $\alpha_i^0 \in (1 - \widehat{\alpha}^0(k), \widehat{\alpha}^0(k))$*, which implies* $k \in (4/7, 2/3)$*, then both monopoly equilibria give rise to a strictly higher consumer surplus than the market sharing equilibrium.*

Proposition 4 states that social welfare is always lower in the market sharing equilibrium when compared with the monopoly outcome as network effects are maximized in the monopoly outcome and switching costs are relatively small in the area where both types of equilibria coexist. Most importantly, Proposition 4 reveals a fundamental conflict between social welfare and consumer surplus. The conflict becomes most obvious in the parameter region where network effects dominate switching costs (i.e., $k \leq 1/2$ holds). In that region consumers strictly prefer market sharing to a monopoly outcome as market sharing minimizes consumers' overall payments to the firms. The result is independent of firms' installed bases, so that even significant consumers' switching in the market sharing equilibrium does not affect the ordering. In the market sharing equilibrium firms' prices and consumers' switching costs are proportional to the switching costs parameter, t, which is low in the area $k \leq 1/2$. On the other hand, in the monopoly equilibrium the monopolist sets the price which allows to expropriate all the network effects from consumers. As a results, in the area $k \leq 1/2$ where network effects are so valuable consumers enjoy larger network effects net of firms' prices in the market sharing equilibrium.

The tension between social welfare and consumer surplus remains to some extent valid in the parameter range, where switching costs become larger (i.e., $1/2 < k < 2/3$). Precisely, we obtain a critical value for firm i's initial market share, $\widehat{\alpha}^0(k)$, such that consumer surplus is maximized under the monopoly outcome (with firm i monopolizing the market) if firm i's initial market share does not fall short of the critical value. Hence, consumers can be better off in the monopoly equilibrium when compared with the market sharing equilibrium if the prospective monopolist has a relatively small installed base and must, therefore, price aggressively (i.e., set a relatively low price) in order to obtain the (expected) monopoly position.

The third part of Proposition 4 compares consumer surplus under the market sharing equilibrium with both monopoly equilibria. The region where the market sharing outcome maximizes consumer surplus when compared with both possible monopoly outcomes vanishes if switching costs become sufficiently large (i.e., at the point $k = 4/7$). In that case the market sharing equilibrium becomes increasingly costly for consumers as it involves substantial switching due to the alternating market sharing pattern. Interestingly, in the interval $k \in (4/7, 2/3)$ there also exists an area for installed bases such that both monopoly outcomes give rise to higher consumer surpluses than the market sharing equilibrium, so that social welfare and consumer surplus maximization are aligned in that area. In the interval $k \in (4/7, 2/3)$ both monopoly equilibria are possible in the area where firms' installed bases are more balanced, which makes the prospective monopolist set a relatively low price to monopolize the market. Moreover, this price reduction is proportional to the switching costs parameter, t, which is relatively high in the interval $k \in (4/7, 2/3)$.

Our results are instructive for recent policy debates that circle around the appropriate application of traditional competition policy instruments in markets with pronounced network effects (see, e.g., OECD, 1997, and FTC, 1996). While some consensus has been reached concerning the desirability of compatibility, the assessment of market outcomes when products are incompatible remains largely unresolved (see also Klemperer, 2005). Incompatibilities give rise to ambiguities as on the one hand pronounced network effects may drive the industry towards monopolization (an obviously unfortunate outcome from a traditional competition policy point of view) while on the other hand under a market sharing outcome where incompatible products compete head-to-head substantial incompatibilities among consumers prevail (an outcome being obviously inefficient).

As Proposition 4 shows, at least some of the ambiguities concerning the policy assessment of competition under incompatible products can be attributed to a fundamental conflict between consumer surplus and social welfare. If network effects are sufficiently large, consumers prefer the market sharing equilibrium (which minimizes their payments) over the monopoly outcome, while a social planer would prefer either one of the monopoly equilibria (where network effects are maximized).[31] Taking a policy-making perspective, our results highlight the trade-off involved with those governmental interventions which aim at picking a winning proprietary technology out of incompatible competitors (e.g., by committing governmental procurement or standard setting to a single technology).[32] While such a policy can be advisable from a social welfare perspective, consumers may be substantially hurt.[33] Our results also show that the conflict tends to vanish when switching costs become relatively more important. Therefore, in industries where both network effects and switching costs are important, a monopoly outcome can be preferable both from a social welfare and a consumer surplus perspective (which may have been the case in the above mentioned DVD format war, where Toshiba decided to pull out recently).

[31] Our finding is related to Farrell and Saloner (1992) who showed in a model of technology competition under network effects that the existence of (imperfect) converters makes a standardization (or, equivalently, a monopoly) outcome less likely, so that overall incompatibilities tend to be larger with converters. They interpret their finding as an inefficiency due to the "irresponsibility of competition"; a phenomenon which occurs quite generally under (incompatible) duopoly competition (see Suleymanova and Wey, 2010).

[32] A recent example for this kind of intervention can be seen in the announcement of the EU to support DVB-H as the mobile-television standard over rival technologies, as e.g., Qualcomm's MediaFLO ("EU Opts for DVB-H as Mobile-TV Standard," The Wall Street Journal Europe, March 18, 2008, p. 5).

[33] One may speculate that our results are somehow supported by the fact that policy makers taking an industrial policy perspective (i.e., focus primarily on profits) tend to prefer to pick a winning technology (out of a set of incompatible alternatives) while in competition policy circles (which are supposed to focus primarily on consumer surplus) a more reticent attitude appears to have gained control (as, e.g., in FTC, 1996).

5 Extensions

In this section we consider several extensions of our basic model. We analyze firms' compatibility incentives and the relationship between switching costs and firms' profits. We also provide equilibrium analysis for the cases where the market expands and firms have different marginal costs. Finally, we analyze a two-period extension where consumers have switching costs only in the second period but can choose freely between the products in the first period.

5.1 Compatibility Incentives

In this section we analyze firms' incentives to make their products compatible, in which case they become perfect substitutes with respect to their associated network effects. We assume that compatibility does not erase switching costs and products remain differentiated for consumers who belong to either one of the installed bases.

We use the superscript "c" to denote the case of compatible products. When products are compatible, the amount of network effects which consumers derive from any of the two products is given by b. The utility from buying the product of firm i for a consumer with address x under compatibility is then given by

$$U_x^{i,c} = \begin{cases} v + b - p_i & \text{if} \quad x \in \alpha_i^0 \\ v + b - p_i - t\left|\alpha_A^0 - x\right| & \text{if} \quad x \in \alpha_j^0, \end{cases} \tag{13}$$

with $i, j = A, B$ and $j \neq i$. From (13) we obtain the demand function $\alpha_i^c(p_i, p_j; \alpha_i^0)$ under compatibility

$$\alpha_i^c(p_i, p_j; \alpha_i^0) = \begin{cases} 0 & \text{if} \quad p_j - p_i \leq -t\alpha_i^0 \\ \alpha_i^0 + \frac{p_j - p_i}{t} & \text{if} \quad -t\alpha_i^0 < p_j - p_i < t(1 - \alpha_i^0) \\ 1 & \text{if} \quad p_j - p_i \geq t(1 - \alpha_i^0), \end{cases} \tag{14}$$

with $i, j = A, B$ and $i \neq j$. The following lemma summarizes the equilibrium outcome of the market game.[34]

[34] The following lemma can be derived from Proposition 1 by setting $b \to 0$. With compatible products both firms provide the same amount of network effects. Hence, they are irrelevant for consumers' choices. As $b \to 0$ implies $k \to \infty$, we get a monotone market sharing pattern as identified in Proposition 2 for $k > 1$.

Lemma 3. *Suppose products are compatible. Then the market sharing equilibrium is the unique equilibrium, where firms' market shares and prices are given by* $\alpha_i^c(\alpha_i^0) = (1+\alpha_i^0)/3$ *and* $p_i^c = t\alpha_i^c$*, respectively. Moreover, monotone market sharing pattern prevails everywhere; i.e.,* $\alpha_i^0 > \alpha_i^c > 1/2 > \alpha_j^c > \alpha_j^0$*, for* $i, j = A, B$ *and* $i \neq j$*.*

Lemma 3 reveals more consistent competitive pattern under compatible products when compared with incompatible products. Monotone market sharing occurs everywhere so that a monopoly outcome is never possible for the case of compatible products. As network effects are irrelevant for consumers' choices under compatibility, we get the same equilibrium pattern as with incompatible products and large switching costs.

We now turn to firms' incentives to make their products compatible in the first place. As in Katz and Shapiro (1985) we distinguish two cases depending on whether firms can make side payments. While firms are able to maximize their joint profits with side payments, firms will only agree on compatibility without side payments whenever compatibility benefits both firms. The next proposition summarizes our results when transfers are ruled out.

Proposition 5. *Firms never agree on making their products compatible with each other if side payments are ruled out. Conflicting incentives arise in the following way:*

i) If under incompatibility the monopoly equilibrium emerges, then the firm which becomes the monopolist loses and the other firm gains from compatibility.

ii) Assume $\alpha_i^0 \neq 1/2$ *(*$i = A, B$*) and suppose that the market sharing outcome emerges under incompatibility. Then, depending on the value of the parameter* k *either the dominant or the smaller rival firm loses under compatibility:*

If $k < \frac{2}{3}$*, then the dominant firm gains and the smaller rival firm loses under compatibility.*

If $k > \frac{2}{3}$*, then the dominant firm loses, while the smaller rival firm gains from compatibility.*

Moreover, if both firms share the market equally (i.e., $\alpha_i^0 = 1/2$*, with* $i = A, B$*), then both firms are indifferent between compatibility and incompatibility.*

The first part of Proposition 5 shows that the firm which becomes the monopolist under incompatibility does not have an incentive to make the products compatible, while the losing rival firm, of course, prefers compatible product designs. This result is closely related to Katz and Shapiro's (1985) finding that the possibility of an asymmetric equilibrium outcome under incompatibility (which corresponds to the monopoly outcome in our model) should lead to a blockage of compatibility by the "large" firm. The second part of Proposition 5 refers to the market sharing equilibrium under incompatibility. To understand the result it is instructive to analyze how firms' market shares change under compatibility and incompatibility (note that firms' profits are monotone in their market shares). From Proposition 2 we know that the initially dominant firm loses its dominant position under incompatibility if $k < 2/3$, while under compatibility the dominant firm keeps its dominant position (according to Lemma 3). Hence, $\alpha_i^c(\alpha_i^0) > \alpha_i^I(\alpha_i^0, k)$ must hold for $\alpha_i^0 > 1/2$, so that the dominant firm gains from compatibility. Obviously, in that region the opposite is true for the initially smaller rival firm which, therefore, has an incentive to block a move towards compatibility.

For $2/3 < k < 1$, we know from Proposition 2 that the dominant firm increases its market share under incompatibility, while (according to Lemma 3) it must decrease under compatibility. Hence, the dominant firm loses from a move towards compatibility, while the opposite must be true for the smaller rival firm.

For $k > 1$ the dominant firm loses market shares but still keeps its dominant position both under compatibility and under incompatibility. A comparison of market shares under compatibility $\alpha_i^c(\alpha_i^0) = (1+\alpha_i^0)/3$ and under incompatibility $\alpha_i^I(\alpha_i^0, k) = (k-1+k\alpha_i^0)/(3k-2)$ yields that $\alpha_i^c(\alpha_i^0) < \alpha_i^I(\alpha_i^0, k)$ holds for all $k > 2/3$ and $\alpha_i^0 > 1/2$. Hence, the dominant firm loses a larger fraction of its market share under compatibility, and, therefore, opposes compatibility.[35] Applying the same logic to the smaller rival firm we obtain conflicting incentives for compatibility.

It is instructive to compare our results with Katz and Shapiro (1985), where it is shown that firms should have an incentive to make their products compatible, whenever under incompatibility the (symmetric) interior solution is realized. In their Cournot model, compatibility leads to an overall expansion of firms' outputs (and, hence, an increase in profits) which is absent in our model. It is an artifact of our model that such a market expansion cannot occur. However, our analysis of asymmetric installed bases reveals that a fundamental conflict of interests between an initially dominant firm and its smaller rival remains valid in the (interior) market sharing

[35] Under incompatibility a larger market share for the dominant firm can be sustained in equilibrium as is provides larger network effects. Moreover, with a decrease in network effects (as k gets larger) the dominant firm's advantage under incompatibility erodes: the difference $\alpha_i^I(\alpha_i^0, k) - \alpha_i^c(\alpha_i^0) = \left(2\alpha_i^0 - 1\right) / [3(3k-2)]$ gets smaller.

outcome. Overall, our results, therefore, increase the bar for possible market expansion effects so as to make compatibility profitable for both firms when switching costs are present and side payments are not feasible.

We now turn to firms' incentives to achieve compatibility when transfers between the firms are feasible.

Proposition 6. *Suppose that both firms can make side payments when deciding about compatibility. Then, the following cases emerge:*

i) Firms do not agree on compatibility if under incompatibility one of the firms obtains a monopoly position.

ii) If $\alpha_i^0 \neq 1/2$ and $k < 1/3$, then firms agree on compatibility if under incompatibility market sharing equilibrium occurs.

iii) If $\alpha_i^0 \neq 1/2$ and $k > 1/3$, then firms do not agree on compatibility if under incompatibility market sharing equilibrium occurs.

Moreover, if $\alpha_i^0 = 1/2$ or if $k = 1/3$, then firms are indifferent between compatibility and incompatibility if market sharing equilibrium prevails under incompatibility.

Proposition 6 shows that firms cannot do jointly better even when side payments are possible, if under incompatibility the monopoly equilibrium emerges. The monopoly equilibrium emerges in the interval where network effects are more important compared to switching costs. As the monopolist sets the price which allows to expropriate all the network effects and under compatibility prices are proportional to the switching costs parameter, t, firms' joint profits are higher in the monopoly equilibrium under incompatibility. Proposition 6 also shows, however, that firms may agree on compatibility when the market sharing equilibrium holds under incompatibility. Namely, if switching costs are relatively low (or, network effects are sufficiently large) such that $k < 1/3$ holds, then firms can increase their joint profits if side payments are feasible. If, to the contrary, $k > 1/3$ holds, then firms can never jointly do better by making their products compatible. As firms' prices in the market sharing equilibrium are proportional to the switching costs parameter, t, both under incompatibility and incompatibility, the comparison between firms' joint profits under the two regimes depends on the exact value of t (k). Firms' joint profits are larger, the more asymmetric their equilibrium market shares are. In the region where network effects are moderate ($1/2 < k < 1$), the asymmetry in firms' market shares is amplified under incompatibility, while it decreases under compatibility, so that joint profits are higher under incompatibility. Although with low network effects ($k > 1$) firms' market shares become more symmetric under both regimes, the dominant firm can sustain a larger market share under incompatibility due to its network effects advantage leading to larger joint profits with incompatible products. When network effects are large ($k < 1/2$), the asymmetry in market shares erodes under both regimes and the comparison depends on the exact value of t (k). Under very large network effects ($k < 1/3$), only a very small market share of the new dominant firm can be sustained in the market sharing equilibrium under incompatibility. As a result, asymmetries vanish more under incompatibility, which makes firms' joint profits larger with compatible products.

We are now interested in consumers' preferences concerning compatibility.

Proposition 7. *Consumers are always better off under compatibility when compared with incompatible products.*

Proposition 7 shows that consumers are always better off when products are compatible. This result is independent of the type of equilibrium that emerges under incompatibility. When network effects are large ($k < 1/2$), under incompatibility consumers prefer the market sharing equilibrium to both monopoly equilibria. In the market sharing equilibrium under compatibility consumers enjoy larger network effects and lower switching costs due to the monotone market sharing pattern compared to the alternating pattern under incompatibility, which makes them prefer compatibility to both equilibria under incompatibility. When switching costs are large ($k > 1$), consumers enjoy larger network effects under compatibility. Moreover, with compatible products firms' market shares become less asymmetric compared to incompatible products, which makes switching less costly under incompatibility, while consumers' payments are lower under compatibility. The trade-off is resolved in a way that the market sharing equilibrium under compatibility is preferred by consumers. When network effects are moderate ($1/2 < k < 1$), depending on firms' installed bases consumers may either prefer market sharing or monopoly equilibrium. In that interval asymmetries in firms' market shares are amplified under incompatibility, which makes both switching more costly and consumers' payments larger in the market sharing equilibrium under incompatibility compared to compatible products. As a monopoly equilibrium in that region delivers larger consumer surplus only when the prospective monopolist's installed base is low enough, consumers' switching costs are larger under incompatibility compared with compatibility.

We conclude our discussion of firms' compatibility incentives with the comparison of social welfare under both regimes.

Proposition 8. *The comparison of social welfare under compatibility and incompatibility depends on the type of equilibrium under incompatibility.*

Case i) Suppose that under incompatibility the market sharing equilibrium emerges. If $k > 5/6$*, then there exists a unique threshold value,* $\mu(k) < \overline{\alpha}^0(k)$*, such that for all* $\alpha_i^0 \in (1-\overline{\alpha}^0(k), 1-\mu(k))$ *and* $\alpha_i^0 \in (\mu(k), \overline{\alpha}^0(k))$ *social welfare is strictly larger under incompatibility than under compatibility, with* $\mu(k) := 1/2 + [3(3k-2)] \,/\, \left[2\sqrt{5k(3k-1)}\right]$*. In all other cases, social welfare is higher under compatibility (with indifference holding if* $\alpha_i^0 \in \{\mu(k), 1-\mu(k)\}$*). Moreover,* $\mu(k)$ *is monotonically increasing and it holds that* $\mu(5/6) = \overline{\alpha}^0(5/6)$ *and* $\mu((103+\sqrt{1105})/132) = 1$*.*

Case ii) Suppose that under incompatibility the monopoly equilibrium emerges. If $\alpha_i^0 < 1/5$ *(*$\alpha_i^0 > 4/5$*), then social welfare is strictly larger in the monopoly equilibrium where firm* j *(firm* i*) becomes the monopolist (*$i, j = A, B$ *and* $i \neq j$*). In all other instances social welfare is larger under compatibility (with indifference holding if* $\alpha_i^0 \in \{1/5, 4/5\}$*).*

Proposition 8 shows that social welfare can be larger under incompatibility than under compatibility. The monopoly outcome under incompatibility appears to be attractive if the initial market share of the firm which becomes the monopolist in equilibrium is already large (larger than four-fifth). In those instances consumers' switching costs are lower with incompatible products, while network effects are maximized in both regimes. This result is related to Klemperer's (1988) finding that new entry into a monopoly market where consumers have switching costs can be detrimental to social welfare. Finally, Proposition 8 also shows the existence of a (small) parameter range where social welfare is higher in the market sharing equilibrium under incompatibility when compared with compatible products. Again, in that interval the relatively higher switching costs incurred under compatibility in connection with relatively high network effects under incompatibility give rise to the surprising result that social welfare can be higher under incompatibility.

5.2 Switching Costs and Firms' Profits

We now analyze how switching costs affect firms' profits. This allows us to examine firms' incentives to raise consumer switching costs. We distinguish two cases: First, we require that switching costs are symmetric as we assumed throughout the analysis. Second, we allow for asymmetric switching costs in the sense that the costs of switching from firm i to j can be different than switching from j to i. In the former case it is natural to assume that an increase in switching costs can only be implemented when both firms benefit from doing so. In the latter case, we allow a firm to raise switching costs unilaterally.

Suppose, switching costs are symmetric (i.e., the parameter t holds for both installed bases). We analyze the incentives to raise switching costs at the margin, so that they follow from the sign of $\partial \pi_i / \partial t$.[36] We first consider the market sharing equilibrium under incompatibility. In that case firms' profits are given by $t\left[\alpha_i^I(\alpha_i^0, k)\right]^2$, so that the direct effect of an increase in switching costs on firms' profits is always positive. However, there is also an indirect effect running through firms' market shares. Taking derivative of firm i's market share, $\alpha_i^I(\alpha_i^0, k)$, with respect to t yields

$$\frac{\partial \alpha_i^I(\alpha_i^0, k)}{\partial t} = \frac{1 - 2\alpha_i^0}{b(3k-2)^2}. \qquad (15)$$

Notice that the numerator of the right-hand side of (15) is negative if firm i is initially dominant ($\alpha_i^0 > 1/2$). Hence, the indirect effect of an increase of switching costs is negative for the initially dominant firm. The opposite holds for the initially smaller firm, so that the smaller firm must always be better off when switching costs increase. This is not necessarily the case for the initially dominant firm as the following proposition shows.

Proposition 9. *Suppose the market sharing equilibrium under incompatibility. Then, the initially smaller firm's profit strictly increases as switching costs increase. For the initially dominant firm, there exists a unique threshold value* $\widetilde{\alpha}^0(k) := [3k(1-k)-2]/[3k(k-2)]$ *such that the profit of the initially dominant firm increases as switching costs increase if* $\alpha_i^0 < \widetilde{\alpha}^0(k)$ *holds, while its profit decreases otherwise (with indifference holding if* $\alpha_i^0 = \widetilde{\alpha}^0(k)$*). The threshold value* $\widetilde{\alpha}^0(k)$ *is strictly convex with* $\partial \widetilde{\alpha}^0(k)/\partial k < 0$ *for all* $k < 2/3$ *and* $\partial \widetilde{\alpha}^0(k)/\partial k > 0$ *for all* $k > 2/3$. *Moreover,* $\widetilde{\alpha}^0(k) = 1$ *if* $k \in \{(9-\sqrt{33})/12, (9+\sqrt{33})/12\}$.

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[36] In our analysis we focus on marginal changes of parameter t (and thus, of parameter k). We, therefore, assume that a change in switching costs does not change the type of equilibrium.

According to Proposition 9 the initially dominant firm has an unambiguous incentive to raise switching costs if its initial market share is not too large. Hence, both firms' interests are always aligned if either network effects are large (so that $k < (9 - \sqrt{33})/12$) or switching costs dominate (such that $k > (9 + \sqrt{33})/12$). In contrast, if switching costs and network effects are more balanced, then a conflict of interests becomes more likely, in particular, whenever firms' installed bases are sufficiently asymmetric. Equation (15) shows that the equilibrium market share of the initially dominant firm decreases more with an increase in t, the larger its initial market share becomes and hence, the more asymmetric firms' installed bases are.

Let us next examine the incentives to increase switching costs whenever the monopoly equilibrium emerges under incompatibility with $\alpha_i^M = 1$. In that case the profit of the monopolist is given by $\pi_i^M(\alpha_i^0, t, b) := b\left[1 - k(1 - \alpha_i^0)\right]$ and the profit of the losing rival firm j is zero. The following result is now immediate.

Proposition 10. *Suppose $\alpha_i^0 < 1$ ($i = A, B$). If the monopoly equilibrium emerges under incompatibility with $\alpha_i^M = 1$, then firm i has no incentives to raise switching costs, while firm j ($j \neq i$) is indifferent in that case. If $\alpha_i^0 = 1$, then both firms do neither gain nor lose from a change in switching costs.*

Proposition 10 shows that a prospective monopolist does not have any incentives to increase switching costs as higher switching costs decrease the equilibrium price. In other words, as it is easier to monopolize the market when switching costs are relatively low, the prospective monopolist has a strict incentive to lower switching costs. Conversely, the losing rival firm finds it increasingly difficult to break consumers' monopolizing expectations the smaller switching costs become.[37]

We now turn to the case where switching costs can be asymmetric and where a firm can unilaterally raise switching costs. Denote the costs of switching from firm i to firm j by $t_i > 0$.[38] Firm i can either raise the costs of switching to the rival firm (t_i) or it can increase the costs of switching from the rival firm to itself (t_j).

[37] Note that we only consider marginal changes of switching costs, so that the type of equilibrium does not change. It then follows that the losing firm does not have a strict incentive to raise switching costs as it cannot change the fact that the other firm will monopolize the market.

[38] Accordingly, define $k_i := t_i/b$.

To keep the analysis tractable, we take the symmetric case ($t_i = t_j$) as the benchmark equilibrium. We concentrate on a market sharing equilibrium. Proposition 2 states for any given α_i^0 and k whether in equilibrium consumers switch to firm i or to firm j. Hence, we know which one of the parameters t_i or t_j matters.

The initially dominant firm, say firm A, always loses market shares in the interior equilibrium, except for $2/3 < k < 1$. Hence, for all $k \notin (2/3, 1)$ the switching costs t_A are critical for firms' equilibrium market shares.[39] In the region $2/3 < k < 1$, the initially dominant firm gains additional market shares, so that the switching cost parameter t_B becomes relevant.

Now recall that the initially smaller firm always gains from an increase in switching costs. Hence, if the smaller firm controls the relevant switching costs, it will always increase them. If, however, the initially dominant firm controls the relevant switching costs, then we obtain the same incentives as implied by Proposition 9.

Precisely, suppose firms control the costs of switching from their own product to the other firm; i.e., firm A controls t_A and firm B controls t_B. Suppose again that firm A is the initially dominant firm. Then firm A controls the relevant switching costs for all $k \notin (2/3, 1)$, and we get the same incentives as under the unanimity rule. If, however, $k \in (2/3, 1)$, such that consumers switch in equilibrium to the initially dominant firm, then the smaller firm B controls the relevant switching costs t_B, which it wants to increase.

If, in contrast, firms control the costs of switching to their own products, then it follows that the initially smaller firm B now controls the relevant switching costs t_A for all $k \notin (2/3, 1)$. Hence, switching costs will be raised by the smaller firm in that parameter region. If, however, $k \in (2/3, 1)$, then the initially dominant firm A controls the relevant switching costs t_B and will decide according to the incentives implied by Proposition 9; i.e., it will increase t_B if its initial market share is not too large.

Summing up, we observe that switching costs are more likely to increase, i) when firms unilaterally control and decide about switching costs and ii) when a firm controls the costs of switching to its own product.

[39] For $\alpha_A^0 > 1/2$ and $k \notin (2/3, 1)$, firms' equilibrium market shares are $\alpha_A^I(\alpha_A^0, k_A) = \left[k_A(1+\alpha_A^0) - 1\right]/(3k_A - 2)$ and $\alpha_B^I(\alpha_B^0, k_A) = \left[k_A(1+\alpha_B^0) - 1\right]/(3k_A - 2)$, where we substituted the symmetric switching cost parameter t by the relevant asymmetric parameter t_A.

We finally compare the incentives to raise switching costs when goods are compatible.

Proposition 11. *Under compatibility, both firms always have strict incentives to increase switching costs.*

Proposition 11 follows immediately from firms' profits under compatibility which are given by $t[(1+\alpha_i^0)/3]^2$ $(i = A, B)$, so that the indirect effect which creates conflicting interests under incompatibility is absent under compatibility. Both firms have always strict incentives to raise switching costs. As network effects are irrelevant for consumers' choices with compatible products, equilibrium prices only depend on the switching costs parameter, t, and get larger when switching becomes more costly.

Our analysis of firms' incentives to raise switching costs reveals a potentially important drawback under compatibility. As compatibility unambiguously aligns both firms' incentives to raise switching costs, markets with compatible products may end up with overall higher switching costs when compared with markets where products remain incompatible. This observation should be particularly true if the market is monopolized under incompatibility as in that case incentives to raise switching costs are completely absent (see Proposition 10).

5.3 Market Expansion, Cost Asymmetries, and Dynamics

We now discuss how market expansion and cost asymmetries among firms affect the equilibrium outcomes. We, finally, present a dynamic extension with two periods where both firms and consumers are forward-looking.[40]

Market expansion. We now allow for entry of new consumers. We call consumers who form installed bases of the firms the "old" consumers. We normalize the size of the old consumers to unity. At the beginning of the period a mass of new consumers of size $\Delta \geq 0$ enters the market.[41] The total market size is then given by $1 + \Delta$.

New consumers are homogenous and do not have to bear switching costs. Hence, all new consumers either buy product A or product B.[42] Without loss of generality we assume that new consumers are expected to buy product i in equilibrium. We treat α_i^e as measuring firm i's

[40] We thank two anonymous referees for suggesting those extensions.

[41] We can interpret Δ as the growth rate of the market.

[42] Using (1) we obtain a new consumers' utility from setting $t = 0$.

market share of old consumers expected by all (old and new) consumers. The demand of firm i can then be written as

$$\alpha_i^1(p_i, p_j, \alpha_i^e; \alpha_i^0, \Delta) =$$

$$\begin{cases} 0 & \text{if} \quad p_j - p_i \leq -t\alpha_i^0 - \tau \\ \alpha_i^0 + \frac{p_j - p_i + \tau}{t} & \text{if} \quad -t\alpha_i^0 - \tau < p_j - p_i < -\tau \\ \alpha_i^0 + \frac{p_j - p_i + \tau}{t} + \Delta & \text{if} \quad -\tau \leq p_j - p_i < -\tau + t(1 - \alpha_i^0) \\ 1 + \Delta & \text{if} \quad p_j - p_i \geq -\tau + t(1 - \alpha_i^0), \end{cases} \tag{16}$$

with $\tau := b(2\alpha_i^e - 1 + \Delta)$.

The two intermediate intervals are derived from the indifference condition of the new consumers.[43] In the second interval none of the new consumers is part of the demand of firm i, while in the third (and fourth) interval all new consumers are part of the demand of firm i. Note, if firm i attracts the new consumers, then it also gains market shares among the old consumers (i.e., $\alpha_i^1 - \Delta \geq \alpha_i^0$). Suppose now a fulfilled expectations equilibrium exists, then prices must lie either in the third or fourth interval of (16).

We first consider the monopoly equilibrium, where firm i serves all the new and old consumers ($\alpha_i^e = 1$). Firm j sets its price equal to zero. Firm i in turn sets the highest possible price, which allows to monopolize the market (given $\alpha_i^e = 1$ and $p_j = 0$), i.e., $p_i^M(\alpha_i^0, \Delta) = b(1+\Delta) - t(1-\alpha_i^0)$. Those prices constitute an equilibrium, when firm i does not have an incentive to increase its price, which yields the condition

$$\left.\frac{\partial \pi_i(p_i, p_j, \alpha_i^e; \alpha_i^0, \Delta)}{\partial p_i}\right|_{p_i = p_i^M(\alpha_i^0, \Delta), p_j = 0, \alpha_i^e = 1} = \frac{t(2 - \alpha_i^0) + t\Delta - b(\Delta + 1)}{t} \leq 0$$

$$\Rightarrow \alpha_i^0 \geq \overline{\alpha}^0(k, \Delta) := 2 + \Delta - (1 + \Delta)/k. \tag{17}$$

If (17) holds, then $p_i^M(\alpha_i^0, \Delta) > 0$. Hence, if Condition (17) is fulfilled, then the described monopoly equilibrium exists. Inspection of (17) reveals that the monopoly equilibrium is more likely when new consumers enter the market. This can be seen by comparing $\overline{\alpha}^0(k, \Delta)$ with the critical value $\overline{\alpha}^0(k)$, which we derived above (with no new consumers).[44] The function $\overline{\alpha}^0(k, \Delta)$

[43] Given that all the new consumers are expected to by from firm i, new consumers are indifferent between both products if $b(\alpha_i^e + \Delta) - p_i = b(1 - \alpha_i^e) - p_j$.

[44] Note, if $\Delta = 0$, then $\overline{\alpha}^0(k, \Delta) = \overline{\alpha}^0(k)$.

is increasing in k. Moreover, $\overline{\alpha}^0(k,\Delta) = 0$ at $k = \widetilde{k}(\Delta) := (1+\Delta)/(2+\Delta)$ and $\overline{\alpha}^0(1,\Delta) = 1$. Inspecting $\widetilde{k}(\Delta)$ yields that the parameter range where the monopoly equilibrium exists increases in Δ.[45] Intuitively, when the market expands, then expectations become more important for pinning down the equilibrium. If consumers expect all the new consumers to join firm i, then switching costs and firms' installed bases become relatively unimportant (which holds if $k \leq 1$, where network effects are relatively large).

Let us next turn to the market sharing equilibrium, where firm i serves all the new consumers and its market share among the old consumers increases (third interval of (16)). Maximization of firms' profits yields the best response functions

$$\begin{aligned} p_i(p_j, \alpha_i^e; \alpha_i^0, \Delta) &= \left[t(\alpha_i^0 + \Delta) + b(2\alpha_i^e - 1 + \Delta) + p_j\right]/2 \text{ and} \\ p_j(p_i, \alpha_i^e; \alpha_i^0, \Delta) &= \left[t(1 - \alpha_i^0) - b(2\alpha_i^e - 1 + \Delta) + p_i\right]/2, \end{aligned}$$

which give the prices

$$\begin{aligned} p_i(\alpha_i^e; \alpha_i^0, \Delta) &= \left[t(1 + \alpha_i^0 + 2\Delta) + b(2\alpha_i^e - 1 + \Delta)\right]/3 \text{ and} \\ p_j(\alpha_i^e; \alpha_i^0, \Delta) &= \left[t(2 - \alpha_i^0 + \Delta) + b(1 - 2\alpha_i^e - \Delta)\right]/3. \end{aligned}$$

Solving the equation $\alpha_i^1(p_i(\cdot), p_j(\cdot), \alpha_i^e; \alpha_i^0, \Delta) = \alpha_i^e + \Delta$ for $\alpha_i^e = \alpha_i^I$ we get firms' equilibrium market shares

$$\alpha_i^I(\alpha_i^0, k, \Delta) = \frac{k(1+\alpha_i^0) - 1 + \Delta(2k-1)}{3k-2} \text{ and } \alpha_j^I(\alpha_j^0, k, \Delta) = \frac{k(1+\alpha_j^0) - 1 + \Delta(k-1)}{3k-2}. \quad (18)$$

Note that the market shares $\alpha_i^I(\alpha_i^0, k, \Delta)$ and $\alpha_j^I(\alpha_j^0, k, \Delta)$ are the same as in (8) for $\Delta = 0$. Firms i and j set prices $p_i^I = t\alpha_i^I$ and $p_j^I = t\alpha_j^I$, respectively.[46] The market shares in (18) can only constitute an equilibrium outcome if $\alpha_i^0 \leq \alpha_i^I(\alpha_i^0, k, \Delta) - \Delta < 1$.[47] Inspection of the inequalities yields the following proposition.

Proposition 12. *Assume that Δ new consumers enter the market and join firm i in equilibrium. If $\Delta \leq 1$, then the market sharing equilibrium emerges in the following cases:*

[45] Formally, $\partial\widetilde{k}(\Delta)/\partial\Delta > 0$ and $\lim_{\Delta\to\infty}\widetilde{k}(\Delta) = 1$, where the latter property says that the monopoly equilibrium exists for all $k \leq 1$ and all α_i^0 when Δ becomes very large.

[46] Firms' optimization problems are well-defined.

[47] As mentioned above, the requirement $\alpha_i^0 \leq \alpha_i^I(\alpha_i^0, k, \Delta) - \Delta$ follows immediately when firm i attracts all new consumers.

i) $k \in (1, \infty)$ *and* $\alpha_i^0 \in [0, (1-\Delta)/2]$,

ii) $k \in (2/3, 1]$ *and* $\alpha_i^0 \in \left[(1-\Delta)/2, \overline{\alpha}^0(k, \Delta)\right)$,

iii) $k \in [(1+\Delta)/(2+\Delta), 2/3)$ and $\alpha_i^0 \in \left(\overline{\alpha}^0(k, \Delta), (1-\Delta)/2\right]$,

iv) $k \in (0, (1+\Delta)/(2+\Delta))$ and $\alpha_i^0 \in [0, (1-\Delta)/2]$.

If $\Delta > 1$, *then the market sharing equilibrium emerges only if* $k \in [(1+\Delta)/(2+\Delta), 1]$ *and* $\alpha_i^0 \in \left[0, \overline{\alpha}^0(k, \Delta)\right)$.

Those results basically mirror the analysis without market expansion, whenever $\Delta \leq 1$ holds, i.e., the size of new consumers is not larger than the size of old consumers. The only difference comes from assuming that the new consumers join firm i in equilibrium. Those expectations can only be fulfilled in equilibrium, when firm i also gains market shares among the old consumers. As a consequence, we have to constraint the installed base of firm i by $(1-\Delta)/2$ (instead of $1/2$). For example, in case *i)* of Proposition 12, all the new consumers can only join firm i in equilibrium if firm i's installed base is smaller than $(1-\Delta)/2$, which assures that its market share among the old consumers increases in equilibrium.

If $\Delta > 1$, then our results differ sharply from our previous analysis without market expansion. The market sharing equilibrium only exists for moderate switching costs ($k \in [(1+\Delta)/(2+\Delta), 1]$). It follows directly from the properties of the function $\overline{\alpha}^0(k, \Delta)$ that if $\Delta \to \infty$, then no market sharing equilibrium exists anymore. Intuitively, a very large expansion of the market means that the expected network value of one of the products becomes very large which rules out a market sharing equilibrium.

In sum, a market expansion tends to make network effects more important, which increases the region where a monopoly equilibrium exists (for $k \leq 1$) and it may fully rule out a market sharing equilibrium, whenever the expansion is sufficiently large ($\Delta \to \infty$).

Cost asymmetries. Assume that firms have different marginal costs $c_i \geq 0$ and $c_j \geq 0$. We suppose that firm i has higher marginal costs than firm j, with $\Delta c := c_i - c_j > 0$. Given the consumer demand (2), following Proposition 1 we get three candidate equilibria. If an equilibrium exists where firm i monopolizes the market ($\alpha_i^1 = 1$), then $p_i^* = b - t(1-\alpha_i^0) + c_j$ and $p_j^* = c_j$. Accordingly, if firm j monopolizes the market ($\alpha_j^1 = 1$), we get $p_j^* = b - t\alpha_i^0 + c_i$ and $p_i^* = c_i$.

If both firms share the market, then $\alpha_i^1 = \alpha_i^I(\alpha_i^0, t, b, \Delta c)$ and $\alpha_j^1 = \alpha_j^I(\alpha_i^0, t, b, \Delta c)$, where

$\alpha_i^I(\alpha_i^0, t, b, \Delta c) := [k(1+\alpha_i^0) - 1 - \Delta c/b]/(3k-2)$ and $\alpha_j^I(\alpha_i^0, t, b, \Delta c) := [k(2-\alpha_i^0) - 1 + \Delta c/b]/(3k-2)$. The prices are $p_i^I = t\alpha_i^I + c_i$ and $p_j^I = t\alpha_j^I + c_j$.

Following again the equilibrium analysis which led us to Proposition 1, we obtain the critical values $\overline{\alpha}^0(t, b, \Delta c) := 2 - 1/k + \Delta c/t$ and $1 - \overline{\alpha}^0(t, b, \Delta c) + 2\Delta c/t$, which replace $\overline{\alpha}^0(k)$ and $1 - \overline{\alpha}^0(k)$, respectively. Inspecting the new critical values we observe that the parameter region increases where firm j monopolizes the market, while the corresponding area decreases for firm i. In fact, when the cost difference becomes sufficiently large, then for any installed base α_i^0, the less efficient firm i can no longer monopolize the market for sure.[48] Moreover, the area where the market sharing equilibrium emerges also gets smaller. We can conclude that cost asymmetries make monopolization by the more efficient firm more likely.

Dynamics. In our basic model we treat firms' installed bases as given. A natural question to ask is how the results of our model might impact on competition in the initial period, where consumers can choose between the offered products without having to bear switching costs. Moreover, both firms and consumers are forward-looking and maximize the discounted sum of their payoffs.

To answer this question we consider a two-period extension of our basic model. In the first period all consumers can freely choose one of the two products and firms set prices simultaneously. Consumers who buy product i in the first period become the installed base of firm i at the beginning of the second period. In the second period consumers must incur switching costs as we have specified in our basic model.

It is convenient to change the timing of the two period game such that consumer expectations about firms' market shares are formed *after* prices have been set. This approach gives rise to a Nash equilibrium demand schedule which does not depend on initial expectations anymore.[49] In the following we also focus on the parameter region where a unique equilibrium exists in the second period and consumer demand is downward-sloping in the first period.[50]

[48] This is the case when $\Delta c \geq t/2$.

[49] The fact that demands do not depend on initial expectations which have to be fulfilled in equilibrium simplifies the analysis. A comparison of the different timings is provided in Suleymanova and Wey (2010). It can be shown that the results do not change qualitatively. However, the analysis is quite different when network effects are large.

[50] By that we do not consider the parameter range where multiple equilibria exist in the second period of the

We indicate first-period and second-period variables by superscripts "1" and "2", respectively; i.e., α_i^1 stands for firm i's market share in the first period, p_i^2 stands for firm i's price in the second period, and so on.

We start with the equilibrium analysis of the second period. The following proposition states that a unique equilibrium exists in the second period for high switching costs ($k > 3$).

Proposition 13. *Assume that consumers form expectations after observing firms' prices. If switching costs are high with $k > 3$, then in the second period there exists a unique equilibrium, which is a market sharing equilibrium, where firms' market shares and prices are given by $\alpha_i^I(\alpha_i^1, k) := [k(\alpha_i^1 + 1) - 3]/[3(k-2)]$ and $p_i^I(\alpha_i^1, t, b) := b(k-2)\alpha_i^I(\alpha_i^1, k)$, respectively. Moreover, monotone market sharing pattern prevails everywhere.*

Proposition 13 obviously corresponds to our previous analysis of fulfilled expectations. Precisely, for $k > 1$ Propositions 1 and 2 state essentially the same results as Proposition 13. When switching costs are high enough, then a unique (market sharing) equilibrium exists and monotone market sharing pattern holds. One consequence is that the second-period equilibrium is always an interior solution even if one firm is able to monopolize the market in the first period when $k > 3$.

We can now analyze firms' and consumers' decisions in the first period. We assume high switching costs ($k > 3$). Consumers maximize the sum of the first-period utility ($v + b\alpha_i^1 - p_i^1$) and the second-period utility (U_x^i as given by (1)) discounted by the consumer discount factor, $\delta_c \in [0, 1]$. Similarly, firms maximize the discounted sum of their profits

$$p_i^1 \alpha_i^1(p_i^1, p_j^1) + \delta_f p_i^I(\alpha_i^1, t, b)\alpha_i^I(\alpha_i^1, k), \tag{19}$$

where $\delta_f \in [0, 1]$ is firms' discount factor. Both consumers and firms predict in the first period how firms' market shares will change in the second period given their first-period market shares.

game (which is the case when network effects are large, i.e., $k < 2$). Analyzing those instances would require to use equilibrium selection criteria which are "reasonable" for both consumers and firms. Moreover, we do not consider cases where the Nash demand schedule is upward-sloping in the first period even though equilibria are unique in the second period (which is the case when network effects are moderate, i.e., $2 < k < 3$). An upward-sloping demand gives rise to multiple equilibria and complicates refinement problems (see Grilo, Shy, and Thisse 2001, who use an invariance axiom to reduce the set of Nash equilibria for large network effects). As is shown below, we avoid all those technical issues by focusing on large switching costs.

Firm i's market share in the second period is $\alpha_i^I(\alpha_i^1, k)$ and its price is $p_i^I(\alpha_i^1, t, b)$ as stated in Proposition 13.

We start with consumer demand in the first period. We first state the condition for firm i to monopolize the market in the first period:

$$b - p_i^1 + \delta_c b\alpha_j^I(0, k) - \delta_c p_j^I(0, t, b) - \delta_c t\alpha_j^I(0, k) \geq -p_j^1 + \delta_c b\alpha_j^I(0, k) - \delta_c p_j^I(0, t, b). \quad (20)$$

If $\alpha_i^1 = 1$, then in the second period firm i will lose the share $\alpha_j^I(0, k)$ of consumers who switch to firm j. $\alpha_i^1 = 1$ requires that not a single consumer wants to choose firm j given that all the others buy product i. It is sufficient to focus on the consumer who is most likely to choose firm j in the first period, i.e., the consumer with the highest switching costs in the second period (given $\alpha_i^1 = 1$). This is the consumer whose switching cost is $t\alpha_j^I(0, k)$. If that consumer buys firm j's product already in the first period, then he does not enjoy any network effects in the first period (as all the other consumers choose firm i), but he does not have to bear switching costs in the second period. The discounted sum of utilities of that consumer if he chooses product j in the first period is stated on the right-hand side of Inequality (20). The left-hand side of Inequality (20) states the discounted sum of that consumer's utilities if he chooses product i in the first period. Rewriting Inequality (20) we get

$$p_j^1 - p_i^1 \geq -b + \delta_c t\alpha_j^I(0, k), \quad (21)$$

which says that to monopolize the market firm i must compensate the consumer with the highest switching costs (in the second period) with a sufficiently high first-period price reduction.

Accordingly, we can find firms' first-period market shares, α_i^1, in an interior solution by solving the indifference condition

$$\begin{aligned} & b\alpha_i^1 - p_i^1 + \delta_c b\alpha_j^I(\alpha_j^1, k) - \delta_c p_j^I(\alpha_j^1, t, b) - \delta_c t\left[\alpha_i^1 - \alpha_i^I(\alpha_i^1, k)\right] \\ = \quad & b\alpha_j^1 - p_j^1 + \delta_c b\alpha_j^I(\alpha_j^1, k) - \delta_c p_j^I(\alpha_j^1, t, b), \end{aligned}$$

which yields the market share

$$\alpha_i^1(p_i^1, p_j^1) = \frac{1}{2} - \frac{3(k-2)(p_i^1 - p_j^1)}{2b\left[3(2-k) - \delta_c k(3-k)\right]}. \quad (22)$$

Summing up (21) and (22), we can summarize consumer first-period demand as

$$\alpha_i^1(p_i^1, p_j^1) = \begin{cases} 1 & \text{if} \quad p_j^1 - p_i^1 \geq -b + \delta_c t \alpha_j^1(0,k) \\ \frac{1}{2} - \frac{3(k-2)(p_i^1 - p_j^1)}{2b[3(2-k) - \delta_c k(3-k)]} & \text{if} \quad b - \delta_c t \alpha_i^1(0,k) < p_j^1 - p_i^1 < -b + \delta_c t \alpha_j^1(0,k) \\ 0 & \text{if} \quad p_j^1 - p_i^1 \leq b - \delta_c t \alpha_i^1(0,k). \end{cases}$$

Consumer demand decreases in a firm's price over the interval $k > 3$ if

$$\delta_c > \frac{3(k-2)}{k(k-3)}, \tag{23}$$

i.e., when consumers are patient enough.[51] Assuming that Inequality (23) holds, we maximize firms' profits (19), which yields the first-period prices in the (symmetric) market sharing equilibrium

$$p_A^1 = p_B^1 = p^I(t, b, \delta_c, \delta_f) = \frac{1}{2} \left(\frac{3(k-2)}{2b\left[3(2-k) - \delta_c k(3-k)\right]} \right)^{-1} - \frac{\delta_f t}{3}. \tag{24}$$

Note that the price $p_i^I(t, b, \delta_c, \delta_f)$ is positive given Condition (23) if δ_f is not too large.[52] Inspecting (24) gives

$$\begin{aligned} \frac{\partial p_i^I(\cdot)}{\partial \delta_c} &> 0 \text{ and} \\ \frac{\partial p_i^I(\cdot)}{\partial \delta_f} &< 0, \end{aligned}$$

which shows how forward-looking behavior by consumers and firms affects first-period competition. Increasing the consumer discount factor, δ_c, makes consumer demand in the first period less elastic, which softens competition. Consumers expect that a firm which gets a larger market share in the first period will set a relatively high price in the second period, which will force some consumers to switch. The latter behavior of the dominant firm affects a consumer's decision in the first period: foreseeing that he has to bear high switching costs or pay a high price "tomorrow" he becomes less responsive to price reductions "today." Hence, a rising consumer discount factor makes consumer demand in the first period less elastic. Aggressive pricing

[51] If consumers discount future too much, network effects become relatively more important than second-period switching costs, which gives rise to an upward-sloping demand schedule and multiple equilibria (see Grilo, Shy, and Thisse, 2001). Note that there always exists $\delta_c \in [0, 1]$ such that Condition (23) holds whenever k is sufficiently large.

[52] First-period prices can be positive or negative depending on the exact values of agents' discount factors. Below-cost prices in the first period are more likely the higher (lower) δ_f (δ_c).

becomes then less attractive in the first period as consumers foresee the exploitative pricing behavior in the second period. As a consequence, competition is softened and equilibrium prices tend to increase.

In contrast, a higher discount factor of the firms makes competition in the first period tougher. Every firm wants to get a larger market share in the first period as this allows to secure higher profits in the second period.[53]

We conclude the analysis with some remarks on values $2 < k < 3$. On this interval, consumer demand is well-behaved in the second period (i.e., demand decreases in a firm's price). Then for any combination of firms' installed bases a unique equilibrium emerges in the second period.[54] However, in the two-period extension demand becomes upward-sloping in the first period which gives rise to a complicated analysis of multiple equilibria. Yet, it is worth noting that there are two reasons which make a monopoly outcome already in the first period highly likely if $2 < k < 3$ (i.e., network effects become larger). *Firstly*, there exists a critical mass effect in the second period, so that a firm with a first-period market share above the critical mass becomes the monopolist in the second period for sure. *Secondly*, the initially dominant firm increases its market share in the market sharing equilibrium of the second period if it could not reach the critical mass in the first period. Both reasons make it less attractive for consumers to buy the good of the smaller firm in the first period as this requires to bear switching costs in the second period (at least for some consumers).

6 Conclusion

We presented a model of duopolistic Bertrand competition in a market where both network effects and consumer switching costs shape competitive outcomes. Our main contribution is the analysis of market outcomes when products are incompatible and network effects and switching

[53] Our results are in line with Klemperer (1987a,b) and Caminal and Matutes (1990) who derive similar dynamic effects in the presence of consumer switching costs.

[54] In fact, under Nash expectations in the second period the interval $2 < k < 3$ corresponds in its equilibrium behavior exactly with the interval $2/3 < k < 1$, which we characterized for the fulfilled expectations case. Most importantly, over that both a critical mass effect exists and monotone monopolization holds in the unique interior equilibrium (which emerges if firms' first-period market shares are similar).

costs are balanced. Our model nests previous results derived in the switching costs and network effects literature and reveals that the delicate interplay of both market forces gives rise to new results; i.e., when both forces are more balanced. In that area we obtained a critical mass effect, such that a region of parameter constellations emerges where the initially dominant firm becomes the monopolist for sure at the end of the period (as a result of a unique equilibrium prediction). Neither large network effects nor large switching costs alone can drive the industry into a monopoly outcome for sure. In the former case the multiplicity of equilibria and in the latter case the unique market sharing equilibrium rule out the establishment of an uncontestable monopoly outcome. We also showed that changes in firms' market shares in the market sharing equilibrium can follow different patterns depending on the relative strength of switching costs to network effects. Most importantly, monopolization pattern (which can be either alternating or monotone) can only emerge when strong network effects are combined with strong enough switching costs. When network effects dominate and switching costs are negligible (or the opposite holds), then the asymmetry in firms' market shares becomes less pronounced.

The comparison of social welfare and consumer surplus under incompatibility in the market sharing equilibrium and the monopoly equilibrium (when both coexist) highlights a fundamental trade-off between both policy goals. While the very existence of network effects dictates a monopoly outcome from a social welfare point of view when switching costs are low, a market sharing outcome is preferred from a consumer perspective. That result may explain why policy makers taking an industrial policy perspective (and, hence, primarily focusing on profits) tend to favor picking a winning standard out of incompatible alternatives whereas in competition policy circles (which are supposed to focus on consumer surplus) a more tentative assessment appears to have gained control.

We analyzed market outcomes when products are compatible. Most importantly, we showed that in contrast to often expressed views concerning the desirability of compatibility social welfare is strictly higher under incompatibility if a prospective monopolist already holds a sufficiently large market share. The reason for this result is that switching costs under compatibility are larger in that case while network effects are maximized under both regimes. Imposing compatibility in a market where one firm already holds a dominant position may, therefore, involve welfare losses which depend on the importance of consumer switching costs.

We also examined incentives to raise switching costs where the main lesson was that under incompatibility firms' interests may not be aligned while under compatibility both firms have strict incentives to increase switching costs so as to lessen competition. Again, that result highlights a possible drawback of promoting compatibility as this may lead to welfare losses caused by higher switching costs in the market.

We showed that our results remain largely valid when we consider market expansion or asymmetries in firms' marginal costs. Only if the market expansion is very large or costs are very asymmetric, then the market sharing equilibrium becomes less likely when compared with a monopoly outcome.

Finally, we considered a two-period extension of our basic market game with endogenous installed bases. We focused on relatively large switching costs which guarantees a unique (market sharing) equilibrium in the second period of the game. We showed that the results critically depend on agents' discounting factors.

Appendix

In this Appendix we provide the omitted proofs.

Proof of Lemma 1. First notice that market shares add up to unit; hence, if $0 < \alpha_i^I(\alpha_i^0, k) < 1$ holds, then $0 < \alpha_j^I(\alpha_j^0, k) < 1$ holds as well, with $i, j = A, B$ and $i \neq j$. Hence, existence of the interior solution $\alpha_i^I(\alpha_i^0, k) = \left[k(1 + \alpha_i^0) - 1\right] / (3k - 2)$ is guaranteed if and only if condition $0 < \alpha_i^I(\alpha_i^0, k) < 1$ holds. Note also that condition $0 < \alpha_i^I(\alpha_i^0, k) < 1$ implies $p_i^I > 0$ $(i = A, B)$. We first prove that for $k < 2/3$ the market sharing equilibrium arises if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1 - \overline{\alpha}^0(k))$. We then prove that for all $k > 2/3$ the market sharing equilibrium exists if $\alpha_i^0 \in (1 - \overline{\alpha}^0(k), \overline{\alpha}^0(k))$.

Case i) $(k < 2/3)$. Applying condition $0 < \alpha_i^I(\alpha_i^0, k) < 1$ gives that $\alpha_i^I > 0 \Leftrightarrow \alpha_i^0 < 1/k - 1$ while $\alpha_i^I < 1 \Leftrightarrow \alpha_i^0 > 2 - 1/k$.

Case ii) $(k > 2/3)$. Again, using condition $0 < \alpha_i^I(\alpha_i^0, k) < 1$ gives that $\alpha_i^I > 0 \Leftrightarrow \alpha_i^0 > 1/k - 1$ and $\alpha_i^I < 1 \Leftrightarrow \alpha_i^0 < 2 - 1/k$.

Differentiation of the threshold value $\overline{\alpha}^0(k)$ gives $1/k^2 > 0$. Finally, uniqueness follows from the concavity of firms' optimization problems over the relevant parameter range. *Q.E.D.*

Proof of Proposition 2. We have to compare α_i^0 with $\alpha_i^I(\alpha_i^0, k)$. Suppose that $\alpha_i^0 \neq 1/2$.

Firm i obtains a dominant position if $\alpha_i^I = \left[k(1+\alpha_i^0)-1\right]/(3k-2) > 1/2$ holds. This can only be the case, if either $\alpha_i^0 > 1/2$ and $k > 2/3$ or $\alpha_i^0 < 1/2$ and $k < 2/3$ hold. Hence, for all $k > 2/3$ ($k < 2/3$) the initially dominant firm keeps (loses) its dominant position. We now examine whether $|\alpha_i^0 - \alpha_j^0| > |\alpha_i^I - \alpha_j^I|$ or $|\alpha_i^0 - \alpha_j^0| < |\alpha_i^I - \alpha_j^I|$ holds. We obtain that $|\alpha_i^0 - \alpha_j^0| > |\alpha_i^I - \alpha_j^I|$ holds if and only if $k < 1/2$ or $k > 1$, while $|\alpha_i^0 - \alpha_j^0| < |\alpha_i^I - \alpha_j^I|$ is true if and only if $1/2 < k < 1$ (note that $k \neq 2/3$). Combining those results, we obtain all four patterns as specified in the proposition. The last part of the proposition follows directly from substituting the specific values into $\alpha_i^I(\alpha_i^0, k)$. *Q.E.D.*

Proof of Proposition 3. Consider first $k > 1$. From Proposition 1 we know that for any $\alpha_i^0 \in [0,1]$ a unique equilibrium emerges, $\alpha_i^I(\alpha_i^0, k)$. Moreover, if $\alpha_i^0 \neq 1/2$, then $\alpha_i^I \neq \alpha_i^0$ and if $\alpha_i^0 = 1/2$, then $\alpha_i^I = \alpha_i^0$. Hence, only $a_i(k) = 1/2$ satisfies the first stability condition. Let $\epsilon(k) = 1/2$, such that any $\alpha_i^0 \in (0,1)$ belongs to the neighborhood of $a_i(k) = 1/2$. It follows from Proposition 2 that for any $\alpha_i^0 \in (0,1)$ and $\alpha_i^0 \neq 1/2$ it holds $\left|\alpha_i^1 - 1/2\right| < \left|\alpha_i^0 - 1/2\right|$. Hence, $a_i(k) = 1/2$ satisfies also the second stability condition.

Consider next $1/2 < k < 1$. Only $a_i(k) = 0$ and $a_i(k) = 1$ satisfy the first stability requirement. We show that they also satisfy the second requirement by defining the proper neighborhoods. For $a_i(k) = 0$ we set $\epsilon(k) = \min\{1 - \overline{\alpha}^0(k), \overline{\alpha}^0(k)\}$. From Proposition 1 we know that for any $\alpha_i^0 \in (0, \min\{1 - \overline{\alpha}^0(k), \overline{\alpha}^0(k)\})$ in the unique equilibrium $\alpha_i^1 = 0$, hence, $\left|\alpha_i^1 - a_i(k)\right| = 0 < \left|\alpha_i^0 - a_i(k)\right| = \alpha_i^0$. For $a_i(k) = 1$ we choose $\epsilon(k) = 1 - \max\{1 - \overline{\alpha}^0(k), \overline{\alpha}^0(k)\}$. From Proposition 1 we know that for any $\alpha_i^0 \in (\max\{1 - \overline{\alpha}^0(k), \overline{\alpha}^0(k)\}), 1)$ in the unique equilibrium $\alpha_i^1 = 1$, hence, $\left|\alpha_i^1(\alpha_i^0, k) - a_i(k)\right| = 0 < \left|\alpha_i^0 - a_i(k)\right| = \left|\alpha_i^0 - 1\right|$.

The nonexistence of stable equilibria for $k \leq 1/2$ follows directly from Proposition 1 as for any $\alpha_i^0 \in [0,1]$ multiple equilibria prevail, hence, the first stability condition is violated. If $k = 1$, then the second stability condition is violated as for any $\alpha_i^0 \in [0,1]$ and any $a_i(k) \in [0,1]$ it holds that $\left|\alpha_i^1(\alpha_i^0, k) - a(k)\right| = \left|\alpha_i^0 - a(k)\right|$. *Q.E.D.*

Proof of Proposition 4. From Proposition 1 we know that monopoly equilibria and the market sharing equilibrium coexist if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1 - \overline{\alpha}^0(k))$, with $i = A, B$, which implies $k < 2/3$. We first examine consumer surplus and then turn to social welfare.

Apart from the stand-alone value, v, consumer surplus consists of three terms; namely, the value of network effects, incurred switching costs, and consumers' overall expenses. In the market sharing equilibrium those terms are given by $b[(\alpha_i^I)^2 + (1-\alpha_i^I)^2]$, $(1/2)\left(\alpha_i^I - \alpha_i^0\right)(b - t)\left(2\alpha_i^I - 1\right)$, and $t[(\alpha_i^I)^2 + (1-\alpha_i^I)^2]$, respectively (for $i = A, B$). Adding all three terms we can (implicitly) express consumer surplus in the market sharing equilibrium as

$$\frac{CS^I(\alpha_i^I, \alpha_i^0, k) - v}{b} = (1-k)\left[2\left(\alpha_i^I\right)^2 - 2\alpha_i^I + 1 - \frac{1}{2}(\alpha_i^I - \alpha_i^0)(2\alpha_i^I - 1)\right]. \tag{25}$$

Substituting $\alpha_i^I(\alpha_i^0, k) = \left[k(1+\alpha_i^0) - 1\right] / (3k-2)$ into (25) we obtain

$$\frac{CS^I(\alpha_i^0, k) - v}{b} = \frac{(1-k)\left[4k(1-2k)\alpha_i^0(1-\alpha_i^0) + 11k^2 - 13k + 4\right]}{2\,(3k-2)^2}. \tag{26}$$

In the monopoly equilibrium with firm i ($i = A, B$) gaining the entire market, consumer surplus is given by $CS_i^M(\alpha_i^0, k) = v + (t/2)\,[1 - (\alpha_i^0)^2]$ which we can re-write as $[CS_i^M(\alpha_i^0, k) - v]/b = (k/2)\,[1 - (\alpha_i^0)^2]$. Thus, the comparison of consumer surpluses under the market sharing and the monopoly equilibrium gives rise to the following expression

$$\frac{CS^I(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k^3\left[\alpha_i^0 - (2k-1)/k\right]\left[\alpha_i^0 - [k(13-10k) - 4]/k^2\right]}{2\,(3k-2)^2}. \tag{27}$$

Defining $\widehat{\alpha}^0(k) := [k(13-10k) - 4]/k^2$ and substituting $\widehat{\alpha}^0(k)$ and $\overline{\alpha}^0(k) := (2k-1)/k$ into the right-hand side of Equation (27) we obtain

$$\frac{CS^I(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k^3}{2\,(3k-2)^2}\left[\alpha_i^0 - \overline{\alpha}^0(k)\right]\left[\alpha_i^0 - \widehat{\alpha}^0(k)\right]. \tag{28}$$

For the case that firm j ($j = A, B$, $j \neq i$) becomes the monopolist in the monopoly equilibrium we obtain the following expression (which follows from replacing $\overline{\alpha}^0(k)$ by $1 - \overline{\alpha}^0(k)$ and $\widehat{\alpha}^0(k)$ by $1 - \widehat{\alpha}^0(k)$ in (28))

$$\frac{CS^I(\alpha_i^0, k) - CS_j^M(\alpha_i^0, k)}{b} = \frac{k^3}{2\,(3k-2)^2}\left[\alpha_i^0 - (1 - \overline{\alpha}^0(k))\right]\left[\alpha_i^0 - (1 - \widehat{\alpha}^0(k))\right]. \tag{29}$$

From Equation (28) we observe that the sign of $CS^I(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)$ is determined by the sign of $[\alpha_i^0 - \overline{\alpha}^0(k)][\alpha_i^0 - \widehat{\alpha}^0(k)]$. We start with the properties of $\widehat{\alpha}^0(k)$. Successive differentiation of $\widehat{\alpha}^0(k)$ yields $\partial\widehat{\alpha}^0(k)/\partial k = -(13k-8)/k^3$ and $\partial^2\widehat{\alpha}^0(k)/\partial k^2 = [2(13k-12)]/k^4$. Note that $\partial^2\widehat{\alpha}^0/\partial k^2 < 0$ for all $k < 2/3$. Hence, $\widehat{\alpha}^0(k)$ is strictly concave over $k \in (0, 2/3)$ and obtains a unique maximum at $k = 8/13$ with $\widehat{\alpha}^0(8/13) = 9/16$. Note further that $\widehat{\alpha}^0(1/2) = 0$. As $\overline{\alpha}^0(k)$ is strictly increasing over $k \in (0, 2/3)$ and obtains a zero at $k = 1/2$, we know that $\widehat{\alpha}^0(k)$ and $\overline{\alpha}^0(k)$ are nonpositive for all $k \leq 1/2$. Hence, for all $k \leq 1/2$ the right-hand side of (28) is strictly positive (except if $k = 1/2$ and $\alpha_i^0 = 0$, in which case $CS^I(\alpha_i^0, k) = CS_i^M(\alpha_i^0, k)$).

Turning to the comparison of consumer surplus when firm j $(j \neq i)$ becomes the monopolist (see Equation (29)), we first notice that $(1-\widehat{\alpha}^0(k))$ is the exact mirror image of $\widehat{\alpha}^0(k)$, so that $(1-\widehat{\alpha}^0(k))$ is strictly convex over $k \in (0,2/3)$, reaches a unique minimum at $k = 8/13$ with $(1-\widehat{\alpha}^0(8/13)) = 7/16$, and obtains the value $(1-\widehat{\alpha}^0(1/2)) = 1$. Moreover, $\widehat{\alpha}^0(k) = (1-\widehat{\alpha}^0(k))$ at $k = 4/7$ and $\lim_{k\to 2/3} \widehat{\alpha}^0(k) = \lim_{k\to 2/3}(1-\widehat{\alpha}^0(k)) = 1/2$. Inspecting (29) we then obtain that $[\alpha_i^0-(1-\overline{\alpha}^0(k))]$ and $[\alpha_i^0-(1-\widehat{\alpha}^0(k))]$ are strictly negative for all α_i^0 if $k \leq 1/2$. Hence, consumer surplus is always larger in the monopoly equilibrium where firm j becomes the monopolist when compared with the market sharing equilibrium (except if $k = 1/2$ and $\alpha_i^0 = 1$, in which case $CS^I(\alpha_i^0, k) = CS_j^M(\alpha_i^0, k)$). This proves part i) of Proposition 4.

In the interval $k \in (1/2, 2/3)$ multiple equilibria emerge only if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$. We first focus on the case when firm i becomes the monopolist where the comparison of consumer surplus depends on Equation (28). We have to analyze how $\widehat{\alpha}^0(k)$ is related to $\overline{\alpha}^0(k)$ and $1-\overline{\alpha}^0(k)$ in the interval $k \in (1/2, 2/3)$. The following claim shows that $\widehat{\alpha}^0(k)$ lies exactly between the upper boundary, $1-\overline{\alpha}^0(k)$, and the lower boundary, $\overline{\alpha}^0(k)$.

Claim 1. *$\widehat{\alpha}^0(k) - \overline{\alpha}^0(k) > 0$ and $1-\overline{\alpha}^0(k) - \widehat{\alpha}^0(k) > 0$ hold for all $k \in (1/2, 2/3)$.*

Proof. Simple calculations give $\widehat{\alpha}^0(k) - \overline{\alpha}^0(k) = -12(k-1/2)(k-2/3)/k^2$ which is clearly strictly positive over the interval $k \in (1/2, 2/3)$. Similarly, we obtain $1-\overline{\alpha}^0(k) - \widehat{\alpha}^0(k) = (3k-2)^2/k^2$ which is obviously strictly positive. This proves Claim 1.

From Claim 1 we know that α_i^0 lies either in the interval $(\overline{\alpha}^0(k), \widehat{\alpha}^0(k))$ or in the interval $(\widehat{\alpha}^0(k), 1-\overline{\alpha}^0(k))$. In the former case $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ and $[\alpha_i^0 - \widehat{\alpha}^0(k)] < 0$, so that the right-hand side of Equation (28) is strictly negative. Hence, consumer surplus is higher in the monopoly equilibrium if $\alpha_i^0 \in (\overline{\alpha}^0(k), \widehat{\alpha}^0(k))$ for $k \in (1/2, 2/3)$. In the latter case with $\alpha_i^0 \in (\widehat{\alpha}^0(k), 1-\overline{\alpha}^0(k))$, where $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ and $[\alpha_i^0 - \widehat{\alpha}^0(k)] > 0$, so that the right-hand side of Equation (28) is strictly positive for all $k \in (1/2, 2/3)$ and consumer surplus, therefore, is strictly larger in the market sharing equilibrium when compared with the monopoly equilibrium where firm i becomes the monopolist.

We now turn to the case where firm j $(j \neq i)$ becomes the monopolist in the monopoly equilibrium in which case the comparison depends on Equation (29). It is immediate from Claim 1 that $(1-\widehat{\alpha}^0(k)) - (1-\overline{\alpha}^0(k)) < 0$ and $\overline{\alpha}^0(k) - (1-\widehat{\alpha}^0(k)) < 0$ hold for all $k \in (1/2, 2/3)$. Inspecting (29) we observe that $[\alpha_i^0 - (1-\overline{\alpha}^0(k))] < 0$ must always hold, so that consumer surplus is larger in the market sharing equilibrium than in the monopoly equilibrium with firm j becoming the monopolist if and only if $[\alpha_i^0 - (1-\widehat{\alpha}^0(k))] < 0$ or $\alpha_i^0 < 1 - \widehat{\alpha}^0(k)$ is fulfilled. This proves part ii) of Proposition 4. Part iii) follows from combining the results derived in part ii).

We turn now to the comparison of social welfare. Social welfare is given by the sum of consumer surplus and firms' profits, where the latter is given by consumers' overall expenses, $t[(\alpha_i^I)^2 + (1-\alpha_i^I)^2]$. Adding firms' profits to (25) we can express social welfare in the market sharing equilibrium (implicitly) as

$$\frac{SW^I(\alpha_i^I, \alpha_i^0, k) - v}{b} = 2\left(\alpha_i^I\right)^2 - 2\alpha_i^I + 1 - \frac{1}{2}(1-k)\left(\alpha_i^I - \alpha_i^0\right)\left(2\alpha_i^I - 1\right). \tag{30}$$

Substituting $\alpha_i^I(\alpha_i^0, k) = \left[k(1+\alpha_i^0) - 1\right] / (3k-2)$ into (30) yields

$$\frac{SW^I(\alpha_i^0, k) - v}{b} = \frac{4k\alpha_i^0\left(\alpha_i^0 - 1\right)[k(3-k)-1] - k^3 + 12k^2 - 13k + 4}{2\left(3k-2\right)^2}. \tag{31}$$

Accordingly, we can express social welfare in the monopoly equilibrium when firm i becomes the monopolist as[55]

$$\frac{SW_i^M(\alpha_i^0, k) - v}{b} = 1 - \frac{k}{2}\left(1-\alpha_i^0\right)^2. \tag{32}$$

Using (31) and (32) we get the difference between social welfare in the market sharing and the monopoly equilibrium (implicitly) given by

$$\frac{SW^I(\alpha_i^0, k) - SW_i^M(\alpha_i^0, k)}{b} = \frac{5k^3}{2\left(3k-2\right)^2}\left[\alpha_i^0 - \frac{2k-1}{k}\right]\left[\alpha_i^0 - \frac{k(4k-7)+4}{5k^2}\right]. \tag{33}$$

Defining $\phi_1(k) := [k(4k-7)+4] / \left(5k^2\right)$ and substituting $\phi_1(k)$ and $\overline{\alpha}^0(k) := (2k-1)/k$ into the right-hand side of Equation (33) we obtain

$$\frac{SW^I(\alpha_i^0, k) - SW_i^M(\alpha_i^0, k)}{b} = \frac{5k^3}{2\left(3k-2\right)^2}\left[\alpha_i^0 - \overline{\alpha}^0(k)\right]\left[\alpha_i^0 - \phi_1(k)\right]. \tag{34}$$

From Equation (34) we observe that the sign of $SW^I(\alpha_i^0, k) - SW_i^M(\alpha_i^0, k)$ is determined by the sign of $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right]\left[\alpha_i^0 - \phi_1(k)\right]$. Let us now examine the properties of $\phi_1(k)$ and how it is related to $\overline{\alpha}^0(k)$. Note first that $\partial\phi_1/\partial k = (7k-8)/(5k^3)$, from which we see directly that $\phi_1(k)$ is strictly decreasing over the interval $k \in (0, 2/3)$. As $\phi_1(1/2) = 6/5 > 1$ holds we know

[55] We omit the proof for the case where firm j ($j \neq i$) becomes the monopolist in the monopoly equilibrium which proceeds analogously.

that $[\alpha_i^0 - \phi_1(k)] < 0$ must hold for all $k \in (0, 1/2]$. As $\overline{\alpha}^0(k) \leq 0$ holds for all $k \in (0, 1/2]$ we know that $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ must be true over that interval (except if $k = 1/2$ and $\alpha_i^0 = 0$). Hence, the right-hand side of Equation (34) is strictly negative over the interval $k \in (0, 1/2]$ which implies that social welfare is higher in the monopoly equilibrium when compared with the market sharing equilibrium.

We now turn to the analysis of the remaining interval $k \in (1/2, 2/3)$, where the market sharing equilibrium only exists if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1 - \overline{\alpha}^0(k))$. As in the first part of the proof we are interested how $\phi_1(k)$ is related to $\overline{\alpha}^0(k)$ and $1 - \overline{\alpha}^0(k)$. The next claim shows that $\phi_1(k) > 1 - \overline{\alpha}^0(k)$, so that $[\alpha_i^0 - \phi_1(k)] < 0$ must hold for all $k \in (1/2, 2/3)$.

Claim 2. $\phi_1(k) - (1 - \overline{\alpha}^0(k)) > 0$ *holds for all* $k \in (1/2, 2/3)$.

Proof. The difference $\phi_1(k) - (1 - \overline{\alpha}^0(k))$ can be re-written as $\phi_1(k) - (1 - \overline{\alpha}^0(k)) = (3k-2)^2 / (5k^2)$ which is clearly strictly positive over the interval $k \in (1/2, 2/3)$. This proves Claim 2.

With Claim 2 at hand we know that for any α_i^0 for which both market sharing and monopoly equilibria emerge, i.e., $\alpha_i^0 \in (\overline{\alpha}^0(k), 1 - \overline{\alpha}^0(k))$, it holds that $[\alpha_i^0 - \phi_1(k)] < 0$. As $\alpha_i^0 > \overline{\alpha}^0(k)$ must hold to ensure that both monopoly equilibria and the market sharing equilibrium coexist, we know that $\left[\alpha_i^0 - \overline{\alpha}^0(k)\right] > 0$ must hold for all $k \in (1/2, 2/3)$. Hence, the right-hand side of Equation (34) is strictly negative for all $k \in (1/2, 2/3)$ which completes the proof of Proposition 4. *Q.E.D.*

Proof of Lemma 3. First, we rule out the existence of a monopoly equilibrium. We proceed by contradiction. Assume that in the monopoly equilibrium $\alpha_i^c(p_i, p_j; \alpha_i^0) = 1$ (with $i, j = A, B$, $j \neq i$). It must then hold that $p_j = 0$, as otherwise (with $p_j > 0$) firm j could increase its profit by decreasing its price. From the demand function $\alpha_i^c(p_i, p_j; \alpha_i^0)$ it follows that $-p_i \geq t(1 - \alpha_i^0)$ must hold which is only feasible if $p_i = 0$ and $\alpha_i^0 = 1$. In a monopoly equilibrium, firm i must not have an incentive to increase its price above $p_i = 0$. By increasing its price firm i faces the demand given by $\alpha_i^c(p_i, p_j; \alpha_i^0) = \alpha_i^0 + (p_j - p_i)/t$, so that $\partial \pi_i^c(p_i, p_j; \alpha_i^0)/\partial p_i = \alpha_i^0 + (p_j - 2p_i)/t$. Evaluating this derivative at $p_A = p_B = 0$ and $\alpha_i^0 = 1$ we obtain $\partial \pi_i^c(p_i, p_j; \alpha_i^0)/\partial p_i = 1$. Hence, the monopoly outcome cannot be an equilibrium under compatibility.

In the market sharing equilibrium firm i's demand is given by $\alpha_i^c(p_i, p_j; \alpha_i^0) = \alpha_i^0 + (p_j - p_i)/t$, with $i, j = A, B$ and $i \neq j$. Solving firms' optimization problems (which are globally concave) we obtain the prices and market shares as unique equilibrium outcomes as stated in the lemma. The last part of the lemma follows from the fact that $\alpha_i^c(\alpha_i^0) = (\alpha_i^0 + 1)/3 > 1/2$ and $\alpha_i^c(\alpha_i^0) = (\alpha_i^0 + 1)/3 < \alpha_i^0$ hold for all $\alpha_i^0 > 1/2$. Hence, we obtain monotone market sharing as the unique market pattern when products are compatible. *Q.E.D.*

Proof of Proposition 5. *Case i).* In the monopoly equilibrium under incompatibility the profit of the monopolist (say, firm $i = A, B$) is given by $\pi_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0)$ and the profit of the rival firm is zero, with $j \neq i$. Clearly, firm j gains from compatibility as $\pi_j^c(\alpha_j^0) = t(1 + \alpha_j^0)^2/9 > 0$ holds. For the monopolist under incompatibility (firm i) we have to compare $\pi_i^c(\alpha_i^0) = t(1 + \alpha_i^0)^2/9$ and $\pi_i^M(\alpha_i^0) = b - t(1 - \alpha_i^0)$. Comparison of the profits reveals that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ is true if and only if $\varphi_1(\alpha_i^0) < 9/k$ with $\varphi_1(\alpha_i^0) := (\alpha_i^0 - 2)(\alpha_i^0 - 5)$. Note that $\varphi_1'(\alpha_i^0) < 0$ for all $\alpha_i^0 \in [0, 1]$. We now analyze different values of k for which the monopoly equilibrium emerges. Consider first $k < 2/3$. If $k < 2/3$, then $9/k > 27/2$. As $\varphi_1(\alpha_i^0)$ obtains its maximum at $\alpha_i^0 = 0$ we get $\varphi_1(\alpha_i^0) \leq \varphi_1(0) = 10 < 27/2 < 9/k$, so that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ must hold for any α_i^0 if $k < 2/3$.

Consider next the interval $2/3 < k \leq 1$. In that region, the monopoly equilibrium only emerges for firm i if α_i^0 fulfills $\alpha_i^0 \in [\overline{\alpha}^0(k), 1]$. Note that $\overline{\alpha}^0(k) > 1/2$ for any $2/3 < k \leq 1$, hence, α_i^0 fulfills $\alpha_i^0 > 1/2$. As $\varphi_1(\alpha_i^0)$ monotonically decreases over the interval $\alpha_i^0 \in [0, 1]$, we have to show that $\varphi_1(1/2) < 9/k$ for $2/3 < k \leq 1$, which proves that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$ holds for any α_i^0 (for which the monopoly equilibrium emerges under $2/3 < k \leq 1$). In fact, evaluating $\varphi_1(\cdot)$ at the point $\alpha_i^0 = 1/2$ we get $\varphi_1(1/2) = 27/4 < 9/k$ if $2/3 < k \leq 1$. Hence for any α_i^0 it holds that $\pi_i^c(\alpha_i^0) < \pi_i^M(\alpha_i^0)$.

Finally, if $k > 1$, a monopoly equilibrium does not exist. Hence, we have proven part i) of the proposition.

Case ii). In the market sharing equilibrium under incompatibility firm i's profit is given by $t(\alpha_i^I)^2$ and under compatibility by $t(\alpha_i^c)^2$. It is then straightforward that $\pi_i^c - \pi_i^I = t(\alpha_i^c - \alpha_i^I)(\alpha_i^c + \alpha_i^I)$. Hence, the sign of the difference $\pi_i^c - \pi_i^I$ is given by the sign of $\alpha_i^c - \alpha_i^I = (1 - 2\alpha_i^0)/[3(3k-2)]$. It is now easily checked that $\alpha_i^c - \alpha_i^I < 0$ holds if either $k < 2/3$ and $\alpha_i^0 < 1/2$ or $k > 2/3$ and $\alpha_i^0 > 1/2$, while in the remaining cases $\alpha_i^c - \alpha_i^I > 0$ holds. If $\alpha_i^0 = 1/2$, then $\pi_i^c = \pi_i^I$. *Q.E.D.*

Proof of Proposition 6. *Case i)* We first analyze the incentives for compatibility when under incompatibility firm i ($i = A, B$) obtains a monopoly position in equilibrium. In this case we have to compare the sum of firms' profits in the monopoly equilibrium under incompatibility, π_i^M, with the sum of firms' profits under compatibility, $\sum_{j=A,B} \pi_j^c$, which are given by $b - t(1-\alpha_i^0)$ and $(t/9)[(1+\alpha_i^0)^2 + (2-\alpha_i^0)^2]$, respectively. The sign of the difference $\sum_{j=A,B} \pi_j^c - \pi_i^M$ is given by the sign of the expression $\psi_1(\alpha_i^0) - 9/k$, with $\psi_1(\alpha_i^0) := 2(\alpha_i^0 - 2)(\alpha_i^0 - 7/2)$. The function $\psi_1(\cdot)$ is monotonically decreasing over the interval $\alpha_i^0 \in [0,1]$, and obtains its maximum at $\alpha_i^0 = 0$ with $\psi_1(0) = 14$ and its minimum at $\alpha_i^0 = 1$ with $\psi_1(1) = 5$. Hence, the range of possible values of the function $\psi_1(\alpha_i^0)$ is given by $5 \leq \psi_1(\alpha_i^0) \leq 14$. From the latter it is straightforward to conclude that for $k \leq 9/14$ (for which $9/k \geq 14$) it holds that $\psi_1(\alpha_i^0) - 9/k \leq 0$ for any α_i^0, so that $\sum_{j=A,B} \pi_j^c - \pi_i^M \leq 0$, which implies that compatibility is not jointly optimal. The values $k > 1$ are irrelevant since for $k > 1$ no monopoly equilibrium under incompatibility emerges.

Thus it is left to consider $9/14 < k < 1$. Then the sign of $\psi_1(\alpha_i^0) - 9/k$ depends on the initial market share of firm i, α_i^0, which becomes the monopolist under incompatibility. Inspecting the difference $\psi_2(\alpha_i^0, k) := \psi_1(\alpha_i^0) - 9/k$ we obtain two zeros: $\psi_2^1(k) := 11/4 - (3/4)\sqrt{(k+8)/k}$ and $\psi_2^2(k) := 11/4 + (3/4)\sqrt{(k+8)/k}$. It is straightforward that $\psi_2^2(k) > 1$ for any k. We next show that $0 < \psi_2^1(k) < 1$. Note that $\psi_2^1(k)$ is strictly increasing in k. At $k = 9/14$ we obtain $\psi_2^1(9/14) = 0$ and for $k = 1$ we obtain $\psi_2^1(1) = 1/2$. As we know that the monopoly equilibrium can emerge for firm i only if $\alpha_i^0 \geq \overline{\alpha}^0(k)$, we have to check whether $\psi_2^1(k) \geq \overline{\alpha}^0(k)$ or $\psi_2^1(k) < \overline{\alpha}^0(k)$ holds. We next show that $\psi_2^1(k) < \overline{\alpha}^0(k)$ holds for $k > 1/3$ and $\psi_2^1(k) \geq \overline{\alpha}^0(k)$ holds for $k \leq 1/3$. In fact, $11/4 - (3/4)\sqrt{(k+8)/k} < 2 - 1/k$ yields $k > 1/3$. Hence, for $9/14 < k < 1$, it holds that $\psi_2^1(k) < \overline{\alpha}^0(k)$. Thus, for any α_i^0 for which the monopoly equilibrium emerges it holds that $\alpha_i^0 \in (\psi_2^1(k), 1]$. Note that for any $\alpha_i^0 \in (\psi_2^1(k), 1]$ the function $\psi_2(\cdot)$ is negative. Hence, $\psi_1(\alpha_i^0) - 9/k < 0$ and $\sum_{j=A,B} \pi_j^c - \pi_i^M < 0$. We have, therefore, shown that for any k and α_i^0 for which the monopoly equilibrium emerges under incompatibility it holds that $\sum_{j=A,B} \pi_j^c - \pi_i^M \leq 0$, which implies that both firms never agree on compatibility. Finally, as $\sum_{j=A,B} \pi_j^c - \pi_i^M \leq 0$ holds for any α_i^0 when firm i obtains the monopoly position, then because of symmetry it follows that the inequality also holds if firm j ($j \neq i$) becomes the monopolist under incompatibility.

Cases ii) and iii). We now analyze the possibility for compatibility when otherwise (under incompatibility) firms would share the market in equilibrium. The sum of firms' profits under incompatibility in the market sharing equilibrium is given by

$$\sum_{j=A,B} \pi_j^I(\alpha_i^0, k) = \frac{t\left[2k^2(\alpha_i^0)^2 - 2k^2\alpha_i^0 + 5k^2 - 6k + 2\right]}{(3k-2)^2}$$

and the sum of firms' profits under compatibility is given by

$$\sum_{j=A,B} \pi_j^c(\alpha_i^0, k) = \frac{t\left[2(\alpha_i^0)^2 - 2\alpha_i^0 + 5\right]}{9}.$$

Then the difference of firms' joint profits under compatibility and incompatibility is given by

$$\sum_{j=A,B} \pi_j^c - \sum_{j=A,B} \pi_j^I = \frac{2(1-3k)(2\alpha_i^0 - 1)^2}{9(3k-2)^2}.$$

Obviously, that difference is positive if $k < 1/3$ and negative if $k > 1/3$ (with equality holding at $k = 1/3$ or $\alpha_i^0 = 1/2$). *Q.E.D.*

Proof of Proposition 7. We start with the comparison of consumer surplus when under incompatibility the market sharing equilibrium emerges (*Case i*) and then proceed with the comparison of consumer surplus when under incompatibility the monopoly equilibrium emerges (*Case ii*).

Case i). Assume that under incompatibility the market sharing equilibrium emerges. We proceed by comparing consumer surplus under compatibility and incompatibility. Apart from the stand-alone value, v, consumer surplus consists of three terms; namely, the value of the network effects, incurred switching costs, and consumers' overall expenses. Under compatibility those terms are given by b, $(1/2)t\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)$, and $t[(\alpha_i^c)^2 + (1-\alpha_i^c)^2]$, respectively (for $i = A, B$), so that consumer surplus under compatibility $CS^c(\alpha_i^c, \alpha_i^0, k)$ can be (implicitly) expressed as

$$\frac{CS^c(\alpha_i^c, \alpha_i^0, k) - v}{b} = 1 - \frac{k\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)}{2} - k[(\alpha_i^c)^2 + (1-\alpha_i^c)^2]. \tag{35}$$

Substituting $\alpha_i^c(\alpha_i^0) = (1 + \alpha_i^0)/3$ into the right-hand side of (35) we obtain

$$\frac{CS^c(\alpha_i^0, k) - v}{b} = \frac{8k\alpha_i^0 - 8k\left(\alpha_i^0\right)^2 - 11k + 18}{18}. \tag{36}$$

Using (36) and (26) we can express the difference between the consumer surpluses as

$$\frac{CS^c(\alpha_i^0, k) - CS^I(\alpha_i^0, k)}{b} = \frac{4k(1-3k)\alpha_i^0(\alpha_i^0 - 1) + 78k^2 - 107k + 36}{18\left(3k-2\right)^2}. \tag{37}$$

One can easily see that the sign of the right-hand side of Equation (37) is given by the sign of the numerator which we define by $\xi_1(\alpha_i^0, k)$. Let us also define $\xi_2(k) := 4k(1-3k)$. Note that $\xi_2(k)$ is positive if $k < 1/3$, zero if $k = 1/3$ and negative otherwise. The discriminant of the function $\xi_1(\cdot)$ is given by $D = 12^2 k\,(3k-1)\,(3k-2)^2$. The discriminant is negative if $k < 1/3$, zero if $k = 1/3$, and positive otherwise. Hence, $\xi_2(\cdot)$ is positive, while the discriminant is negative for $k < 1/3$, which implies that $\xi_1(\cdot)$ is positive for any α_i^0. Hence, consumer surplus is higher under compatibility than in the market sharing equilibrium under incompatibility in that region. If $k = 1/3$, then $\xi_1(\cdot) = 9$ for any α_i^0, and consumer surplus is again higher under compatibility. Consider now $k > 1/3$, for which the function $\xi_1(\alpha_i^0, 1/3)$ has two roots, namely, $\alpha_1(k) = 1/2 + [3/(2k)]\,|3k-2|\,\sqrt{k(3k-1)}/(3k-1)$ and $\alpha_2(k) = 1/2 - [3/(2k)]\,|3k-2|\,\sqrt{k(3k-1)}/(3k-1)$, it is straightforward that $\alpha_1(k) > \alpha_2(k)$ for any $k > 1/3$. The following claim shows how $\alpha_1(k)$ and $\alpha_2(k)$ are related to $\overline{\alpha}^0(k)$ and $1 - \overline{\alpha}^0(k)$.

Claim 3. *It holds that* $\alpha_1(k) > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ *and* $\alpha_2(k) < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ *for any* $k > 1/3$.

Proof. We first show that $\max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\} = 1/2 + |3k-2|/(2k)$ and $\min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\} = 1/2 - |3k-2|/(2k)$. If $k < 2/3$, then $\max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\} = 1/k - 1$ and $1/2 + |3k-2|/(2k) = 1/2 - (3k-2)/(2k) = 1/k - 1$ and if $k > 2/3$, then $\max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\} = 2 - 1/k$ and $1/2 + |3k-2|/(2k) = 1/2 + (3k-2)/2k = 2 - 1/k$. The proof for $\min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\} = 1/2 - |3k-2|/(2k)$ proceeds in the same way. Consider now the difference $\alpha_1(k) - \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))\}$ which has the same sign as the expression $3\sqrt{k(3k-1)}/(3k-1) - 1$. The latter is positive if $(3k-1)(6k+1) > 0$ which is true for any $k > 1/3$. Hence, $\alpha_1(k) > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$. Consider now the difference $\alpha_2(k) - \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ which has the sign opposite to the sign of the expression $3\sqrt{k(3k-1)}/(3k-1) - 1$. As we have shown that $3\sqrt{k(3k-1)}/(3k-1) - 1$ is positive for any $k > 1/3$, we can then conclude that $\alpha_2(k) < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ must hold. This completes the proof of Claim 3.

As the roots of the function $\xi_1(\cdot)$ are such that $\alpha_1(k) > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ and $\alpha_2(k) < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ and $4k(1-3k) < 0$ holds for $k > 1/3$, it follows that for any α_i^0 (for which the market sharing equilibrium under incompatibility emerges) $\xi_1(\cdot)$ takes only positive values. Hence, for any $k > 1/3$ consumers are better off under compatibility than in the market sharing equilibrium under incompatibility.

Case ii) Assume now that under incompatibility the monopoly equilibrium emerges with firm i gaining the monopoly position. Using (36) and the formula for consumer surplus under the monopoly equilibrium (which is given by $CS_i^M(\alpha_i^0, k) = v + (t/2)\left[1 - (\alpha_i^0)^2\right]$) we express the difference between the consumer surpluses as

$$\frac{CS^c(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)}{b} = \frac{k\left(\alpha_i^0\right)^2 + 8k\alpha_i^0 - 20k + 18}{18}. \tag{38}$$

The sign of the difference $CS^c(\alpha_i^0, k) - CS_i^M(\alpha_i^0, k)$ is given by the sign of the nominator which we define as $\xi_3(\alpha_i^0, k)$. The discriminant of the function $\xi_3(\cdot)$ is given by $D = 72k\,(2k-1)$, which is negative for $k < 1/2$, zero if $k = 1/2$ and positive otherwise. Hence, for $k < 1/2$ the function $\xi_3(\cdot)$ takes only positive values and consumers are better off under compatibility than in the monopoly equilibrium with firm i being the monopolist. Consider $k = 1/2$ for which $\xi_3(\alpha_i^0, 1/2) = \left(\alpha_i^0 + 4\right)^2/2$, which is positive for any α_i^0. Consider finally $k > 1/2$. The roots of the function $\xi_3(\cdot)$ are given by $\beta_1(k) := -4 + 3\sqrt{2k(2k-1)}/k$ and $\beta_2(k) := -4 - 3\sqrt{2k(2k-1)}/k$. It is straightforward that $\beta_2(k) < \beta_1(k)$ for any $k > 1/2$. We show that $\beta_1(k)$ is such that $\beta_1(k) < \overline{\alpha}^0(k)$. Solving $\beta_1(k) < \overline{\alpha}^0(k)$, we get $3\sqrt{2k(2k-1)} < 6k - 1$, which can be simplified to $-6k < 1$. For any k the inequality $-6k < 1$ is true, hence, $\beta_1(k) < \overline{\alpha}^0(k)$ follows. As the roots of the function $\xi_3(\alpha_i^0, k)$ are such that $\beta_2(k) < \beta_1(k) < \overline{\alpha}^0(k)$, then for any α_i^0 for which the monopoly equilibrium with firm i gaining the market emerges ($\alpha_i^0 \geq \overline{\alpha}^0(k)$) the function $\xi_3(\cdot)$ takes only positive values and consumers are better off under compatibility than in the monopoly equilibrium with firm i being the monopolist.

Assume now that under incompatibility the monopoly equilibrium emerges with firm j gaining the monopoly position in which case consumer surplus is given by $CS_j^M(\alpha_i^0, k) = v + (t/2)\left[1 - (1 - \alpha_i^0)^2\right]$. Note now that $CS^c(\alpha_i^0, k) = CS^c(1 - \alpha_i^0, k)$ and $CS_j^M(\alpha_i^0, k) = CS_i^M(1 - \alpha_i^0, k)$. As $CS^c(\alpha_i^0, k) > CS_i^M(\alpha_i^0, k)$ holds for any α_i^0, then because of symmetry consumers must also be better off for any α_i^0 if firm j ($j \neq i$) becomes the monopolist under incompatibility. *Q.E.D.*

Proof of Proposition 8. *Case i).* We compare social welfare under compatibility and incompatibility. Apart from the stand-alone value, v, under compatibility social welfare is given by the value of the network effects, b, and incurred switching costs, $(t/2)\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)$. So that social welfare under compatibility can be (implicitly) expressed as

$$\frac{SW^c(\alpha_i^c, \alpha_i^0, k) - v}{b} = 1 - \frac{k\left(\alpha_i^c - \alpha_i^0\right)(1 - 2\alpha_i^c)}{2}. \tag{39}$$

Substituting $\alpha_i^c(\alpha_i^0) = (1+\alpha_i^0)/3$ into (39) yields

$$\frac{SW^c(\alpha_i^0,k)-v}{b} = \frac{4k\alpha_i^0(1-\alpha_i^0)-k+18}{18}. \tag{40}$$

Using (40) and (31) we can write the difference between social welfare under compatibility and social welfare in the market sharing equilibrium under incompatibility as

$$\frac{SW^c(\alpha_i^0,k)-SW^I(\alpha_i^0,k)}{b} = \frac{20k\alpha_i^0(\alpha_i^0-1)(1-3k)+66k^2-103k+36}{18\,(3k-2)^2}. \tag{41}$$

Define the numerator of (41) as $\varsigma_1(\alpha_i^0,k)$, which determines the sign of the right hand-side of Equation (41). The discriminant of $\varsigma_1(\cdot)$ is given by $720k\,(3k-1)\,(3k-2)^2$, which is negative for $k<1/3$, zero if $k=1/3$ and positive otherwise. Note that $20k(1-3k)$ is positive if $k<1/3$, zero if $k=1/3$ and negative otherwise. Hence, for $k<1/3$ the function $\varsigma_1(\cdot)$ takes positive values for any α_i^0 and social welfare is higher under compatibility. If $k=1/3$, then $\varsigma_1(\alpha_i^0,1/3)=9$ and social welfare is again higher under compatibility. Consider next $k>1/3$. The roots of the function $\varsigma_1(\cdot)$ are given by $\mu_1(k) := 1/2+3\,|3k-2|\,\sqrt{5k(3k-1)}/\,[10k(3k-1)]$ and $\mu_2(k) := 1/2-3\,|3k-2|\,\sqrt{5k(3k-1)}/\,[10k(3k-1)]$ with $\mu_2(k)=1-\mu_1(k)$. In the following claim we describe the properties of those roots.

Claim 4. *The roots of the function $\varsigma_1(\alpha_i^0,k)$ have the following properties. If $1/3<k<5/6$, then $\mu_1(k) > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ and $\mu_2(k) < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$. If $k=5/6$, then $\mu_1(k)=\overline{\alpha}^0(k)$ and $\mu_2(k)=1-\overline{\alpha}^0(k)$. If $5/6<k\le 1$, then $\mu_1(k)<\overline{\alpha}^0(k)$ and $\mu_2(k)>1-\overline{\alpha}^0(k)$. If $1<k<(103+\sqrt{1105})/132$, then $\mu_1(k)<1$ and $\mu_2(k)>0$. If $k=(103+\sqrt{1105})/132$, then $\mu_1(k)=1$ and $\mu_2(k)=0$. If $k>(103+\sqrt{1105})/132$, then $\mu_1(k)>1$ and $\mu_2(k)<0$.*

Proof. Recall that $\max\{\overline{\alpha}^0(k),1-\overline{\alpha}^0(k)\} = 1/2+|3k-2|\,/\,(2k)$ and $\min\{\overline{\alpha}^0(k),1-\overline{\alpha}^0(k)\} = 1/2-|3k-2|\,/\,(2k)$. Solving $\mu_1(k) > \max\{\overline{\alpha}^0(k),1-\overline{\alpha}^0(k)\}$ for k we get $3\sqrt{5k(3k-1)} > 5(3k-1)$. The latter inequality can be simplified to $k<5/6$, while for $k>5/6$ the opposite holds. For $k=5/6$ we get $\mu_1(k)=\max\{\overline{\alpha}^0(k),1-\overline{\alpha}^0(k)\}$. Solving $\mu_2(k)<\min\{\overline{\alpha}^0(k),1-\overline{\alpha}^0(k)\}$ for k we get $3\sqrt{5k(3k-1)} > 5(3k-1)$, what we showed to be true if $k<5/6$, while for $k>5/6$ the opposite holds. This proves the first part of the claim. Consider now $k>1$, for which we have to know how $\mu_1(k)$ and $\mu_2(k)$ are related to 1 and 0, respectively. Solving $\mu_1(k)>1$ we get $9(3k-2)^2 > 5k(3k-1)$, which holds for $k>(103+\sqrt{1105})/132>1$, while for $k<(103+\sqrt{1105})/132$ the opposite is true and if $k=(103+\sqrt{1105})/132$, then $\mu_1(k)=1$. Solving $\mu_2(k)<0$ is equivalent to solving $\mu_1(k)>1$. This completes the proof of Claim 4.

We can now determine the sign of $\varsigma_1(\cdot)$. Consider first $1/3 < k < 5/6$. By Claim 4 we know that for $1/3 < k < 5/6$ $\mu_1(k)$ and $\mu_2(k)$ are such that $\mu_1(k) > \max\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$ and $\mu_2(k) < \min\{\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k)\}$. Hence, for any α_i^0 for which the market sharing equilibrium under incompatibility emerges $\varsigma_1(\cdot)$ takes only positive values as $20k(1-3k) < 0$ and social welfare is higher under compatibility. If $k = 5/6$, then $\varsigma_1(\alpha_i^0, k) = 0$ if $\alpha_i^0 = \overline{\alpha}^0(k)$ or if $\alpha_i^0 = 1-\overline{\alpha}^0(k)$ and $\varsigma_1(\alpha_i^0, k)$ is positive for all α_i^0 for which the market sharing equilibrium under incompatibility emerges. Consider now $5/6 < k \leq 1$ for which $\mu_1(k) < \overline{\alpha}^0(k)$ and $\mu_2(k) > 1-\overline{\alpha}^0(k)$ hold. Then $\varsigma_1(\cdot)$ is positive if $\alpha_i^0 \in (\mu_2(k), \mu_1(k))$, while $\varsigma_1(\cdot) = 0$ if $\alpha_i^0 = \mu_2(k)$ or if $\alpha_i^0 = \mu_1(k)$, and $\varsigma_1(\cdot)$ is negative if $\alpha_i^0 \in (1-\overline{\alpha}^0(k), \mu_2(k))$ or if $\alpha_i^0 \in (\mu_1(k), \overline{\alpha}^0(k))$. Consider $k > (103+\sqrt{1105})/132$ for which $\mu_1(k) > 1$ and $\mu_2(k) < 0$. Hence, for any α_i^0 it follows that $\varsigma_1(\cdot) > 0$. Consider now $k = (103+\sqrt{1105})/132$ for which $\mu_1(k) = 1$ and $\mu_2(k) = 0$. Hence, $\varsigma_1(\cdot) > 0$ for any $\alpha_i^0 \notin \{0, 1\}$, and $\varsigma_1(\cdot) = 0$ for $\alpha_i^0 \in \{0, 1\}$. Consider finally $1 < k < (103+\sqrt{1105})/(132)$ for which $\mu_1(k) < 1$ and $\mu_2(k) > 0$. Then $\varsigma_1(\cdot)$ is positive if $\alpha_i^0 \in (\mu_2(k), \mu_1(k))$, and $\varsigma_1(\cdot) = 0$ if $\alpha_i^0 = \mu_2(k)$ or if $\alpha_i^0 = \mu_1(k)$, while $\varsigma_1(\cdot)$ is negative if $\alpha_i^0 \in [0, \mu_2(k))$ or if $\alpha_i^0 \in (\mu_1(k), 1]$.

Case ii). Consider now the case that under incompatibility the monopoly equilibrium emerges. Using (32) and (40) we get the difference between social welfare under compatibility and under the monopoly equilibrium with firm i being the monopolist under incompatibility

$$\frac{SW^c(\alpha_i^0, k) - SW_i^M(\alpha_i^0, k)}{b} = \frac{k(\alpha_i^0 - 4/5)(\alpha_i^0 - 2)}{18}, \tag{42}$$

from which the result stated in the proposition follows immediately. *Q.E.D.*

Proof of Proposition 9. From $\alpha_i^I(\alpha_i^0, k) = \left[k(1+\alpha_i^0) - 1\right] / (3k-2)$ and the fact that in the market sharing equilibrium firms' prices are given by $p_i(\alpha_i^0, k) = kb\alpha_i^I(\alpha_i^0, k)$ we get firm i's profit in the market sharing equilibrium as

$$\pi_i^I(\alpha_i^0, k) = kb\left(\frac{k-1+k\alpha_i^0}{3k-2}\right)^2. \tag{43}$$

Taking derivative of (43) with respect to t we obtain

$$\frac{\partial \pi_i^I(\alpha_i^0, k)}{\partial t} = \frac{\partial \pi_i^I(\alpha_i^0, k)}{\partial k}\frac{\partial k}{\partial t} = \frac{(k-1+k\alpha_i^0)\left[3k\alpha_i^0(k-2) + 3k(k-1) + 2\right]}{(3k-2)^3}. \tag{44}$$

Consider first all $k \neq 2$. Defining $\widetilde{\alpha}^0(k) := [3k(1-k) - 2]/[3k(k-2)]$ and substituting $\widetilde{\alpha}^0(k)$ and $1 - \overline{\alpha}^0(k) = (1-k)k$ into the right-hand side of Equation (44) yields

$$\frac{\partial \pi_i(\alpha_i^0, k)}{\partial t} = \frac{3k^2(k-2)}{(3k-2)^3}\left[\alpha_i^0 - (1-\overline{\alpha}^0(k))\right]\left[\alpha_i^0 - \widetilde{\alpha}^0(k)\right]. \tag{45}$$

From Equation (45) we observe that the sign of $\partial \pi_i(\alpha_i^0, k)/\partial t$ is given by the sign of $[(k-2)/(3k-2)^3][\alpha_i^0 - (1-\overline{\alpha}^0(k))][\alpha_i^0 - \widetilde{\alpha}^0(k)]$. Let us now examine the properties of $\widetilde{\alpha}^0(k)$. Successive differentiation of $\widetilde{\alpha}^0(k)$ yields $\partial\widetilde{\alpha}^0(k)/\partial k = 3(k-2/3)(k+2)/\left[3k^2(k-2)^2\right]$ and $\partial^2\widetilde{\alpha}^0(k)/\partial k^2 = -2\left(3k^3+6k^2-12k+8\right)/[3k^3\left(k-2\right)^3]$. Note that $\partial\widetilde{\alpha}^0(k)/\partial k < 0$ if $k < 2/3$ and $\partial\widetilde{\alpha}^0(k)/\partial k > 0$ if $2/3 < k < 2$ and $k > 2$. We get $\lim_{k\to 2/3}\widetilde{\alpha}^0(k) = 1/2$ and for any $k \neq 2$ it holds $\widetilde{\alpha}^0(k) > 1/2$. Solving $\widetilde{\alpha}^0(k) = 1$, we obtain $k_1 = (1/12)(9-\sqrt{33})$ and $k_2 = (1/12)(9+\sqrt{33})$ with $k_1 < 1/2$ and $k_2 < 4/3$. Taking the limit we obtain $\lim_{k\to\infty}\widetilde{\alpha}^0(k) = -1$. Hence, $\widetilde{\alpha}^0(k) \in (1/2, 1]$ if $k \in \{(1/12)(9-\sqrt{33}), 2/3) \cup (2/3, (1/12)(9+\sqrt{33})\}$ and for any other k it holds that either $\widetilde{\alpha}^0(k) > 1$ or $\widetilde{\alpha}^0(k) < 0$. In the intervals $k \in [1/2, 2/3)$ and $k \in (2/3, 1]$ the market sharing equilibrium only exist if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$ or $\alpha_i^0 \in (1-\overline{\alpha}^0(k), \overline{\alpha}^0(k))$ holds, respectively. We, therefore, have to analyze how $\widetilde{\alpha}^0(k)$ is related to $\overline{\alpha}^0(k)$ and $1-\overline{\alpha}^0(k)$ in those intervals. The following claim shows that for $k \in [1/2, 2/3)$ it is true that $\widetilde{\alpha}^0(k) \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$, while for $k \in (2/3, 1]$ it holds that $\widetilde{\alpha}^0(k) \in (1-\overline{\alpha}^0(k), \overline{\alpha}^0(k))$.

Claim 5. *It holds that* $\widetilde{\alpha}^0(k) - \overline{\alpha}^0(k) > 0$ *and* $1-\overline{\alpha}^0(k) - \widetilde{\alpha}^0(k) > 0$ *for all* $k \in [1/2, 2/3)$, *while for all* $k \in (2/3, 1]$ *it holds that* $\overline{\alpha}^0(k) - \widetilde{\alpha}^0(k) > 0$ *and* $\widetilde{\alpha}^0(k) - (1-\overline{\alpha}^0(k)) > 0$.

Proof. Simple calculations give $1-\overline{\alpha}^0(k)-\widetilde{\alpha}^0(k) = 2(3k-2)/\left[3k(k-2)\right]$ which is strictly positive over the interval $k \in [1/2, 2/3)$ and negative over the interval $k \in (2/3, 1]$. Similarly, we obtain $\widetilde{\alpha}^0(k) - \overline{\alpha}^0(k) = -3(k-2/3)(k-4/3)/\left[k(k-2)\right]$, which is strictly positive over the interval $k \in [1/2, 2/3)$ and negative over the interval $k \in (2/3, 1]$. This completes the proof of Claim 5.

For $k \in [1/2, 2/3)$ the market sharing equilibrium exists if $\alpha_i^0 \in (\overline{\alpha}^0(k), 1-\overline{\alpha}^0(k))$. From Claim 5 we know that α_i^0 lies either in the interval $(\overline{\alpha}^0(k), \widetilde{\alpha}^0(k))$ or in the interval $(\widetilde{\alpha}^0(k), 1-\overline{\alpha}^0(k))$ for $k \in [1/2, 2/3)$. In the former case $\alpha_i^0 - (1-\overline{\alpha}^0(k)) < 0$ and $\alpha_i^0 - \widetilde{\alpha}^0(k) < 0$, so that the right-hand side of Equation (45) is strictly positive as both $k-2 < 0$ and $3k-2 < 0$ hold. Hence, a firm's profit increases as switching costs increase if $\alpha_i^0 \in (\overline{\alpha}^0(k), \widetilde{\alpha}^0(k))$ for $k \in (0, 2/3)$. Consider now the other case with $\alpha_i^0 \in (\widetilde{\alpha}^0(k), 1-\overline{\alpha}^0(k))$, where $\alpha_i^0 - (1-\overline{\alpha}^0(k)) < 0$ and $\alpha_i^0 - \widetilde{\alpha}^0(k) > 0$, so that the the right-hand side of Equation (45) is strictly negative. Note now that for $k \in (2/3, 1]$ the market sharing equilibrium emerges if $\alpha_i^0 \in (1-\overline{\alpha}^0(k), \overline{\alpha}^0(k))$. From Claim 5 we know that α_i^0 lies either in the interval $(1-\overline{\alpha}^0(k), \widetilde{\alpha}^0(k))$ or in the interval $(\widetilde{\alpha}^0(k), \overline{\alpha}^0(k))$ for $k \in (2/3, 1)$. Proceeding as before we get again that firm i's profit increases as switching costs increase if $\alpha_i^0 < \widetilde{\alpha}^0(k)$, whereas its profit decreases if $\alpha_i^0 > \widetilde{\alpha}^0(k)$ holds.

If $k = 2$, then the right-hand side of Equation (44) is given by $(1 + 2\alpha_i^0)/8 > 0$ for any α_i^0. *Q.E.D.*

Proof of Proposition 13. Given that in every period consumers form expectations after observing firms' prices, firm i's demand in period 2, $\alpha_i^2(p_i^2, p_j^2; \alpha_i^1)$, takes the form:

$$\alpha_i^2(p_i^2, p_j^2; \alpha_i^1) = \begin{cases} 1 & \text{if} \quad p_j^2 - p_i^2 \geq t(1-\alpha_i^1) - b \\ \frac{p_j^2 - p_i^2 + t\alpha_i^1 - b}{(k-2)b} & \text{if} \quad -t\alpha_i^1 + b < p_j^2 - p_i^2 < t(1-\alpha_i^1) - b \\ 0 & \text{if} \quad p_j^2 - p_i^2 \leq -t\alpha_i^1 + b. \end{cases} \tag{46}$$

Note that as $k > 3$, the demand function (46) is downward-sloping. We start with the market sharing equilibrium. Maximizing firm i's profit, $\pi_i^2(p_i^2, p_j^2; \alpha_i^1) = p_i^2 \alpha_i^2(p_i^2, p_j^2; \alpha_i^1)$, with respect to p_i^2 we obtain the best response function $p_i^2(p_j^2; \alpha_i^1) = \left(p_j^2 + t\alpha_i^1 - b\right)/2$. Solving firms' best response functions yields profit maximizing prices $p_i^I(\alpha_i^1, t, b) = t(1 + \alpha_i^1)/3 - b$. Plugging these prices into (46) we obtain the equilibrium market share of firm i: $\alpha_i^I(\alpha_i^1, k) = \left[k(1 + \alpha_i^1) - 3\right] / [3(k-2)]$. Existence of the market sharing equilibrium is guaranteed if and only if $0 < \alpha_i^I(\alpha_i^1, k) < 1$ yielding $3/k - 1 < \alpha_i^1 < 2 - 3/k$, which is fulfilled for any $\alpha_i^1 \in [0, 1]$ if $k > 3$. Moreover, firms' equilibrium prices are positive: re-writing firm i's price as $p_i^I(\alpha_i^I) = b(k-2)\alpha_i^I$ shows that $p_i^I(\alpha_i^I) > 0$ holds for any $k > 3$. We next show that for $k > 3$ monotone market sharing pattern prevails everywhere. The comparison of α_i^I and α_i^1 yields $\alpha_i^I - \alpha_i^1 = (k-3)(1 - 2\alpha_i^1)/ [3(k-2)]$. It follows that $\alpha_i^I > \alpha_i^1$ ($\alpha_i^I < \alpha_i^1$) provided $\alpha_i^1 < 1/2$ ($\alpha_i^1 > 1/2$). Hence, a firm with a larger installed base loses market shares. The comparison of α_i^I and $1/2$ yields $\alpha_i^I - 1/2 = k(2\alpha_i^1 - 1)/ [3(k-2)]$, such that $\alpha_i^I > 1/2$ ($\alpha_i^I < 1/2$) if $\alpha_i^1 > 1/2$ ($\alpha_i^1 < 1/2$), which implies that dominance is never alternated.

We finally rule out the existence of a monopoly equilibrium. The highest price which allows firm i to monopolize the market is $p_i^2 = p_i^M = p_j^2 - t(1 - \alpha_i^1) + b$. It must hold that $p_j^2 = 0$, which gives $p_i^M = b - t(1 - \alpha_i^1)$. Firm i does not have an incentive to increase its price above p_i^M if

$$\left.\frac{\partial \pi_i^2(p_i^2, p_j^2; \alpha_i^1)}{\partial p_i}\right|_{p_i^2 = p_i^M, p_j^2 = 0} = \frac{-3 + k(2 - \alpha_i^1)}{k-2} \leq 0,$$

which yields $\alpha_i^1 \geq 2 - 3/k$. Moreover, requiring $p_i^M \geq 0$ implies $\alpha_i^1 \geq 1 - 1/k$. Given $k > 3$ the former condition is stronger than the latter. Note, finally, that $2 - 3/k > 1$ holds for any $k > 3$, which implies that $\alpha_i^1 \geq 2 - 3/k$ is never fulfilled and there exists no monopoly equilibrium. *Q.E.D.*

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